Questions about vagueness

Cian Dorr

August 31, 2005

1 The original sorites paradox

The philosophy of vagueness begins with the sorites ('heaper') paradox, attributed to Eubulides of Miletus, an adversary of Aristotle active in the 4th century B.C. The ancients tended to conceive of the paradox in "dialectical" terms, as a strategy for winning debates conducted according to certain rules (see Williamson, chapter 1). The strategy was simple: start by asking 'Could one grain of wheat make a heap?' Then ask 'Could two grains of wheat make a heap?' And so on.

- Williamson reports the form of the questions as '*Does n* grains of wheat make a heap?' I've substituted 'could' to head off the answer, 'That depends on how they are arranged, what size and shape they are, etc.'

How is this supposed to help one win a debate? One way it could do so is by leading one's interlocutor to say something manifestly false, e.g. by answering 'No' to 'Could 10,000 grains make a heap?' This could end up happening for the same kind of reason that frogs can be boiled by putting them into a pot of cold water and gradually heating it up.

But that can't be the whole story. It also seems to be part of the idea that if your interlocutor says 'yes' in answer to one question, and 'no' in answer to the next question, you will have trapped him. But how, exactly? It does seem like a pretty bad idea to, say, answer 'no' to the question 'Could 86 grains of wheat make a heap?', and then answer 'yes' to the question 'Could 87 grains of wheat make a heap?'. But it's controversial exactly what sort of bad position one puts oneself in by answering in this way.

Anyhow, if you want to avoid being put this absurd position, your counterstrategy for dealing with sorites questioners is going to have to involve sometimes doing something other than saying 'yes' or 'no' in answer to a sorites question.

2 What does it mean for an object to be a borderline case of a predicate?

In fact, there is a certain kind of answer that we find very natural to give for many of the sorites questions. We want to say that so many grains of wheat would (at best) be a *borderline case* of the predicate "heap". This is a kind of thing we very often want to say in ordinary life, not just when we are involved in debates with sorites questioners.

- We seem to find 'it's a borderline case' satisfying as an answer to a yes-no question. Think of how strange it would be if, after having answered 'borderline' to the question 'Could *n* grains make a heap?', the questioner were to make the following rejoinder: 'You haven't answered my question: I didn't ask you whether *n* grains could make a *borderline case* of the predicate 'heap', I asked you whether they could make a heap.' It's an interesting and difficult question why this should be the case.

A central question in the philosophy of vagueness: what is it for an object to be a borderline case of a predicate [in a given language]?

When *x* is a borderline case of a predicate '*F*', and '*a*' is a name for *x*, the sentence '*a* is [an] *F*' is a *borderline*, or *indefinite* sentence. Another central question: what is it for a sentence to be borderline [in a given language]?

- Not all indefinite sentences are of the form '*a* is *F*', where *a* is a name for an object that is a borderline case of '*F*'. Examples: 'John is thin and Frank is bald'; 'Some members of the Society for the Borderline Bald are bald'; ? 'Frank is bald or not bald'; ? 'Jupiter weighs more than *n* grams'; ? 'Princeton is Princeton Borough'....

3 A canonical answer

For a sentence to be indefinite is for it to be neither true nor false. For an object to be a borderline case of a predicate is for the predicate to be neither true of the object nor false of the object.

- This might need to be modified if we wanted to count the kind of "borderline" status associated with vagueness as only one way for a sentence to lack a truth value, where others might include phenomena like presupposition failure and reference failure.
- N.B.: those who endorse this answer must give up the universal intersubstitutability of ' "*P*" is true' and '*P*'. For universal intersubstitutaibility lets us infer from ' "*P*" is not true and "Not *P*" is not true' to 'Not *P* and not not *P*': a contradiction.

Suppose this is correct. Then we still face the questions: 'What is it for a sentence to be true?' 'What is it for a sentence to be false?' 'What is it for a predicate to be true of an object?' 'What is it for a predicate to be false of an object?' It will turn out that in answering these questions we will face many of the same choices we would have faced had we not adopted the canonical answer.

4 Vagueness and precision

In ordinary language, 'vague' and its antonym 'precise' have several uses. Sometimes 'vague' seems to mean something like 'unspecific': one can be accused of being unduly vague if one answers the question 'How tall are you' with 'between four and eight feet'.

