Non-symmetric Relations

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1 Primitive predicates

Let us say that a relation r is *symmetric* iff whenever x bears r to y, y bears r to x; otherwise, r is *non-symmetric*.[1] In this paper, I will argue for the thesis that necessarily, there are no non-symmetric relations.

What is this predicate '... bears ... to ...' in terms of which 'symmetric' was defined? According to one important theory of relations, this predicate is *primitive*, in the same sense in which the two-place predicate 'instantiates' ('has', 'exemplifies') is primitive according to some theories of properties. When x bears r to y, this is not so *in virtue of* any more basic truths involving x, r and y; there is no interesting answer to the question '*What is it* for x to bear r to y?' But it is not uncontroversial that 'bears' is primitive in this sense. Various analyses of this predicate have been proposed. For example, a believer in states of affairs might analyse 'x bears r to y' as 'there is a state of affairs s such that s relates x to y, and r is the universal component of s.' The predicates '... relates ... to ...' and '... is a universal

[1]Russell (1903, p. 25), by contrast, rather misleadingly defines '*not-symmetric*' to mean: not symmetric, and also not *asymmetric*—where an asymmetric relation is one for which there are no x and y such that x bears it to y and y bears it to x.

component of . . . ' might be taken as primitive, or analysed still further.

I regard the question which predicates are primitive in this sense as the most fundamental question of metaphysics. Traditionally, metaphysics aspires not just to answer a long list of questions of the form 'are there Fs?', but to understand the *ultimate structure of reality.* This aspiration should be understood, I think, as equivalent to the quest for a complete list of primitive predicates.[2]

But what do we mean when we ask whether some predicate, like 'bears', is primitive? 'Primitive' means 'unanalysable'; but what does 'unanalysable' mean? Unless we are extremely optimistic about conceptual analysis, we should not take it to mean '*conceptually* unanalysable'. For there are a great many predicates—'is an umbrella', 'is a dog', 'loves', 'is in pain'. . . —which only an extreme optimist would deny are primitive in *this* sense.[3] The project of listing all these predicates has nothing in common with the traditional metaphysical enterprise of understanding the ultimate structure of reality. Fortunately, we seem to understand a less demanding notion of analysis, better suited to the purposes of foundational metaphysics. We report such "analyses" when we say things like 'to be made of water is to be composed of H_2O molecules', or 'for one thing to be hotter than another is for the former to have a greater mean molecular kinetic energy than the latter'. As these examples make clear, analyses in this sense need not be accessible a priori. They also can be "de re", as when we say 'to be Napoleonic is to resemble a certain person, namely Napoleon,' or 'to be

[2]I don't mean to presuppose that it is always a *determinate* question whether a predicate is primitive. Certain views lead naturally to the conclusion that there is a certain amount of indeterminacy in the question. For example, surely there can be no determinate fact of the matter as regards whether it is 'instantiates' or 'is instantiated by' that is primitive: if one of them is primitive, it is indeterminate which it is. I discuss the issue of indeterminacy further in Dorr MS.

[3]For the optimistic view, see Jackson 1998.

an electron is to instantiate a certain universal, namely Electronhood.'[4]

Does this notion of analysis give us the notion of primitiveness we were looking for, or is the list of unanalysable predicates still too long and miscellaneous to be metaphysically significant? I will briefly mention two categories of predicates which might be thought to resist analysis without being primitive in the metaphysically interesting sense. First, there are *vague* predicates. The vagueness of 'small', for example, seems to prevent us from stating any interesting truth of the form 'to be small is to. . . '. One way to respond to this challenge would be to allow the notion of analysis to have a vagueness correlative to the vagueness of the expressions being analysed. Thus, there might be a range of precise predicates having to do with an object's mass, volume, etc. such that it's determinate that one of them counts as an analysis of 'is small', although there is no specific one of them such that it's determinate that *it* is an analysis of 'is small'.

Second, there is the category of "semantically defective" predicates, comprising various predicates introduced by myths, fictions and false theories, like 'is a unicorn', 'is a friend of Sherlock Holmes' ', 'is phlogiston', and perhaps 'is a Form' and 'is a monad'. It is hard to see how any of these predicates could be analysed, but clearly we don't want to count them all as primitive. We could deal with this problem by arbitrarily selecting some arbitrary logical contradiction (like 'is non-self-identical') to be the analysis of all such predicates.

[4]As this last example shows, the question whether a predicate is *primitive* is quite different from the question whether a predicate *corresponds to a universal.* If this analysis is correct, then 'is an electron' corresponds to a universal but is not primitive: the whole point of positing Electronhood is to allow us to give a straightforward analysis of 'is an electron'. And conversely, if 'instantiates' is primitive, there is no corresponding reason to posit a corresponding binary relation of Instantiation. And even if we decide that 'instantiates' is not primitive, surely we should not analyse 'x instantiates y' as 'x bears Instantiation to y'. If we do, we will have taken the first step on Bradley's regress: unless we stop the regress arbitrarily, we will find ourselves analysing 'bears' in turn in terms of a ternary Bearing relation, and so on (Bradley 1897, chapter 3).

Alternatively, we could simply redefine 'primitive' as 'unanalysable, and not semantically defective'.

The notion of analysis I have been describing is naturally associated with a notion of analyticity. Say that a sentence (lacking quotational and intensional contexts) is *metaphysically analytic* iff it can be transformed into a logical truth by

(i) substituting analyses for analysanda;

(ii) substituting co-referential proper names; and

(iii) replacing semantically defective predicates with logically contradictory ones.

Unlike sentences that are analytic in the traditional sense, metaphysically analytic sentences can be a posteriori, like 'everything which is made of water is made of H_2O molecules', 'Hesperus is Phosphorus' and 'there are no unicorns.'[5] Nevertheless, metaphysical analyticity is a stronger notion than metaphysical necessity, at least as the latter notion is generally understood. All metaphysically analytic sentences are metaphysically necessary,[6] but the reverse is not true: 'If Socrates exists, Socrates is human' is generally thought to be metaphysically necessary, but it is clearly not metaphysically analytic, since Socrates does not need to be mentioned in the analysis of 'is human'.

