Reducing possibilist discourse
Modality seminar
February 14th, 2005

1   Different versions of "possibilist discourse"

—With or without non-world-relativised predications, as in 'some possible individual is a donkey'.

—With or without modal operators in addition to quantification over worlds.

—With or without transworld identity.

—With or without transworld sets.

2   Questions for reductionists

Suppose we want to analyse each sentence in some *target language* by associating it with a sentence in some *reducing language*, to which the sentence in the target language reduces. (We'll be focusing on the example of reducing possibilist language, but hoping to draw general morals.) What choices do we face?

(1)   *Can the analyses be specified recursively according to the logical structure of the target sentences?*
(I.e.: in such a way that the analysis of any sentence is built up from the analyses of its constituents.)

If 'yes', we have two further questions:

(2)   *Are the constituents in the reducing sentences that correspond to quantifiers in the target sentences themselves quantifiers (perhaps restricted ones)?*

(3)   *Does vocabulary common to the target language and the reducing language get altered in the reduction?*

- Lewis's 'ersatzers'—and those of Fine's 'proxy' reductionists according to whom "each entity is traded in for itself—answer 'yes' to all three of (1), (2) and (3).

  Fine objects: some possibilia are possibly people; but no abstract objects are possibly people.
  — But the version of possibilist discourse which Lewis's ersatzers are trying to reduce doesn't contain non-world-relativised predications. And it may or may not contain modal operators in addition to world quantifiers.
  What's true is that some possibilia are people *at* (or according to) some worlds. Remember this means for the ersatzers: the world *represents* that something *represented* by the possibilium is a person. If worlds are propositions and possibilia are properties: the world *entails* that something that *instantiates* the possibilium is a person.

'All possibilia are abstract' comes out *true* for the ersatzers. It's not a sentence of the target discourse. In principle we could resist counting this as true by stipulating that sentences that contain the word 'possibilia' but are not part of the target discourse are meaningless; but this would be silly and arbitrary. However, 'All possibilia are abstract at the actual world' is false.

- The other sort of 'proxy reduction' considered in Fine's section 1 answers 'yes' to (1) and (2), and 'no' to (3).

Some examples to give the idea:

'Some possibilium is a person' is reduced to 'Some property (of the right sort) is instantiated by a person'.

'Some possibilium possibly is a person' is reduced to 'Some property (of the right sort) is possibly instantiated by a person'.

'All possibilia are abstract objects' is reduced to 'All properties (of the right sort) are instantiated by abstract objects.' — which is false.

Should we think of this view as saying that 'person' shifts its meaning when we start engaging in possibilist discourse, coming to mean what 'property instantiated by a person' normally means? This might lead to problems with the messy ways in which we use fragments of possibilist talk in everyday life? (E.g.: 'I am necessarily human if I exist; and so are all my possible brothers and sisters.)

Fine's objection to these proxy reductions: they can't account for *actualistically indiscernable* worlds or possibilia.

EG: can't explain the truth of 'there are two worlds w_1 and w_2 and two non-actual possibilia a and b such that the only difference between w_1 and w_2 is that the qualitative role played by a in w_1 is played by b in w_2, and vice-versa.'

— Perhaps to make the case clearer, we need to add: '...and such that for any world where either a or b exist, there is another world just like it except that a and b swap roles.'

Possible responses we'll talk more about later: (i) give up on the kind of possibilist discourse in which you have to say the hard things about indiscernables, and offer some replacement such as talk of 'possibilities'; (ii) be an "anti-existentialist" like Plantinga.

3 Fine's non-proxy reductions

Examples of reductionist projects that answer 'yes' to (1) (and (3)) while answering 'no' to (2):

- the reduction of ordered-pairs mentioned by Fine at the beginning of section 3 (quantifiers are replaced by pairs of quantifiers).

- Fine's basic reduction of possibilist discourse (quantifiers are replaced by combinations of modal operators and quantifiers).

- Fine's reduction of set-theoretic possibilist discourse (quantifiers over transworld sets are replaced by giant infinitary blocks of modal operators and quantifiers).

First version of Fine's account of quantification over possibilia: 'there is a possibilium x such that...' becomes 'possibly there is an x such that...'.

> Problem with this (my version, less easily misunderstood than Fine's): 'There is a possibilium x such that x is square and there is a possibilium y such that y is not square and $x = y$' comes out true.

