Expressivist theories of chance and lawhood

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1. A really bad kind of theory

A natural first thought about what an expressivist theory of chance might look like: `the chance that P is x' expresses the mental state *believing that P to degree x*. One "believes" that the chance that P is x [to degree 1?] iff one's degree of belief that P is x.

- ***First problem:*** intuitively, it can happen quite easily that one's degree of belief that P is x without believing that the chance that P is x—even while believing that it isn't!
- ***Second problem***: what's this a theory of? If it's time-dependent chance, where's the time index? If it's ur-chance, the first problem is extra bad, and also arises in reverse: one can believe firmly that the ur-chance that P is x without believing that P to any degree even close to x (because one has evidence about how things actually turned out.)
- ***Third problem***: what do we do about embeddings? On this theory everyone is necessarily completely opinionated about what the chances are; so we can say that someone believes a disjunction of claims about chance iff one believes either disjunct, and that someone believes the negation of a claim about chance iff they don't believe the claim. But these are intuitively completely implausible.

2. Objective chance as objectivised credence

A more promising analysis-schema: with each sense of 'chance', we associate a *preferred partition* of the space of possible worlds: a set of propositions any two of which are inconsistent and whose disjunction is equivalent to a logical truth. To 'believe that the chance that A is x' is to have a high degree of belief in a disjunction of preferred propositions H each of which is such that one's conditional credence in A given H is x.

- More generally: to "believe to degree y that the chance that A is x" is to have degree of belief y in the disjunction of all the preferred propositions H for which one's conditional credence in A given H is x.
- More generally still: let S be any sentence which may contain various embedded clauses of the form 'the chance that A is x'. To "believe to degree y that S" is to believe to degree y that S*, where S* is the proposition that is expressed by S relative to an interpretation on which 'the chance that A is x' is taken to express the disjunction of all those H for which one's conditional credence in A given H is x.

Skyrms suggests an analysis of this sort for claims about time-relativised chance, in the sense in which chance is incompatible with determinism: when we're talking about chances at *t*, the preferred propositions are complete descriptions of history up to *t*.

- ***Problem:*** this gives bad results when *t* is earlier than the present. If I saw the coin land heads, my conditional credence in 'the coin landed heads' given every complete description of history up to some long-ago time *t* may be high, even thoug○h my credence that there was a high chance at *t* that the coin would land heads is low.
    - Natural solution: don't look at my *credences* conditional on H. Look instead at my *prior* credences conditional on H.
        - Requires a picture on which it makes sense to think of each person—at least, each procedurally rational person—as having a prior credence distribution, which encodes their dispositions to conform their credences to their evidence.
- ***Problem:*** while this allows for people to be uncertain about what the chances were at *t*, it entails that everyone is *conditionally* certain about the chances about *t* conditional on any complete description about history up to *t*. But intuitively, uncertainty about the chances at *t* may survive coming to be certain about history up to *t*: one may instead be uncertain about the history-to-chance conditionals.
    - Skyrms's proposal basically amounts to adopting the bad kind of theory from section 1 above as a theory of ur-chance, and then analysing time-dependent chance in terms of ur-chance in the usual way.
- We could try to remedy this by associating a non-trivial preferred partition even with claims about ur-chance. For example, we could allow propositions which describe the relative frequencies of various kinds of outcomes into the preferred partition.
    - But then it'll turn out that everyone is certain about what the chances are, conditional on any hypothesis about the relative frequencies. Whereas intuitively, different theories about what the ur-chances are are consistent with the same relative frequencies, so that one won't be certain what the ur-chances are even given the truth about relative frequencies.
    - Moreover, on this proposal it would turn out that everyone is certain that, if the relative frequencies are such-and-such, there is an ur-chance of 1 that the relative frequencies would be such-and-such. Whereas intuitively, at least in finite cases, people are often pretty close to being certain that this *isn't* the case.
        - Family resemblance between this proposal and the 'L*-chances' discussed by Arntzenius and Hall.

3. Interlude: Skyrms on propensities

Suppose we have a probability distribution **P** over propositions, in which the proposition 'there are finitely many things' has probability 1. Then there's a straightforward way to turn it into a probability distribution **P*** over properties belonging to entities of a certain category: let **P***(F) = the **P**-expectation for the random variable whose value at a world is the fraction of the total number of things of the right category at that world that are F.

- When P assigns nonzero probability to there being infinitely many things in some category, however, mere counting won't do: we need a *measure* on the set of objects of that category at each possible world.

Lots of chance-talk seems to have to do with properties rather than propositions: 'the chance [propensity] that something is F given that it is G is x'.