2 Relations

I will argue that all relations are necessarily symmetric by arguing that 'all relations are symmetric' is a metaphysically analytic truth. One way to argue for this thesis would be

[5]Cf. Kripke 1972, pp. 102–105, 126–127, 156–158.

[6]The only potential counterexamples I can think of are sentences like 'Napoleon exists', which count as logical truths according to standard logic, but are not metaphysically necessary. This difficulty is easily resolved by adopting some version of free logic as our characterisation of logical truth.

to argue for some analysis of 'bears' on which 'x bears r to y' and 'y bears r to x' turn out to be analytically equivalent. Another way to argue for the thesis would be to argue that the only primitive predicates are those on a certain list, where the predicates on the list are such that the only plausible analyses of 'bears' in terms of them render 'all relations are symmetric' analytic. Consider, for example, what I will call the *simple theory of states of affairs*. According to this theory there are just two primitive predicates, 'is a particular component of' and 'is a universal component of'. Given just these two predicates to work with, it is overwhelmingly natural to analyse 'x bears r to y' as 'there is an s such that x and y are the only particular components of s, and r is the only universal component of s'; and of course, under this analysis, 'all relations are symmetric' is analytic. We *could* give an analysis of 'x bears r to y' on which it is not equivalent to 'y bears r to x'—for example, we could analyse it as 'x is a particular component of r and r is a universal component of y'—but all such analyses are completely implausible.

A third way to argue for the thesis would be to argue that 'bears' is a semantically defective predicate. If it is, then 'there are no x, y, r such that x bears r to y' and hence 'there are no non-symmetric relations' count as metaphysically analytic, for the same reason that 'there are no unicorns' counts as metaphysically analytic. This would be a natural view for a Nominalist—a denier of the existence of universals, including relations—to hold. From a Nominalist point of view, predicates like 'is a universal', 'is a relation', 'instantiates' and 'bears' belong in the same category as 'is phlogiston': the theories that introduce these predicates are so radically unlike the truth that there is no way for the predicates to acquire any non-defective meaning.

In fact, I will not argue for any analysis of 'bears', or for any specific list of primitive predicates, Nominalist or Realist in character. Instead, I will defend the logically weaker claim that whatever the true primitive predicates might be, they do not allow for any credible analysis of 'there are non-symmetric relations' on which this claim is consistent.

Before I proceed with this argument, I should briefly address the most common objection to the thesis. It is obvious that there are non-symmetric *predicates*—alas, we sometimes love those who do not love us. If the claim that there are no non-symmetric relations seems obviously false, that may be because it seems obvious that all (or most) two-place predicates correspond to relations. ('F' corresponds to r iff for any x and y, x bears r to y iff Fxy.) Obvious though this may seem, I am committed to denying it. In my view, properties and relations are *sparse* as compared with predicates: in fact, it is an open question whether *any* of our current predicates correspond to relations.

This claim deserves a more sustained defence than I can provide in this paper.[7] For what it's worth, I do have a strategy for "explaining away" the appeal of the idea that all predicates correspond to properties or relations. In my view, our ordinary talk about these sorts of entities is governed by a certain fiction, "the fiction of abundant attributes". It is true according to this fiction that all predicates correspond to properties or relations; hence, we can *correctly* speak of non-symmetric relations like *loving*, *being taller than* and so forth, even though in reality there are no such things. But even those who find this unsatisfactory might be able to make some other sort of sense of the thought that the properties and relations which correspond to most predicates are "derivative" entities rather than "fundamental" ones. If you think you can make sense of this thought, you probably should interpret my thesis—and my quantifiers more generally, at least in formal contexts—as restricted to "fundamental" entities.[8]

[7]See Ramsey 1925; Armstrong 1978, chapter 13; Mellor 1991; Dorr 2002, chapters 4–6.

[8]If there are universals, it will sometimes happen that x instantiates y but y does not instantiate x. Hence, the predicate 'instantiates' is an example of a non-symmetric predicate. So if my thesis is true, it must be an example of a predicate which corresponds to no relation.

So far, I have been considering only binary relations; but the discussion extends in a predictable way to relations of higher degree. The only slight difficulty in the way of stating the thesis is the fact that the three-place predicate 'bears' doesn't have any natural counterparts in English which are appropriate for talking about relations of higher degree. We can say 'r holds among x, y and z' or 'x, y and z instantiate r'; but these expressions appear grammatically to contain the plural referring term 'x, y and z'. Hence, just as 'Tom, Dick and Harry lifted a piano' is logically equivalent to 'Dick, Harry and Tom lifted a piano', 'r holds among x, y and z' should, properly speaking, be logically equivalent to 'r holds among y, z and x'. To get around this difficulty, the best the proponent of non-symmetric relations can do is to say things like 'r holds among x, y and z, in that order' or 'x, y and z instantiate r, in that order'; and it isn't very satisfactory to have to use such quasi-quotational devices to express purportedly fundamental metaphysical facts.[9] Since I don't want to rest my case on these facts about English, it seems best to state my thesis using an artificial notation. Let us abbreviate 'x bears r to y' as '$xy\Delta r$'; and let us stipulate that the $n+1$-place predicate '$x_1 \dots x_n \Delta r$' is to be understood as a natural generalisation of this predicate, playing the same role for n-ary relations that '$xy\Delta r$' plays for binary relations. Then my claim is that the order of the first n arguments in the formula '$x_1 \dots x_n \Delta r$' is semantically irrelevant:

This is unproblematic: as we have already seen in footnote 4 (p. 3 above), the analysis of 'x instantiates y' as 'x bears Instantiation to y' seems like a bad idea for independent reasons. I am arguing for the view that there are no non-symmetric *relations*, not for the quite different view that there are no non-symmetric *primitive predicates*.