— We might think to fix this by using 'possibly there is an x such that actually...' instead of 'possibly there is an x such that...'. But then we won't get 'Possibly there is a possibilium x such that x is a talking donkey' coming out true, though we will get 'There is a possibilium x such that x is possibly a talking donkey'.

— What we need is something like 'actually' except that it only "undoes" the effect of its *local* modal operator, instead of undoing the effect of *all* modal operators in whose scope it occurs. Harold Hodes calls this the 'backspace' operator.

— Fine's strategy: achieve the same effect using a metaphysics of worlds.

Fine's metaphysics of worlds:

(i)   Necessarily, there is exactly one world.

(ii) Necessarily, if φ, then for all worlds w, necessarily, if w exists then φ.

Final version of Fine's account of quantification over possibilia: 'there is a possibilium x such that ...' becomes 'there is a world w such that possibly there is an x such that possibly, w exists and...'

Quantification over possible worlds is handled similarly: 'there is a possible world w such that...' becomes 'there is a world w' such that possibly there is a world w such that possibly, w' exists and...'.

What about quantification over *sets* of possibilia? Fine attempts to handle that using an infinitary language for the reduction—one that permits conjunctions and disjunctions of any number of formulae, and arbitrarily long blocks of quantifiers ($L_{\infty,infty}$).

> The simplest case: sets of non-sets. Proposal: replace 'there is a set s such that...' with '$\exists x_1 \exists x_2 \ldots$' (a list of α possibilist existential quantifiers). Replace '$x \in s$ with '$x = x_1 \lor x = x_2 \lor \ldots$'. This gives the effect of sets with cardinality up to α.

> This can be iterated to give sets of sets, etc.

> Can this be used as a general reduction for set theory? Fine says no: 'This reduction does not allow us to eliminate reference to sets altogether, since the definition of truth requires the full resources of set theory.' Why does he say that?

— Important distinction: reductions to an infinitary language vs. reductions to a finite set-theoretic language in which we can *describe* the sentences of the infinitary language.

> One thing you arguably can't have on this approach: a class of all non-set pos-

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sibilia. Suppose that there's some two-place predicate such that possibility for sentences involving *it* coincides with logical possibility. Then for any *logically consistent* claim about how many sets there are, there is a corresponding *true* claim about how many possible individuals there are in some world.

— Would you *want* a set of all non-set possibilia? Yes if you want Lewis-style set-theoretic posssible worlds semantics.

4   Reductions without structural isomorphism

Let's turn now to reductions that answer 'no' to question (1).

A simple example: specify the reduction of any sentence S as 'According to the possible worlds fiction, S'.

A more complicated sort of case: first transform S according to some recursively-specified rules, then add certain specified material at the beginning and at the end of the result.

Questions (2) and (3) will arise again concerning the recursively-specified component of this.

Sider's reduction (model-theoretic version).

First step: replace S with a set-theoretic description D_S of a corresponding sentence in a Lagadonian language. So 'Some possibilium x is red' becomes 'The sequence $\langle$'$\exists$', 'x', 'x', 'instantiates', redness$\rangle$.

The reduction of S is: 'D_S is true$_p$ in all realistic models'.

A realistic model is, loosely speaking, a Kripke model which reflects all the modal truths. More precisely: all and only the true sentences in an infinitary, modal, Lagadonian language are true$_m$ in the model.

The cardinality objection:

General idea: if a certain rather attractive thesis about possibility is true, there won't be any realistic models, since the sets that exist at the actual world will not be numerous enough.

What is the thesis about possibility that leads to the problem?

Fine states it in possibilist terms, using transworld sets: 'there is no one-one correspondence between the possible individuals/worlds and the items that exist in the actual world'. But it's not obvious what sentence in an actualist language would be equivalent to that.

Sider: 'there is no upper bound, not even an infinite upper bound, to how many (concrete) individuals may exist'. Sounds like a modal thesis, perhaps this: [necessarily,] for any pure set s, it is possible that there is a set s' of concreta such that s' is strictly bigger than s. (But if this is the thesis, it's not clear to me how Sider's attempted solution accommodates it.)

Fine's version of the modal fictionalist reduction: reduce S to 'According to possible worlds semantics, S'.

Advantage over Fine's view: no need for special treatment of quantification over sets.

Problem for views of this kind: will the negation of the analysis of S be equivalent to the analysis of the negation? Will the disjunction of the analyses of S and T be equivalent to the analysis of the disjunction?

If not, wouldn't that mean that 'not' and 'or' are not being used in the discourse to mean *negation* and *disjunction*?