But in philosophy we are concerned with a sense of 'vague' on which it is intimately connected with the notion of a borderline case—in this sense, 'between four and eight feet' is just as precise as 'between six feet and six foot one'. As Williamson shows, distinguishing this sense from the one connected with unspecificity was an important achievement that took quite a while.

What exactly is the intimate connection between 'vague' and borderline cases? The following definitions are often suggested: A predicate is vague iff it is possible for it to have borderline cases; a sentence is vague iff it is possible for it to be borderline.

- One problem with this: obviously it's not sufficient for 'electron' to be vague that it could have been used in the same way that 'heap' is actually used.... How to word the definition in such a way as to get the effect of "holding meaning fixed"?
- Is it really sufficient for a predicate to be vague for it to have possible or actual borderline cases? One class of putative counterexamples comprises predicates introduced by partial

definitions, such as the following: 'Let "swinky" apply to any number that is less than 96; let it fail to apply to any number that is greater than 103.' Arguably, numbers between 95 and 104 are borderline cases of 'swinky'. But do we want to count 'swinky' as vague?

- Note that the definition allows a sentence to count as precise even if it has constituents which are vague: e.g. 'Some round squares are bald'; '1+1 = 3 and Frank is bald'.

- We should (arguably) want a definition of 'vague' that allows expressions that are neither predicates nor sentences—such as 'Princeton' (?)—to count as "vague". But it is controversial how such a definition should go, if at all.

- Note that 'vague' so understood applies only to bits of language. There are further debates about whether there is any reasonable way to classify things of other sorts as "vague" or "precise". For example, there is a debate about whether there is any interesting distinction to be made between two different kinds of *objects*, the "vague" and the "precise", and if so how the distinction should be drawn. (These debates are sometimes conducted in the material mode: 'So-and-so holds that some objects are vague, but I hold that all objects are precise.' But this is generally misleading, suggesting as it does that there agreement on the question what "vague object" and "precise object".)

5 Non-metalinguistic ways of talking about borderline cases

I've been focusing so far on questions about bits of language: predicates and sentences. But when we're asked 'Is this thing an *F*?' and the object in question is a borderline case, it's very natural to excuse one's failure to answer 'Yes' or 'No' in terms that don't mention language at all. One can say: 'It is indefinite/unclear/indeterminate whether it is an *F*'—here one is using, rather than mentioning, the predicate '*F*'.

- It is often convenient to work with the operator 'definitely/clearly/determinately' rather than 'it is indefinite/unclear/indeterminate whether'. These are interdefinable: 'It is indefinite whether *P*' is equivalent to 'Not definitely *P* and not definitely not *P*'; 'Definitely *P*' is equivalent to '*P* and it is not indefinite whether *P*'.

We also want to get a philosophical grip on this. What is it, in general, for it to be indefinite/unclear/indeterminate whether such-and-such?

There is an important and difficult question about the relative "priority" of the metalinguistic and operator forms of reports about borderline cases. Those who give priority to the operators can use them to give a very straightforward account of the metalinguistic notions: an object is a borderline case of a predicate iff it is indefinite whether the predicate applies to an object; a sentence is borderline iff it is indefinite whether it is true.

- Note that these answers seem to be inconsistent with the "canonical" answers discussed above. At least, there's something odd going on if one says that a sentence is neither true nor false and also says that it's indefinite whether it's true—just as it would be odd to utter 'He's bald, although he's not definitely bald'.

No comparably simple analysis is available to those who regard the metalinguistic forms as primary.

6 Vague properties, concepts, propositions, etc.

Another non-metalinguistic way of talking about borderline cases is to talk about abstract entities like *properties*, *concepts* and *propositions* rather than to predicates and sentences. One says something like 'Frank is a borderline case of *baldness*' or 'The proposition that Frank is bald is indefinite'.

- It is controversial whether there is any such phenomenon as this. Some maintain that every proposition there is is either determinately true or determinately false, and that for each object and property, the object either determinately has or determinately does not have the property. (Similarly for concepts, contents, sets, etc.)
  - — You might put this by saying 'Propositions and properties are precise'. But anyone who held this view would find this misleading: on this view, propositions and properties are not the sort of things that can sensibly be classified as precise or vague, any more than rocks are.
- Does the view seem obviously false? Compare: 'Although it is indeterminate whether Jupiter has a mass of more than *n* grams, every object there is either determinately does or determinately does not have a mass of more than *n* grams.