[9]In practice, of course, the qualifier 'in that order' doesn't have to be added every time. Nevertheless, in treating the order of the constituents of a plural referring term like 'x, y and z' as significant, we are definitely bending the ordinary rules governing such expressions. On the ordinary rules, 'r holds between x, y and z but does not hold between y, z and x' should entail 'there are some things such that r holds among them, and some other things such that r does not hold among them, and each of the former things is identical to one of the latter things, and each of the latter things is identical to one of the former things', which is surely inconsistent.

in other words, when $k_1 \dots k_n$ is some permutation of the first n integers, '$x_1 \dots x_n \Delta r$' is metaphysically analytically equivalent to '$x_{k_1} \dots x_{k_n} \Delta r$'.

The plan for the rest of the paper is as follows. In sections 3 and 4, I present an argument that 'bears' is not primitive. In sections 5, 6 and 7, I present and then argue against a range of other systems of primitive predicates which can allow for non-symmetric relations. In sections 8 and 9 I say more in defence of the thesis that all relations must be symmetric. Finally, in section 10 I present a new argument for the thesis, in the hope that it will convince some of those who reject the central premise of the main argument.

3 Converse relations and brute necessities

Consider the following rather plausible-looking principle:

> CONVERSES For every r, there is an r' such that for any x and y, x bears r' to y iff y bears r to x.

To put it more succinctly, everything has a converse, where a converse of some entity r is any entity r' such that for all x, y, x bears r' to y iff y bears r to x. (Assuming that 'x bears r to y' entails 'r is a relation', everything that is not a relation is automatically one of its own converses, and so CONVERSES is equivalent to the claim that every *relation* has a converse.)

CONVERSES is not a *logical* truth in the narrow sense—as witness the fact that it becomes false if we replace the nonlogical predicate 'bears' with some other predicate, like 'gives'. This fact is not very interesting on its own, but it takes on a new significance if we assume that 'bears' is primitive. For then, it follows that CONVERSES is not metaphysically analytic. In no sense is there any *hidden contradiction* in the claim that there counterexamples to CONVERSES. This seems to me to be a very significant fact, as far as the modal and epistemological status of CONVERSES is concerned. I will defend the following two claims: first, if 'bears' is primitive, CONVERSES is not metaphysically necessary; second, if 'bears' is primitive, CONVERSES cannot be known for certain a priori.

The first of these claims follows from the following principle:

> POSSIBILITY If a sentence S is logically consistent, and the only nonlogical vocabulary in S consists of primitive predicates, then ⌜It is metaphysically possible that S⌝ is true.

The thought behind POSSIBILITY is that metaphysical necessity is never "brute": when a logically contingent sentence is metaphysically necessary, there is always some *explanation* for this fact. One sort of explanation involves the notion of metaphysical analyticity. It's metaphysically necessary that everything is made of water is made of H_2O molecules: this is explained by the fact that to be made of water just is to be made of H_2O molecules. But this sort of explanation is only available when a sentence contains some *non*-primitive expressions which have such analyses. Another, quite different sort of explanation involves the notion of essence. It's metaphysically necessary that if Socrates exists, Socrates is human: this is explained by the fact that Socrates is *essentially* human. But this sort of explanation is only available when a sentence contains some rigid referring terms, or words (like "Napoleonic" and perhaps "electron") whose analysis involves such terms. It is not available for what we might call *purely non-referential* sentences, which contain no such expressions; hence, it is not available for sentences which contain only primitive predicates and logical vocabulary. Thus, if 'bears' is primitive the metaphysical necessity of CONVERSES cannot be explained in either of these two standard ways. At the very least, the burden of proof is on someone who wants to maintain the necessity of CONVERSES to come up with some explanation of a different sort.

The second claim—that if 'bears' is primitive, CONVERSES cannot be known for certain a priori—follows from the following principle:

> KNOWABILITY If a sentence S is logically consistent, and the only nonlogical vocabulary in S consists of primitive predicates, then $\ulcorner$No-one could know for certain a priori that not-$S\urcorner$ is true.

This claim seems to me to follow from the kernel of truth in the traditional empiricist claim that only analytic truths can be known a priori. The traditional empiricist claim needs to be weakened in several ways. First, we should replace 'known' with 'known for certain': if induction is rational, it seems that we must be be able to have a priori knowledge of some general principles like 'the future resembles the past' or 'good explanations are generally true ones', though we cannot justifiably be *certain* of such principles. Second, we should understand 'analytic' not as 'conceptually analytic', but as 'metaphysically analytic': this should be enough to take care of alleged counterexamples like 'nothing is both reddish and greenish all over', or 'any being who feels pain is conscious', or 'anyone who knows that snow is white believes that snow is white'. Third, we should restrict the empiricist thesis to *purely non-referential* sentences, which neither explicitly nor implicitly contain any referring expressions—this will take care of the cases Kripke discussed under the heading of the "contingent a priori", like 'If m exists, then m is a metre long' (where m is the standard metre). But even when the empiricist doctrine is weakened in all these ways, it still entails KNOWABILITY: for a logically contingent sentence which contains no nonlogical vocabulary besides primitive predicates cannot be metaphysically analytic, and must be purely de dicto.

The most important potential counterexamples to POSSIBILITY and KNOWABILITY are the theorems of mathematics, which are often taken to be both metaphysically necessary and a priori. However, many philosophers will be able to find some sense in which they can endorse the claims that numbers and other mathematical entities are "derivative" entities rather than "fundamental" ones, and that this fact is central to a proper understanding of the modal and epistemological status of the truths of mathematics.[10] Hence, if the example of mathematics prompts us to reject certain instances of POSSIBILITY and KNOWABILITY, we can still accept the restrictions of these claims to sentences in which all quantifiers are restricted to fundamental entities. But on its intended interpretation, the quantification in CONVERSES *is* restricted to fundamental entities (see p. 6 above). So even the restricted versions of POSSIBILITY and KNOWABILITY suffice to entail that if 'bears' is primitive, CONVERSES is neither metaphysically necessary nor a priori.

