Seminar on Context-Sensitivity
Week Nine

1 The Liar: basics

Q Q is not true.

(1) Q = 'Q is not true'

(2) Q is true ↔ 'Q is not true' is true

(3) [Q is true ∧ 'Q is not true' is true] ∨ [Q is not true ∧ 'Q is not true' is not true]

2 Options for the theorist

(i) Accept that Q is not true (≈ "classical gap theory")

(ii) Accept that Q is true (≈ "classical glut theory")

(iii) Accept that Q is either true or not true, but refuse to believe that it is true and refuse to believe that it isn't true (≈ "weakly classical theory")

(iv) Refuse to accept that Q is either true or not true (paracomplete theory)

(v) Accept both that Q is true and that it isn't (dialethism)

3 Warnings

4 Context-sensitivity: the indexical model

Q^* Q^* is not true in any context.

Q_c Q_c is not true in my present context.

5 Context-sensitivity: expressing multiple propositions

(T) The proposition that ϕ is true iff ϕ.

(E) 'ϕ' expresses the proposition that ϕ.

$Q_\forall$ $Q_\forall$ expresses no true proposition.

$Q_\exists$ $Q_\exists$ expresses some proposition that isn't true.

Argument that $Q_\exists$ expresses more than one proposition:

(1) $Q_\exists$ expresses the proposition that $Q_\exists$ expresses some proposition that isn't true. ((E))

(2) Suppose $Q_\exists$ expressed only true propositions.

(3) Then the proposition that $Q_\exists$ expresses some proposition that isn't true would be true. ((1), (2))

(4) Then $Q_\exists$ would express some proposition that isn't true. ((3), (T))

(5) So $Q_\exists$ expresses some proposition that isn't true. ((4))

(6) So the proposition that $Q_\exists$ expresses some proposition that isn't true is true. ((5), (T))

(7) So $Q_\exists$ expresses at least one true proposition. ((1),(6))

(8) So $Q_\exists$ expresses at least two propositions. ((4), (7))

6 Asserting multiple propositions

$Q_\forall{}^*$ I am now asserting nothing true.

$Q_\exists{}^*$ I am now asserting at least one untruth.

7 Analogy: clubs

(1*) Michael is the secretary of a club whose members are exactly those who are secretary to some club of which they are not a member.

(2*) Suppose Michael were a member of every club of which he is a secretary.

(3*) Then Michael would be a member of a club whose members are exactly those who are secretary to some club of which they are not a member ((1*), (2*))

(4*) Then Michael would be secretary to some club of which he was not a member. ((3*), (T))

(5*) So Michael is secretary to some club of which he is not a member. ((4*))

(6*) So Michael is a member of every club whose members are exactly those who are secretary to some club of which they are not a member. ((5*))

(7*) So Michael is secretary to a club of which he is a member. ((1*),(6*))

(8*) So Michael is secretary to at least two clubs. ((4*), (7*))

8 Montague's theorem

Factivity $T(`\phi') \to \phi$

Closure $(T(`\phi_1') \wedge \ldots \wedge T(`\phi_n')) \to T(`\psi')$ whenever ψ follows from $\phi_1 \ldots \phi_n$ in predicate logic

Second-level factivity $T(`T(`\phi') \to \phi')$

λ $\neg T(\lambda)$

(1) $T(`\lambda = \neg T(\lambda)')$ (premise)

(2) $T(`T(`\lambda') \to T(`\neg T(`\lambda')')')$ ((1), Closure)

(3) $T(`T(`\neg T(`\lambda')') \to \neg T(`\lambda')')$ (Second-level factivity)

(4) $T(`T(`\lambda') \to \neg T(`\lambda')')$ ((2), (3), Closure)

(5) $T(`\neg T(`\lambda')')$ ((4), Closure)

(6) $\neg T(`\lambda')$ ((5), Factivity)

(7) $\lambda = `\neg T(`\lambda')'$ ((1), Factivity)

(8) $\neg T(`\neg T(`\lambda')')$ ((6), (7))

Upshot: (E) has instances that don't express only truths.

9 "Strengthened" Liars

Natural thought: say that a sentence ϕ *standardly* expresses a proposition p iff ϕ expresses p, and there is no instance ψ of (E) and false proposition q such that $\ulcorner \phi \wedge \psi \urcorner$ expresses the conjunction of p and q.

$Q_\exists+$ $Q_\exists+$ standardly expresses at least one untruth.

$Q_\forall+$ $Q_\forall+$ standardly expresses nothing true.

(E)+ ϕ standardly expresses the proposition that ϕ

Further upshot: (E)+ has instances that don't standardly express only truths. . . .