Some 'object *x* is a borderline case of property *y*' as 'it is indeterminate whether *x* has *y*', and '*x* is an indeterminate proposition' as 'it is indeterminate whether *x* is true'. Others go in the other direction, analysing 'it is indeterminate whether *P*' as 'the proposition that *P* is indeterminate'.

7 Kinds of answers to these questions

1. Linguistic theories. What it is for an object to be a borderline case of a predicate, and for a sentence to be indefinite, is ultimately to be explained in terms of the way the linguistic community uses the predicate or sentence and its constitutent words. (Understand the notion of 'use' very broadly so as to include behavioural dispositions, psychological attitudes, causal relations, conventions and practices, etc.) Linguistic theorists will probably want to take the metalinguistic forms ways of talking about vagueness as primary.
2. "Psychological" theories. For them, the central level of explanation lies not at the level of public language but at the level of the mental states of individuals. Often these are talked about in the language of "concepts". What it is for an object to be a borderline case of a concept is to be explained in terms of what it takes to believe contents involving that concept.
3. Epistemicism. To say that it is indeterminate whether *P* is to say that it is impossible [for beings of such-and-such sort] to know whether *P* [for such-and-such reasons]. This is Williamson's view.
   - Terminological note: "Epistemicism" is generally used in the literature for the conjunction of the view just described and some claim to the effect that "classical logic and semantics apply to vague language".

- Note: as you're reading through the Williamson, you'll see that he takes it as obvious that when it's indeterminate whether *P*, no human being (at least) knows that *P*. But this isn't obvious to me at least—I argue against it in a paper we'll be reading later on in the semester.

4. Operator primitivism. There is nothing non-trivial to be said about what it is for it to be indefinite whether *P* / definite that *P*. Analogous to a popular view according to which operators like 'necessarily' and 'possibly' are primitive.

5. Metaphysical theories. Explain what it is for it to be indeterminate whether *P* by appeal to some kind of claim about propositions/properties/facts/states of affairs/…; take these claims to be explanatorily basic.

6. Expressivism. There is nothing non-trivial to be said about what it is for it to be indefinite whether *P*, but for the same reason that if emotivism about 'good' ('*x* is good' means 'hurray for *x*!') is correct, there is nothing non-trivial to be said about what it is for something to be good. Claims about indefiniteness aren't strictly speaking in the business of "stating facts"; the right way to understand them is to understand the rules of the linguistic practice in which they are involved.

8 Vagueness and logic

Even if we haven't settled on an answer to the question what it means to call a sentence indefinite, or to say that an object is a borderline case of a predicate, we can still ask questions about how the "definiteness status" of complex sentences and expressions depends on the "definiteness status" of their simpler constituents, such as the following:

(i) Supposing '*P*' and '*Q*' are two indefinite sentences: must the disjunction '*P* or *Q*' also be indefinite?

(ii) Supposing that for every object *x*, either *x* is a borderline case of '*F*' or '*F*' definitely fails to apply to *x*: could the sentence 'Something is *F*' be definitely true?

(iii) Could a sentence of the form '*P* or not *P*' be indefinite?

Questions like these loom very large in the literature on vagueness: since they structure Williamson's discussion of his opponents' views, they will also structure our discussion. But we will try as much as possible to focus on the way these questions bear on the (more fundamental) questions I have already introduced.

9 Validity

The question 'What is the correct logic for vagueness?' doesn't wear its meaning on its sleeve. In part, it amounts to a list of questions like (i)–(iii). But even if we had settled the answers to such questions, there would remain the question which arguments (conceived of as sequences of sentences possibly involving vague terms) are *valid*.

'Valid' is normally explained to beginning students in logic in terms of 'true', in one of the following two ways: (i) for an argument to be valid is for it to be the case that, if its premises are all

true, its conclusion must also be true. (ii) for an argument to be valid is for it to be the case that its premises couldn't all be true without its conclusion being true.

Some think that vagueness should lead us to give up some or all of these definitions, and put something more complicated in their place.