Suppose we could somehow show that CONVERSES *is* metaphysically necessary, or a priori; then, by *modus tollens*, we could conclude that 'bears' is not primitive. This argument exemplifies the important role POSSIBILITY and KNOWABILITY (even in their restricted versions) can play in allowing us to make progress in finding out which predicates are primitive. Using these principles, we can use what we know about metaphysical necessity and knowability a priori to rule out a great many hypotheses about primitive predicates. Since I will be making use of this sort of argument time and again, it will be useful to have a simple formula which encapsulates the conclusions of this section. Let us say that a sentence S is a *brute necessity* iff (i) S is not a logical truth; (ii) the only nonlogical vocabulary in S consists of primitive predicates; (iii) ⌜S is metaphysically necessary⌝ is true, (iv) ⌜One can know for certain a priori that S⌝ is true, and (v) all quantifiers in S are restricted to fundamental entities. Then the principle which we need can be stated succinctly as follows: there are no brute necessities.[11]

[10] I myself like to make sense of this sort of talk by interpreting "fundamental" as "real" and "derivative" as "fictional". But there is no need to insist on this.

[11] This principle is weaker than either POSSIBILITY or KNOWABILITY, since it only concerns sentences which are both necessary and a priori. But it is strong enough for our

4 Spurious distinctions

In this section, I will complete the argument that 'bears' is not primitive. I will not actually argue for the claim that Converses is necessary and a priori, but for the weaker claim that *either* Converses is necessary and a priori, *or* 'bears' is not primitive: this weaker premise is still sufficient, given the results of the previous section, to establish the conclusion that 'bears' is not primitive. My aim, in other words, is to argue against the conjunction of the following two claims: first, 'bears' is primitive; second, Converses is not necessary or not a priori. The problem I see with this combination of views is that it forces us to draw *spurious distinctions* between the possibilities (metaphysical or epistemic possibilities, it doesn't matter which) in which Converses fails. We have to distinguish different "possible worlds" where intuitively there should be only one.

Imagine a world containing (among other things) a series of simple particulars, linearly ordered by exactly two independent simple relations, r_1 and r_2. The two relations generate exactly the same order among the objects in the series. (Outside the series, however, the two relations are not linked by any particularly simple laws.) Do they order the series in the same, or in opposite directions? In other words, which of the following is the case?

(i) For any distinct x and y in the series, x bears either r_1 or r_2, but not both, to y.

(ii) For any distinct x and y in the series, x either bears both r_1 and r_2 to y, or bears neither r_1 nor r_2 to y.

At any world containing such a series, exactly one of these hypotheses is true. If the actual world satisfied the description, we could ask which of the two possibilities obtained, and assuming that 'bears' is primitive, there would be a determinately right answer. But I say

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purposes.

that there can be no determinately right answer, because the question is not a legitimate one. There is nothing here for us to be ignorant about; no genuine respect in which two possible worlds might be dissimilar.

Let us consider a concrete example of this kind of situation. Suppose we discover that our talk of the charges of particles was actually talk about two different magnitudes: charge varies in discrete steps from -1 to +1, and charge* also varies from -1 to +1. But in our region of the universe, charge and charge* are always the same (except in the case of a few very rare kinds of particle that only exist inside cyclotrons). If there could be non-symmetric relations, then one way for the theory of charge and charge* to be true would be for there to be two non-symmetric relations such that 'the charge of x is greater than that of y' reports the holding of one of them, and 'the charge* of x is greater than that of y' reports the holding of the other.[12] If CONVERSES were true, there would have to be a total of four non-symmetric relations involved, corresponding to the four predicates 'is of greater charge than', 'is of less charge than', 'is of greater charge* than', and 'is of less charge* than'. But if CONVERSES is false, there might be only two relations: one for comparisons of charge, and one for comparisons of charge*. If this were so, a question would arise: does the correlation of charge with charge* in our region of the universe consist in the fact that for any x and y in our region, x either bears both relations or neither to y, or does it rather consist in the fact for any x and y in our region, x bears exactly one of the relations to y? Do the relations

[12] This might seem like a rather unlikely way for the theory to be true. It entails that comparisons of charge and charge* are *external*; whereas we might have thought they were internal, true in virtue of the (intrinsic) properties of the particles. In that case there would be a property (unary universal) corresponding to each distinct value for charge and charge*; the fundamental relations underlying the comparisons would hold not between the particles themselves but between the properties. But the same question about the meaningfulness of comparisons of the "directions" of the two relations will arise whether the relata are properties or particulars.

"point in the same direction", or in opposite directions?

Surely there are not two structurally different possibilities here. If there were a distant region of the universe in which the correlation went the other way, there would be no real question whether it is we, or the inhabitants of that other region, for whom the two relations coincide. Our scientists assign numbers to charges and charges* in such a way that the numbers for particles in our region are the same, and the numbers for particles in their region are of different signs. Their scientists of course do the opposite: they think of their region as the one in which charges and charges* are equal, and of ours as the one in which charges and charges* are opposite. One need hardly be a verificationist to feel that this difference is purely a matter of convention: neither system of notation is in any way "better" than the other, as far as the metaphysics of the situation is concerned. If non-symmetric relations are possible at all, distinct non-symmetric relations order their relata in different ways.

We might put this point metaphorically by saying that there is no non-arbitrary sense to be made of the way numbers are assigned to the argument places of non-symmetric relations. If we were to assign the number 1 to the 'lower charge' and 'lower charge*' argument places of the two binary relations in the example, and 2 to the 'higher charge' and 'higher charge*' argument places, this would be arbitrary. We might as well assign 2 to 'lower charge' and 'lower charge*', or assign 1 to 'lower charge' and 2 to 'lower charge*', or 2 to 'lower charge' and 1 to 'lower charge*'. In reality, there are just four different argument places: two belonging to the relation 'comparison of charge', and two belonging to the relation 'comparison of charge*'.[13] That is why it makes no sense to ask whether two particulars are related in the same way or in opposite ways by these two relations, unless the question is intended to be

[13]Cf. Williamson 1985, p. 260.

relative to some particular way of making the arbitrary choices.[14]

The hypothesis that 'bears' is primitive must be rejected, since it entails that there is a deep metaphysical fact in these situations, where clearly there is nothing but arbitrariness.[15] Unless 'bears' is semantically defective, it must somehow be analysable.

