Reasons to doubt the existence of numbers
November 20th, 2002

1. **Nominalism and Field's "fictionalism"**

   Nominalism: someone who believes that there are no numbers.
   "Fictionalist": someone who does not believe mathematical sentences (and
      hence, presumably, does not believe that there are numbers.)

   One can argue against the Platonist without arguing for nominalism by
   arguing that there is no good reason to believe in numbers. The arguments
   we'll be looking at today will mostly be of this sort.

2. **An argument by analogy**

   Imagine you grew up in a tribe where everyone believed, and was taught
   from earliest infancy, a very elaborate theory about angels (or if you prefer,
   invisible goblins). According to angel theory, there are infinitely many
   angels; angels are unchangeable; angels never interact causally with the
   material world; angels form a hierarchy with a certain complex pattern.

   Would it be reasonable of you to accept this theory, once you have a chance to
   reflect on it? Or would it be more reasonable to suspend judgment, or even to
   believe that there aren't any angels (goblins)? If the latter, why?

   Nominalists and fictionalists will think that the analogy with our actual
   situation is close; any argument you come up with in the angel case, they will
   want to adapt to the number case.

3. **An attractive but too-simple argument**

   Suppose there weren't any numbers—suppose, if you like, that there were
   only concrete entities. This wouldn't make any difference to the concrete
   world; in particular, things would *look* (and sound, taste…) just the same to
   us. Hence, we have no good reason to think that there are numbers.

   The problem with this argument is that it is too close to certain arguments for
   radical scepticism, e.g. :
   > If the universe had just popped into existence in its present form 5
   > minutes ago, everything would appear just as it actually does; hence, we
   > have no good reason to think that the universe did not just pop into
   > existence 5 minutes ago.
   > If I were being deceived by a malicious demon into thinking there were
   > material objects, everything would appear just as it actually does to me;
   > hence, I have no good reason to think that I am not being deceived by a
   > malicious demon into thinking there are material objects.

4. **The causal inertness of numbers**

   There's an important difference between things that existed more than 5
   minutes ago and material objects, on the one hand, and numbers, on the

other hand: the former do, whereas the latter do not, play a role in *causing* my experiences. Could we appeal to this fact in explaining why the argument of the previous section works for numbers and not for those other things?

Not precisely: consider another sceptical hypothesis, according to which the world is going to blink out of existence 5 minutes hence. Surely I have good reason to believe that this is not the case; but future objects do not (barring time travel) play any role in causing my present experiences.

Still, they do cause *something*—namely, events in the further future—whereas numbers cause nothing at all. Perhaps we could appeal to a general epistemological principle: in general, one should not believe in objects which are causally inefficacious unless one has special reason to do so.

But this doesn't seem very promising as an argument against the Platonist, since Platonists will presumably reject any such epistemological principle.

- Also, Platonists might deny that all mathematical entities are causally inert. It's hard to see how this would go with regard to numbers; but someone might maintain that when, e.g., four men lift a piano together, the movement of the piano is caused by the set that has the four men as members.

5. Field's [version of Benacerraf's] argument

According to Platonism, there is the following very striking fact: mathematicians are (by and large) *reliable* in what they say about mathematical entities; in other words, almost always, the following schema holds:

If mathematicians accept 'p', then p.

Field argues that if Platonism is to be credible, the Platonist must have some sort of *explanation* of this fact: it would be incredible if such a striking fact was just a *coincidence.* He suggests that the Platonist will at least find it very hard to come up with such an explanation.

What about the following style of explanation: the mathematical facts are what they are because [insert deduction of mathematical claims from axioms]. Mathematicians accept what they do because [insert sociological/historical claims about the processes which led mathematicians to accept the axioms]. Hence, the sentences mathematicians accept are true.

But "explanations" of *this* sort don't do anything to show that the facts "explained" are not coincidences. It is still a coincidence if I bump into you in Poland and we are both wearing the same clothes and reading the same book, even if there is a perfectly good explanation of my presence there and of your presence there, and of my wearing that outfit and of your wearing that outfit, etc.

What would a genuine, coincidence-avoiding explanation look like?

— one could claim that we have an extrasensory faculty of "mathematical intuition" that puts us somehow "in touch with" the mathematical realm (causal touch?)
— one could claim that the mathematical realm is the way it is because of our judgments about it (idealism). But there'll be a residual problem: explain the reliability of our judgments about the way the mathematical realm depends on us.
— one could defend the interpretative claim that *any* bunch of sentences we accepted about abstract entities [needs explanation!] would automatically be true: as well as the numbers and the sets, there are "schnumbers" and "schmets": enough abstract entities to make true any conceivable theory of them. But there'll be a residual problem: explain the reliability of our belief that abstract entities are abundant in this way.