Skyrms: for it to be true that the propensity that something is G given that it is F is high, it is not enough for the objective probability of Gness given Fness to be high—it must be *resilently* high, which is to say that for some wide range of other properties Q_1, Q_2, Q_3..., closed under conjunction and negation, the objective probability of Gness given F-and-Q_n must be high (if it is defined).

- Definition: the resiliency of Pr(F) being α = x over domain D iff

$$x = 1 - \mathrm{Max} \mid \alpha - \Pr(F \mid Q_i) \mid$$

- Skyrms thinks that claims about numerical propensities are to be cashed out in terms of claims about high resiliency. But it's actually not so clear how this could go. Surely the claim that the propensity of an F thing to be G = α must be inconsistent with the claim that the propensity of an F thing to be G = β whenever $\alpha \neq \beta$, even if α and β are very close. But if α is close to b and the resilency of Pr(F) being α over some domain is high, then the resiliency of Pr(F) being β over that same domain will also be pretty high.

Another claim of Skyrms: 'it's a law that Fs are Gs' has an important reading on which it's equivalent to the claim that the resiliency of Pr(G-if-F) being 1 is high, over an appropriately broad domain that includes F-ness and G-ness.

- On this reading, 'it's a law that Fs are Gs' doesn't entail 'All Fs are Gs'. (EG: 'ice cubes placed in glasses of warm water melt.')
    - Does it entail 'Fs are Gs' (i.e. Fs are *typically*, or *generically* G)?
- This is an interesting category of law-claim that we have barely touched on. Skyrms is forced to focus on it to the exclusion of by the fact that his expressivist account yields a very implausible theory when applied to the kind of notion of lawhood on which 'it's a law that P' entails 'the ur-chance that P is 1'.

4. A better form of expressivism

We are given a person's credence distribution and prior credence distribution over genuine propositions: to a first approximation, take these to be sets of possible worlds, and assume that any rational person's credences are derived from their prior credences by conditionalising on a certain proposition (evidence). We want to define the person's "quasi-credence" distribution, and "prior quasi-credence" distribution over "quasi-propositions". To a first approximation, take these to be sets of "quasi-worlds", where a quasi-world is an ordered pair $<w,\mathbf{P}>$, w a possible world, **P** a probability distribution over sets of possible worlds.

- Perhaps we should require that $\mathbf{P}(w)$ be positive (perhaps infinitesimal).

Sentences about chance and ur-chance are analysed straightforwardly as "expressing" sets of quasi-worlds [or if you prefer, states of having high quasi-credence in such sets]. The crucial clause: 'The ur-chance that A is x' expresses the set of quasi-worlds $<w, \mathbf{P}>$ in which $\mathbf{P}^{+}(A) = x$, where $\mathbf{P}^{*}$ is the natural extension of **P** to a distribution over quasi-worlds: $\mathbf{P}^{*}(S) = \mathbf{P}\{w \mid <w,\mathbf{P}> \in S)$. This extends in the natural way to all sentences in the language.

- The most natural clause for modal operators: 'Possibly, S' expresses the universal set if S expresses a nonempty set, otherwise the null set.

The idea: suppose a person's prior credence distribution admits of a unique best decomposition as a weighted sum of relatively simple probability distributions:

$$a_1\mathbf{P_1} + a_2\mathbf{P_2} + a_3\mathbf{P_3} + \ldots.$$

Then that person's quasi-prior credence distribution will be the weighted sum

$$a_1\mathbf{P_1}^{*} + a_2\mathbf{P_2}^{*} + a_3\mathbf{P_3}^{*} + \ldots.$$

where $\mathbf{P_i}^{*}(S) = \mathbf{P_i}(\{w \mid <w, \mathbf{P_i}> \in S)$.

And the person's quasi-credence distribution will be the result of conditionalising this distribution on the person's evidence.

- Note that this gives us the Principal Principle for free.

What about those for whom there is no unique best decomposition of their priors? To the extent that this is so, we can say that it's vague what their quasi-credences are.

- Is this a problem? No: there's already a ton of vagueness as regards peoples' credences in factual propositions.

Challenges:

- Extend this to agents whose degrees of belief aren't probabilistically coherent, or whose inductive dispositions are too unstable to be encoded by any single prior credence function.
- (A general objection to many expressivist theories:) What about agents who get to have beliefs (loosely speaking) about the relevant subject matter (in this case, chance) by picking them up from others without fully understanding the words, and in consequence believe all sorts of crazy things about chance?
- To what extent does expressivism about chance and law require expressivism about lots of other subject matters?