There are two ways in which a revealing account of what it is for something to bear a relation to something else could help us deal with the present argument. First, the analysis might block the argument for the contingency of CONVERSES by revealing CONVERSES to be metaphysically analytic. However, as far as I can see, the only credible analysis on which CONVERSES is analytic are those on which 'all relations are symmetric' also comes out analytic. The only way I can think of to render CONVERSES analytic without ruling out non-symmetric relations is to build something tantamount to CONVERSES directly into the analysis of 'bears'. Thus, for example, someone might propose taking some new predicate 'F' as primitive, analysing 'a bears r to b' as '$Fabr$, and there is an r' such that for all x, y, $Fxyr$ iff $Fyxr'$'. But this suggestion is not only incredible but pointless: the spurious structural distinctions between possible worlds are no more palatable when expressed using 'F' than when expressed using 'bears'.

[14]This talk of entities called "argument places" doesn't *have* to be regarded as metaphorical. In section 7 I will consider several theories in which argument places are taken seriously as a fundamental ontological category.

[15]In footnote 2 (p. 2 above), I pointed out that if either 'instantiates' or 'is instantiated by' is primitive, it should be taken as indeterminate which is the primitive one. Likewise, no-one should regard it as a determinate fact that 'x bears r to y' rather than 'y is borne r to by x' is primitive. Thus, it is purely a matter of convention which of these two predicates we should adopt: we could just as well have chosen to mean by 'x bears r to y' what we actually mean by 'y bears r to x'. But this fact doesn't help us to do away with the invidious distinctions that are forced upon us when we take either of these notions as primitive. For no matter which of the two equally good meanings for 'bears' we adopt, the answer to the question 'Does x (i) bear both or neither of r_1 and r_2 to y, or does it rather (ii) bear exactly one of r_1 and r_2 to y?' will be the same.

Second, the analysis might allow us to accept the contingency of Converses, and embrace the conclusion that in the imagined scenario, either (i) or (ii) is true, while denying that this constitutes a deep structural difference. The idea is that the notion of bearing is relative to a series of arbitrary choices, one for each relation. We might have defined our word 'bears' in such a way as to make (i) true; but someone else could just as well choose a different definition on which (ii) comes out true. And if our use of the word 'bears' doesn't decide the question, it will be *indeterminate* which of these two perfectly good notions is expressed by 'bears', and hence indeterminate which of (i) and (ii) is the true hypothesis and which the false one.[16]

In the next three sections I will consider several systems of primitive predicates which allow 'bears' to be defined only by means of such arbitrary choices.

5 Primitive like-relatedness

The first system I will consider is also the most straightforward. We want it to make sense to say 'a is related to b by r in the same way that c is related to d by r', but not to make sense to say 'a is related to b by r_1 in the same way that c is related to d by r_2'. Well, let's just take the kind of comparison we do want to make sense as our new primitive predicate, rather than defining it in terms of 'bears'. We will symbolise the first sentence as '$ab \underset{r}{\uparrow} cd$'. There is no natural analysis of the second sentence: '$ab \underset{r_1 r_2}{\uparrow} cd$' is just ill-formed.

[16]Williamson (1985) holds a view very similar to this, although he allows that *some* sets of relations might have naturally corresponding argument places, so that a single arbitrary choice would suffice to render the notion of bearing determinate for all the members of such a set. Fine (2000) holds essentially the same view as regards what he calls "neutral" relations, although he also believes in less fundamental "biased" relations for which the notion of bearing is unproblematic. Williamson's and Fine's arguments, however, are quite different from mine.

Even if it is the notion of like-relatedness that is primitive, and the notion of bearing that is defined, still there is surely an intimate relation between these two notions, to which we can appeal in informally explaining the intended meaning of '$ab\underset{r}{\uparrow}cd$'. Abbreviating '$x$ bears r to y' as '$xy\Delta r$', the intended connection is given by the following biconditional:

$$\textsc{Bicond} \qquad xy\underset{r}{\uparrow}zw \equiv (xy\Delta r \wedge zw\Delta r) \vee (yx\Delta r \wedge wz\Delta r)$$

If we were taking 'bears' as primitive, it would be natural to take Bicond as a definition of '$\uparrow$'. As it is, given that we are trying to reverse the order of definition, Bicond does not tell us how to go about defining 'Δ' in terms of '$\uparrow$'. It does, however, tell us what conditions have to hold for it to be *possible* to construct any such definition. Bicond has many logical consequences in which '$\uparrow$' is the only nonlogical predicate: only if all these consequences are true will it be *possible* to introduce the predicate 'Δ' in such a way as to make Bicond true. Let us call the totality of such claims the *theory of like-relatedness*.

It is interesting to investigate how we might axiomatise the theory of like-relatedness using the predicate '$\uparrow$' alone. The crucial notion we need is that of a *paradigm-pair* for a relation. Let us say that $\langle a, b\rangle$ is a paradigm-pair for r iff

$$\forall xy(xy\Delta r \equiv xy\underset{r}{\uparrow}ab).$$

Bicond entails that every relation—in fact, everything whatsoever—has at least one paradigm-pair. For any r, there are three possibilities: (i) r is uninstantiated (i.e. there are no x and y such that $xy\Delta r$), in which case any $\langle a, b\rangle$ is a paradigm-pair for r. (ii) r is both symmetric and instantiated, in which case any $\langle a, b\rangle$ such that $ab\Delta r$ is a paradigm-pair for r. (iii) r is non-symmetric and instantiated, in which case any $\langle a, b\rangle$ such that $ab\Delta r$ and not $ba\Delta r$ is a paradigm-pair for r. Hence, Bicond entails the following principle in which the predicate 'Δ' does not occur:

PARADIGMS $\quad \forall r \exists ab \forall xyzw(xy\underset{r}{\uparrow}zw \equiv (xy\underset{r}{\uparrow}ab \wedge zw\underset{r}{\uparrow}ab) \vee (yx\underset{r}{\uparrow}ab \wedge wz\underset{r}{\uparrow}ab))$

PARADIGMS suffices to axiomatise the theory of like-relatedness: that is, BICOND and PARADIGMS have the same logical consequences in which the only predicate is '$\uparrow$'. To prove this, note that any model PARADIGMS can be extended into a model of BICOND by interpreting '$xy\Delta r$' as '$xy\underset{r}{\uparrow}f(r)g(r)$', where f and g are functions which map each r to an a and b as described in PARADIGMS. Hence, any sentence involving only the predicate '$\uparrow$' which is consistent with PARADIGMS is also consistent with BICOND.

