TT-consequence and logical consequence, redux

- Why does TT-consequence suffice for logical consequence?
    - Suppose that Q is *not* a logical consequence of {P1,...,Pn}. Then there is some possible situation in which P1...Pn are true and Q is false.
    - But every possible situation corresponds to some line in the joint truth-table

Informal proofs and the Boolean connectives

- Cardinal rule: in an informal proof, you're allowed to make any inference whose validity you can legitimately assume to be obvious to your audience.

Some *very* obviously valid inferences

- Conjunction introduction: from any two premises P, Q, you may infer the conclusion $P \wedge Q$.
- Conjunction elimination: from the single premise $P \wedge Q$, you may infer either P or Q.
- Disjunction introduction: from any premise P, you may infer $P \vee Q$ for *any* Q.

Other useful valid inferences

- Important tautological equivalences, such as
  - De Morgan's Laws:
    - $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
    - $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
  - Idempotence:
    - $P \wedge P \Leftrightarrow P \Leftrightarrow P \vee P$
  - Distribution:
    - $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
    - $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

- Important tautologies, such as $P \lor \neg P$
  - These can be introduced into your informal proofs at any time, since a logical truth follows from everything.

Indirect proof: proof by cases

- Suppose we want to prove ¬(a = b), given the premise (Cube(a) ∧ Tet(b)) ∨ (Tet(a) ∧ Dodec(b)). How do we do it?
- **Proof:** Suppose Cube(a) ∧ Tet(b). Then it follows that ¬(a = b), since nothing can be both a cube and a tetrahedron. Suppose on the other hand that Tet(a) ∧ Dodec(b). Then again, it follows that ¬(a = b), since nothing can be both a tetrahedron and a dodecahedron. So in either case, ¬(a = b).

- What we're relying on here is the following fact: if a sentence follows from P together with certain other premises, and the same sentence follows from Q together with those premises, then it follows from $P \vee Q$ and those same premises.
- When we say 'Suppose...', we're beginning a *subproof*.
- An indirect proof is a proof that uses subproofs.