The material conditional →

- → is a binary truth-functional connective.
- It has the following truth-table:

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

- The sentence on the left of → is called the *antecedent*, the one on the right is called the *consequent.*

- $P \to Q$ is tautologically equivalent to $\neg P \vee Q$, as can be easily verified using a truth table.
  - So having $\to$ in the language doesn't let us express anything we couldn't have expressed without it—but it's convenient nevertheless.

Translations

- Suppose I say: 'If I left my scarf in the coffee shop, I left my cellphone there too'
  - If I left the scarf there and didn't leave the cellphone there, it's clear that I've said something false. If I left both of them there, it seems pretty clear that I haven't.
  - What if it turns out I didn't leave the scarf there? In this case it sounds a bit odd to suggest that I've said something *false*: I might have had no good *reason* to say what I said, but that's not the same thing.

- So there's a case to be made that 'P → Q' is a correct translation into FOL of an English sentence 'If P, then Q'.
  - Think of it in terms of what one *rules out*: in saying 'If P then Q', one is ruling out the case where P is true and Q isn't, and it's not clear that one is ruling out anything else.
- When we're doing translations in this course, we will translate 'If..then...' using the material conditional.

- But is this really correct? If it were, the following sentences would all be true:
  - ‘If pigs can fly, the moon is made of green cheese’
  - ‘If pigs can fly, the moon isn’t made of green cheese’
  - ‘If pigs can fly, pigs can’t fly’
- This seems pretty strange!

- On the other hand, there's some evidence that 'if...then...' really does express the material conditional.
  - The argument 'P or Q; therefore if not-P, then Q' seems valid.
  - But if this is valid, so is 'not-P or Q; therefore if P then Q'. So the English conditional is true whenever the material conditional is.
- A vexed question in 'philosophical logic'.

- Other English expressions we'll translated using 'P→Q':
  - Q if P (this is obviously equivalent to 'If P then Q'
  - Q provided that P
  - P only if Q
    - 'You will pass the course only if you pass the final exam'
- 'Unless P, Q' and 'Q unless P' are translated as '¬P→Q'

- It's important to distinguish the conditional symbol —which is part of FOL—from the notion of *logical consequence* which is a relation between sentences of FOL.
- A conditional can be true even if the consequent is not a logical consequence of the antecedent.
- However, for a conditional is *logically* true, the consequent does have to be a logical consequence of the antecedent.

The material biconditional ↔

- → is a binary truth-functional connective.
- It has the following truth-table:

P	Q	P↔Q
T	T	T
T	F	F
F	T	F
F	F	T

- The biconditional is true when the left hand side and right hand side have the same truth-value; otherwise it's false.

- $P \leftrightarrow Q$ is tautologically equivalent to $(P \rightarrow Q) \wedge (Q \rightarrow P)$.
    - It's also tautologically equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.

- We use ‘$\leftrightarrow$’ to translate the English expression ‘if and only if’, often abbreviated by mathematicians and philosophers as ‘**iff**’.
  - ‘Iff’ is sometimes read as ‘just in case’—a special bit of jargon.