PARADIGMS shows us how we might go about making the arbitrary choices that are required in order to assign a determinate extension to the predicate 'bears': provided that PARADIGMS is true, it will always be possible to find functions f and g such that 'x bears r to y' can appropriately be defined as '$xy\underset{r}{\uparrow}f(r)g(r)$'. In practice, of course, we haven't bothered to define any general notion of this sort. Rather than using names for relations and the single predicate 'bears', we use many different predicates, often associating more than one with a given relation. Thus, if there is a single non-symmetric binary relation t which underlies facts about temporal order, we have not decided whether to say that x bears it to y when x is earlier than y, or when y is earlier than x. But we evidently have somehow managed to make a distinction between 'x is earlier than y' and 'x is later than y'. One way we might have done this is to stipulate, concerning a paradigm pair $\langle a, b\rangle$ of times or events related by t, that 'x is earlier than y' shall be true just in case $xy\underset{t}{\uparrow}ab$, and '$x$ is later than y' shall be true just in case $xy\underset{t}{\uparrow}ba$.[17] Of course the actual process whereby our predicates come

[17] If we had wanted to, we could have introduced two predicates 'bears_{ab}' and 'bears_{ba}', stipulating that 'x bears_{ab} t to y' shall be true iff $xy\underset{t}{\uparrow}ab$, and that '$x$ bears_{ba} t to y' shall be true iff $xy\underset{t}{\uparrow}ba$. We could, furthermore, have introduced two more names for t, 'beforeness'

to mean what they do would have to be much more complicated than this in all sorts of ways. But this simple story at least gives us a sense of what we might expect to be involved in the analysis of especially basic non-symmetric predicates, if like-relatedness is primitive.[18]

The strategy of taking like-relatedness as primitive can be generalised to relations of higher degree. Just we might introduce an $n+1$-place predicate $x_1 \ldots x_n \Delta r$ to play the same role for n-ary relations that 'bears' plays for binary relations, likewise we might introduce a $2n+1$-place predicate $x_1 \ldots x_n \underset{r}{\uparrow} y_1 \ldots y_n$ to play the same role for n-ary relations that '$\uparrow$' plays for binary relations. The intended connection between these notions can be expressed using a generalised version of Bicond:

Bicond$_n$ $$x_1 \ldots x_n \underset{r}{\uparrow} y_1 \ldots y_n \equiv \bigvee_{k_1 \ldots k_n} (x_{k_1} \ldots x_{k_n} \Delta r \wedge y_{k_1} \ldots y_{k_n} \Delta r)$$

(Here the right-hand side is a disjunction with $n!$ disjuncts, one for each way of assigning

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and 'afterness', together with the following convention of disambiguation: when we refer to t as 'beforeness' we are to interpret 'bears' as 'bears$_{ab}$'; when we refer to it as 'afterness' we are to interpret 'bears' as 'bears$_{ba}$'. We could even allow different disambiguations within the same sentence. If we did, 'x bears beforeness to y but does not bear afterness to y' could be true despite the truth of 'beforeness = afterness'. This would be a way to make non-contradictory sense of the puzzling doctrine, defended by Armstrong (1978, p. 42) and Williamson (1985), that non-symmetric relations have converses, but are identical to their converses.

[18]Like all analyses which make use of paradigms, the suggested analyses of 'before' and 'after' are subject to an important modal objection: if 'x is before y' is analysed as '$xy \underset{t}{\uparrow} ab$', it follows that if a or b didn't exist, nothing could be before anything. This difficulty can be circumvented to some extent by switching to a disjunctive analysis, and by allowing qualitatively similar "counterparts" of the paradigms to count as surrogates for the paradigms themselves; but intuitively, it seems that some things could have been before other things even if *no* actual particulars, or anything like them, had existed. One way to approach this problem would be to look to the direction-of-time literature for a satisfying theoretical characterisation of the difference between past and future. Another would be to allow it to be an indeterminate matter which direction is which at such distant worlds, just as it is often thought to be an indeterminate matter whether a hand-shaped object in a world sufficiently unlike the actual world is left or right.

$k_1 \ldots k_n$ to some permutation of the first n positive integers.)

Can the set of logical consequences of $\textsc{Bicond}_n$ in which '$\uparrow$' is the only non-logical predicate be axiomatised using only the predicate '$\uparrow$'? The most straightforward generalisation of $\textsc{Paradigms}$ would be the following claim:

$$\forall r \exists a_1 \ldots a_n \forall x_1 \ldots x_n y_1 \ldots y_n$$
$$\big(x_1 \ldots x_n \underset{r}{\uparrow} y_1 \ldots y_n \equiv \bigvee_{k_1 \ldots k_n} (x_{k_1} \ldots x_{k_n} \underset{r}{\uparrow} a_1 \ldots a_n \wedge y_{k_1} \ldots y_{k_n} \underset{r}{\uparrow} a_1 \ldots a_n)\big)$$

But this claim is not a consequence of $\textsc{Bicond}_n$. It fails, for example, in a model of $\textsc{Bicond}_n$ in which these are the only true atomic sentences involving the predicate 'Δ':

$$\begin{array}{ll} abc\Delta r & def\Delta r \\ bca\Delta r & dfe\Delta r \\ cab\Delta r & \end{array}$$