- It can be very hard to get a grip on what's at stake in these debates. 'Valid' in its philosophical use doesn't seem to be a term of ordinary language: some will protest that their only understanding of this term is by way of a definition along the lines of (i) or (ii).

The warning about the unclarity of the question 'What is the correct logic for vagueness?' also applies to the question 'What is the correct semantics for vagueness?' Let's make this a bit more personal: I really hate the word 'semantics' and advise you not to use it unless absolutely necessary.

10 The sorites paradox

A paradox, in the contemporary sense: a set of sentences each of which seems true, but which seem inconsistent.

A paradoxical argument: an argument each of which premises seems true, and whose conclusion seems not to be true, but which seems valid.

The sorites paradox can be formulated as a paradoxical argument:

(1)	1 grain cannot make a heap.
(2)	If 1 grain cannot make a heap, then 2 grains cannot make a heap.
(3)	If 2 grains cannot make a heap, then 3 grains cannot make a heap.
⋮	⋮
(i)	If $i-1$ grains cannot make a heap, then i grains cannot make a heap.
⋮	⋮
(10,000)	If 9,999 grains cannot make a heap, then 10,000 grains cannot make a heap.
(10,001)	10,000 grains cannot make a heap.

Seems valid: can be proved using 10,000 applications of modus ponens.

(1) seems obviously true. (10,001) seems obviously false. And (2)–(10,000) also seem true—at least, many philosophers report this intuition.

11 Philosophy and the sorites paradox

For many philosophers working on vagueness, the central task is to "resolve" the sorites paradox.

What exactly does it mean to resolve a paradox? In other areas of philosophy, resolving a paradox is generally taken to mean something like: either saying that the sentences that comprise the paradox are not in fact inconsistent, while explaining why they seem inconsistent, or saying of some

of the sentences that they are not true while explaining why they seem true. But this seems like a lot—maybe too much—to expect philosophers to do for the sorites paradox.

- I should mention that there is also the option of embracing all the appearances: the sentences are all true *and* they are inconsistent / the argument is valid *and* all its premises are true and its conclusion is not true. Dialetheists (according to whom some sentences are both true and not true) do this for many paradoxes. But it seems unpromising as an approach to the sorites.

12 Variants of the sorites argument

Some philosophers claim that the ordinary English conditional is the *material conditional*: 'If *P*, then *Q*' means 'Either not-*P* or *Q*', or 'Not both *P* and not *Q*'. (These proposals are not often distinguished since they are widely held to be equivalent.) Whether or not these philosophers are right, we can consider variants of the sorites argument that use these putative equivalents in place of the conditional.

(1a)	1 grain cannot make a heap.	(1b)	1 grain cannot make a heap.
(2a)	Either 1 grain can make a heap, or 2 grains cannot make a heap.	(2b)	It is not the case that 1 grain of wheat cannot make a heap and 2 grains can make a heap.
⋮	⋮	⋮	⋮
(10,000a)	Either 9,999 grains can make a heap, or 10,000 grains cannot make a heap.	(10,000b)	It is not the case that 9,999 grains cannot make a heap and 10,000 grains can make a heap.
(10,0001a)	10,000 grains can make a heap.	(10,0001b)	10,000 grains can make a heap.

Several philosophers have noticed an interesting asymmetry between these versions of the paradox: many of (2a)–(10,000a) "seem true" only to a slight extent, if at all, in the absence of some argument that they are true; whereas (2b)–(10,000b) strike us even more forcefully as true than the premises of the original sorites argument.

- What rules of inference are needed to establish the validity of these two variant arguments?

There are also versions of these paradoxes that condense all the "conditional" premises into one quantified premise. Here are two:

(1c)	1 grain cannot make a heap.
(2c)	For any n, if n grains cannot make a heap, then $n + 1$ grains cannot make a heap.
(3c)	10,000 grains cannot make a heap.

(1d)	1 grain cannot make a heap.
(2d)	There is no number n such that n grains cannot make a heap and $n + 1$ grains can make a heap.
(3d)	10,000 grains cannot make a heap.

- Both of these arguments are valid in classical logic. In natural deduction systems, one can establish the validity of (1c)–(3c) using 10,000 applications each of universal elimination and modus ponens. (1d)–(3d) requires something a bit more complicated—e.g. the rules of *reductio ad absurdum* (negation introduction), conjunction introduction, and existential introduction.