Translation and the quantifiers

- The four Aristotelian forms, and their standard translations.

All Fs are Gs Every F is a G	$\forall x(F(x) \rightarrow G(x))$
Some Fs are Gs Some F is a G	$\exists x(F(x) \land G(x))$
No Fs are Gs No F is a G	$\neg\exists x(F(x) \land G(x))$ $\forall x(F(x) \rightarrow \neg G(x))$
Some Fs are not Gs Some F is not a G	$\exists x(F(x) \land \neg G(x))$

- Do not make the common mistake of translating ‘Some Fs are Gs’ by ‘$\exists x(F(x) \rightarrow G(x))$’.
  - The latter sentence is true if there is *any* object that is either not an F, or a G!

- Some points to note
  - 'Some Fs are Gs' is treated as equivalent to 'Some F is a G', despite the fact that some people have the intuition that 'Some Fs are Gs' would be false if only one F was a G.

    Are they right?

  - '$\exists x(F(x) \wedge G(x))$' and '$\forall x(F(x) \rightarrow G(x)$' are not inconsistent: both could easily be true. But some people have the intuition that 'Some Fs are Gs' and 'All Fs are Gs' are inconsistent.

    Are they right?

- If ‘$\forall x(\neg F(x))$’ is true, ‘$\forall x(F(x)\rightarrow G(x))$’ and ‘$\forall x(F(x)\rightarrow\neg G(x))$’ are both true — in this case they are called *vacuously* true. So ‘$\forall x(F(x)\rightarrow G(x))$’ does not entail ‘$\exists x(F(x)\wedge G(x))$’. But some people have the intuition that ‘All Fs are Gs’ and ‘No Fs are Gs’ are inconsistent, and the ‘All Fs are Gs’ entails ‘Some F is a G’.

  Are they right?

- Arguably, none of the intuitions I’ve just been talking about is right: they all arise from confusing *conversational implicature* with *entailment*.

- Sometimes the roles of ‘F’ and ‘G’ in the translations we’ve just been looking at will be played by *complex* predicates like ‘happy dog’ or ‘black dog owned by Clinton’. So in our translations, the role of F(x) will be played by a *complex* open formula.

Every happy black dog is owned by Clinton	$\forall x((Happy(x) \land Black(x) \land Dog(x)) \rightarrow Owns(clinton,x))$
Some happy black dog is owned by Clinton	$\exists x(Happy(x) \land Black(x) \land Dog(x) \land Owns(clinton,x))$
Clinton owns a happy black dog	$\exists x(Happy(x) \land Black(x) \land Dog(x) \land Owns(clinton,x))$