$\langle a, b, c \rangle$, for example, won't do as a paradigm sequence in this model: for we have $abc \underset{r}{\uparrow} def$, and $abc \underset{r}{\uparrow} efd$, but not $def \underset{r}{\uparrow} efd$. $\langle d, e, f \rangle$ is also ruled out: for we have $def \underset{r}{\uparrow} abc$, and $def \underset{r}{\uparrow} acb$, but not $abc \underset{r}{\uparrow} acb$. However, it can easily be verified that in this model,

$$\forall xyz(xyz\Delta r \equiv (xyz \underset{r}{\uparrow} abc \wedge xyz \underset{r}{\uparrow} def),$$

so the set $\{\langle a, b, c \rangle, \langle d, e, f \rangle\}$ can do the work that neither sequence can do on its own. $\textsc{Bicond}_n$ entails that it will always be possible to associate each relation with an appropriate set of paradigm n-tuples; moreover, at most $n! - 1$ paradigm n-tuples are needed for each

n-ary relation.[19] So a counterpart of PARADIGMS for n-ary relations can be stated as follows:

$$\text{PARADIGMS}_n \quad \forall r\ \exists a_1^1 \ldots a_n^1 a_1^2 \ldots a_n^2 \ldots\ldots a_1^{n!-1} \ldots a_n^{n!-1}\ \forall x_1 \ldots x_n y_1 \ldots y_n$$

$$\big(x_1 \ldots x_n \underset{r}{\uparrow} y_1 \ldots y_n \equiv \bigvee_{k_1 \ldots k_n} \bigwedge_{0<i<n!} (x_{k_1} \ldots x_{k_n} \underset{r}{\uparrow} a_1^i \ldots a_n^i \wedge y_{k_1} \ldots y_{k_n} \underset{r}{\uparrow} a_1^i \ldots a_n^i)\big)$$

(Again, '$k_1 \ldots k_n$' ranges over all permutations of the first n positive integers.) This is a certainly a mouthful, but it is the simplest first-order axiomatisation I have been able to find.[20]

[19] Sketch of proof: let the *extension* of r be the set $\{\langle x_1, \ldots, x_n\rangle | x_1 \ldots x_n \Delta r\}$. Let the *image* of any sequence $\langle a_1, \ldots, a_n\rangle$ be the set $\{\langle x_1, \ldots, x_n\rangle | x_1 \ldots x_n \underset{r}{\uparrow} a_1 \ldots a_n\}$. Let the *expanded extension* of r be the intersection of all the images that contain the extension of r. It is straightforward to show, using BICOND_n, that the expanded extension of r is equivalent to the extension of r for the purposes of defining like-relatedness: that is, $x_1 \ldots x_n \underset{r}{\uparrow} y_1 \ldots y_n$ is true iff $\langle x_{k_1}, \ldots, x_{k_n}\rangle$ and $\langle y_{k_1}, \ldots, y_{k_n}\rangle$ are both members of the expanded extension of r, for some permutation $k_1 \ldots k_n$. Moreover, BICOND_n entails that every image is a union of some of the permutations of the extension of r, i.e. the sets $\{\langle x_1, \ldots, x_n\rangle | x_{k_1} \ldots x_{k_n} \Delta r\}$. There are at most $n!$ such sets. Hence it is always possible to find some set containing at most $n! - 1$ images whose intersection is the expanded extension of r.

[20] There is another disanalogy between the case of binary relations and the general case which is worth noting. In the case of binary relations, the distinctions between possibilities described using the predicate 'bears' which we get to dismiss as spurious when we take '$xy \underset{r}{\uparrow} zw$' as primitive are exactly those I argued against in section 4. But in general, if we take '$x_1 \ldots x_n \underset{r}{\uparrow} y_1 \ldots y_n$' as primitive we will have to classify as spurious certain distinctions between possibilities which could be perfectly genuine as far as the arguments of section 4 are concerned. Consider, for example, two worlds, each of which contains just three particulars, a, b and c, and a ternary relation r:

w_1	w_2
$abc\Delta r$	$abc\Delta r$
$bca\Delta r$	$bca\Delta r$
$cab\Delta r$	

BICOND_n entails that exactly the same sentences of the form $x_1x_2x_3 \underset{r}{\uparrow} y_1y_2y_3$ hold at w_1 and w_2. So if 'Δ' is analysed in terms of '$\uparrow$', we will have to say that w_1 and w_2 are not really two different possibilities, but one possibility described in two different ways.

6 Against primitive like-relatedness

It is time to take stock. We were led to consider taking the notion of like-relatedness as primitive by the following argument: if 'bears' is primitive, Converses is a brute necessity; but there are no brute necessities; therefore 'bears' is not primitive. Could this be a good reason to adopt like-relatedness as a primitive instead? Surely not. For even if Bicond is not an analysis of '↑' in terms of 'bears', it certainly seems both necessary and a priori. If it is, then so are its logical consequences, including those whose only non-logical vocabulary is the predicate '↑'—what I have been calling the theory of like-relatedness. Hence, if '↑' is primitive, all of these consequences of Bicond are brute necessities, except for those which are logical truths. But the theory of like-relatedness includes many sentences—for example, Paradigms—which are not logical truths. Hence, since there are no brute necessities, '↑' is not primitive.

Someone might respond to this argument by denying that the theory of like-relatedness is metaphysically necessary and/or a priori. The problem with this response is analogous to the problem which I raised in section 4 for someone who holds that 'bears' is primitive and denies that Converses is necessary or a priori. If we take seriously the (metaphysical or epistemic) possibility that the theory of like-relatedness is false, we are forced to see distinctions between possibilities which even the most stalwart anti-verificationist should baulk at accepting as genuine matters of (metaphysically or epistemically) contingent fact.

Here is an example of such a distinction. Take some world w at which the theory of like-relatedness is true, where there are several binary relations $r_1 \ldots r_n$, at most one of which holds between any two objects. Let w' be a world derived from w by replacing all these relations with a single binary relation r, in such a way that $ab \underset{r}{\uparrow} cd$ is true at w' iff $ab \underset{r_i}{\uparrow} cd$ is true at w for any r_i. The theory of like-relatedness will be false at w'. Suppose that at w, a and b are related by r_1 ($ab\underset{r_1}{\uparrow}ab$), and c and d are related by r_2: then at w', a and b are related by r, and c and d are related by r, but these two pairs are *incomparable* as regards the direction in which r holds: '$ab\underset{r}{\uparrow}cd$' and '$ab\underset{r}{\uparrow}dc$' are both false. Here, then, is one example of the sort of question that strikes me as "metaphysical" in the pejorative sense: do we live at a world with several binary relations, like w, or should we instead say that there is just one relation, holding in many different, incomparable ways, as at w'?

