Are there numbers?

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1. More about nominalism

Two questions:

(i) What is the property had by ‘2+2=4’ and not ‘2+2=5’ that makes it a good idea in most ordinary contexts to assert the former and not the latter, given that both are false?

- Field: *truth according to standard mathematics*
  (This needs to be understood in such a way that ‘The number of people in this room is greater than 20’ is true according to standard mathematics.)

(ii) What is it about the ‘story’ of standard mathematics that makes it a ‘good’ story? Why not talk according to some sort of non-standard mathematics instead?

- Partial answer: it’s true in virtue of the meaning of words like ‘2’, ‘+’, ‘4’, ‘number’, that *if* there are numbers, *then* 2+2=4. So some non-standard mathematical theories—e.g. one in which 2+2=5—are *inconsistent*.
- Another partial answer: standard mathematical theories are ‘natural’ in various ways.

2. The appeal to mathematial proof

Haven’t mathematicians *proved* many claims that obviously entail that there are numbers—e.g. that there are infinitely many prime numbers?

When you look at the most rigorous formulations of mathematical proofs, you find that they rely on *axioms*. A theorem is considered to be “proved” whenever it is logically derived from the axioms. Such a “proof” gives us excellent reason to think that *if* the axioms are true, the theorem is true; but someone who rejects the axioms won’t find that convincing (though it might be very interesting in other ways).

Famous sets of axioms you might hear mentioned:

The *Peano axioms* are the standard axioms for the theory of the natural numbers (0, 1, 2...)
Zermelo-Fränkel Set Theory (ZF) is the standard axiomatic set theory.

3. The appeal to mathematical practice

The fact remains that mathematicians *do* seem to believe these axioms and the theorems which they derive from them. And they seem to know their stuff pretty well! Isn't this a good reason for the rest of us to believe what they tell us, including what they tell us about the existence of numbers?

Lewis's credo.

Problems with this argument.

- Many mathematicians are amateur philosophers; they say all sorts of things about the question whether numbers exist (sometimes what they say strikes professional philosophers as confused, but let's not dwell on that). So the fact that they go round asserting, publishing, announcing that they have proved, etc., claims that entail the existence of numbers or sets, doesn't seem like conclusive evidence that they believe that those things exist.
- Anyhow, the thing mathematicians are unimpugnably good at is knowing what does and doesn't follow from axioms.

4. The appeal to science

The foregoing argument can be broadened out a bit. It's a fact that almost all of modern science makes constant use of mathematics. Just about any scientific theory you care to name—especially in physics—entails the existence of numbers and various other mathematical entities.

It's more compelling that we have good reason to believe the theories that are believed by empirical scientists than that we have good reason to believe the theories that are believed by mathematicians: the empirical sciences, after all, are supposed to be founded in the experimental method. And most people—including most nominalists—think that the empirical sciences have given us excellent reason to believe in all sorts of entities: electrons, viruses, black holes... We seem to take the fact that the existence of electrons is entailed by our best physical theories as an excellent reason to believe in electrons; why should we draw the line at numbers?

However, there are enormous differences between the cases. For whatever reason, most scientists seem quite uninterested in the task of finding theories that *don't* entail the existence of numbers. So relatively little ingenuity has been spent on developing and testing such theories.

5. The indispensability argument

The nominalist who disbelieves standard Platonistic scientific theories faces a challenge: if you reject these theories, what theories do you put in their place? How do you explain all the phenomena these theories are meant to explain?

Inference to the Best Explanation.

6. A quick response to the indispensability argument

The nominalist can say: give me your best Platonistic explanation of the phenomena: let's call it T. Then I'll put forth the following alternative theory:

"As far as the facts about concrete things are concerned, it is exactly as if it were the case that T". Call this theory T*. T* is entailed by T, so you must agree that there is good reason to believe T*. T* strikes me as an excellent explanation of the phenomena all on its own: it seems to me that T's commitment to numbers makes it a worse theory—a worse explanation of the phenomena—than T*.

But notice that a similar move is available to people (such as the philosopher of science Bas van Fraassen) who deny that physics gives us any reason to believe in electrons or other unobservable entities. Take any standard theory T; let $T^{\dagger}$ be the theory 'As far as observable entities are concerned, it is just as if T'.

Many scientific realists (who think that there is good reason to believe in unobservables) draw the moral from this that these "as if" theories, and other similar "parasitic" theories, are in general bad from a scientific point of view: they are bad explanations, or no explanations at all, of the phenomena.

The nominalist has three options at this point:

(i) Accept that there is no reason—or at least, no inference-to-the-best-explanation type of reason—to believe in unobservable entities.
(ii) Try to find some principled reason for thinking that T* (or something like it) is not a bad explanation in the way that $T^{\dagger}$ seems to be.
(iii) Try to find some other nominalistically acceptable theory that won't be parasitic on Platonist theories (this is Field's way).