Indirect proof 2: proof by contradiction

- An example: we want to prove ¬(SameRow(a, b) ∨ SameCol(a, b)) from the premises LeftOf(a, b) and FrontOf(a, b).
- **Proof:** To prove ¬(SameRow(a, b) ∨ SameCol(a, b)), we assume (SameRow(a, b) ∨ SameCol(a, b) and derive a contradiction. Suppose first that SameRow(a, b): this contradicts the premise FrontOf(a, b). Suppose on the other hand that SameCol(a, b): this contradicts the premise LeftOf(a, b). So in each case we have a contradiction, which means the original assumption that SameRow(a, b) ∨ SameCol(a, b) must be false.

Proof by contradiction

- If we can derive a *contradiction* from a certain assumption, together with other premises, we can infer the negation of that assumption from those premises.
- A *contradiction* is something that is obviously *logically impossible*, i.e. logically *can't* be true.
  - For example: anything of the form $P \wedge \neg P$.
  - The 'contradiction' symbol

More examples

- An informal proof of one direction of one of De Morgan's Laws: from $(\neg P \vee \neg Q)$, infer $\neg(P \wedge Q)$.
- **Proof:** Suppose for *reductio* that $P \wedge Q$. Given the premise that $(\neg P \vee \neg Q)$, there are two cases. Case 1: $\neg P$; this contradicts the first conjunct of the assumption that $P \wedge Q$. Case 2: $\neg Q$; this contradicts the second conjunct. So in either case we have a contradiction. So we can conclude that $\neg(P \wedge Q)$.

For next week

- Read: Chapter 5; optionally, chapter 6.
- *By hand*, use the truth-table method to determine whether the conclusion of each of the following arguments is a tautological consequence of the premises:
  - $\neg(\neg A \land B)$; $A \lor B$; therefore $\neg A$ (10%)
  - $A \lor B$; $\neg B \land (C \lor \neg A)$; therefore C (10%)
- Do: exercises 4.24 (20%); 5.8, 5.15, 5.17, 5.18 (15% each)

Tautological consequence in Fitch