Some of the distinctions between possibilities in which the theory of like-relatedness fails have the same symmetric character as the distinctions which I discussed in section 4, which makes them especially difficult to take seriously. Suppose Metaphysician A and Metaphysician B agree about everything except for one relation r, as regards which they have the following disagreement: whenever Metaphysician A says that $ab\underset{r}{\uparrow}cd$, Metaphysician B says that $ab\underset{r}{\uparrow}dc$, and vice-versa. Given that they disagree in this way, they cannot both accept the theory of like-relatedness.[21] There is something especially repugnant, I think, about the conclusion that this is a determinate, objective dispute which cannot be resolved by a priori means. For given the entirely symmetric character of the dispute, it seems clear that unless at least one of the hypotheses can be conclusively ruled out a priori, there is no basis at all for assigning one of them higher credence than the other.[22]

[21]Suppose they did. Then whenever A says that $ab\underset{r}{\uparrow}cd$, by Paradigms, A must say that $ab\underset{r}{\uparrow}ab$ and $cd\underset{r}{\uparrow}cd$; hence B will say that $ab\underset{r}{\uparrow}ba$ and $cd\underset{r}{\uparrow}dc$; so by Paradigms, B too must say that $ab\underset{r}{\uparrow}cd$.

[22]Paradigms logically entails that the predicate $\uparrow$ is subject to certain symmetries:

$$ab\underset{r}{\uparrow}cd \equiv cd\underset{r}{\uparrow}ab$$
$$ab\underset{r}{\uparrow}cd \equiv ba\underset{r}{\uparrow}dc$$

Even if '$\uparrow$' were primitive, I think we could still accept these biconditionals as necessary and

These examples fall short of establishing that the whole of the theory of like-relatedness, as opposed to some substantial fragment of it, is necessary and a priori. From the point of view of the argument, this doesn't matter: a weaker brute necessity is just as unacceptable as a strong one. But in fact, I doubt there is any stopping point short of the whole theory: for *any* description of a situation in which the theory fails, there will be some description of a situation in which it holds, such that the distinction between the two descriptions has the spurious character to which I have been objecting.

7 Analysing like-relatedness

So far, I have argued that neither bearing nor like-relatedness is primitive. In this section, I will survey several more systems of primitive predicates. In each of these systems, '$\uparrow$' can be analysed in a natural and general way, whereas 'bears' can be analysed only relative to some arbitrary choices.

My arguments against these systems will be similar to those which I gave against the primitiveness of 'bears' and '$\uparrow$'. First, none of the new systems of primitive predicates allows us to give a natural analysis of '$\uparrow$' on which the theory of like-relatedness turns out to be analytic. (By contrast, if we take 'bears' as primitive, the natural analysis of '$\uparrow$'—i.e. Bicond—does render the theory of like-relatedness analytic.) If I am right that the theory of like-relatedness is necessary and a priori, this means that each of the new systems requires

a priori truths. In fact, they should be classified as *logical* truths, even though they are not theorems of standard logic. They merely express the fact that no semantic significance attaches to certain facts about the order in which the arguments of $\uparrow$ are written down, just as no semantic significance attaches to the choice of a typeface or to the speed with which they are pronounced. It is just an artefact of our linear language that it forces us to choose a unique order for the four arguments of '$\uparrow$'. However, this way of explaining how a sentence involving only primitive predicates could be a logical truth only makes sense for biconditionals with atomic formulae on either side, and their logical consequences.

us to posit brute necessities.

Even if I am wrong that the entire theory of like-relatedness, as opposed to some fragment of it, is necessary and a priori, it doesn't matter much: the new systems of primitive predicates are also objectionable for a quite different reason. Whereas, if we take '$\uparrow$' as our only primitive, there is simply no way to make sense of the question 'Do r_1 and r_2 hold between x and y in the same direction or in opposite directions?', in each of the new systems there *is* at least one natural translation of this question, on which both answers are logically consistent. In section 4, I argued that such cross-relational comparisons are illegitimate, in the sense that there are no pairs of (epistemic or metaphysical) possibilities which differ only in the answers to questions of this sort. Thus, each of the new systems succumbs to the same objection as the view that 'bears' is primitive.

For the sake of brevity, I will present each of the new systems in a condensed format. In each case, I will begin by stating the natural analyses of like-relatedness and bearing in terms of the primitive predicates of the system. Next, I will list some plausible-looking axioms from which the theory of like-relatedness can be derived, given the analysis of like-relatedness. (These axioms will often be somewhat stronger, but also somewhat simpler, than the weakest possible set of axioms needed to derive the theory of like-relatedness.) Finally, I will present a second list of axioms, which are the counterparts of CONVERSES for the given system: only if they were necessary and a priori could spurious cross-relational comparisons be ruled out; but they cannot be necessary and a priori, since if they were they would be brute necessities. The point of stating all these axioms is to help make it clear just what sorts of bizarre possibilities one would have to countenance if one accepted any of these systems of predicates as primitive. Readers who are satisfied with the point should feel free to skip over them.

The first system is what we would end up with if we took the apparent quantification over "ways" in the expression 'a is related by r to b in the same way that c is related by r to d' at face value, as committing us to a new ontological category of *ways* alongside particulars and universals.

System A: Ways of being related

Primitive predicate:

'w is a way in which a is related by r to b' ('$ab\underset{w}{\Delta}r$')

Definition of like-relatedness:

(A1) $$ab\underset{r}{\uparrow}cd =_{df} \exists w(ab\underset{w}{\Delta}r \wedge cd\underset{w}{\Delta}r)$$

Definition of bearing:

(A2) $$ab\Delta r =_{df} ab\underset{f(r)}{\Delta}r$$

where f is a function that associates each relation with one of its ways.

Axiom needed to derive theory of like-relatedness:

(A3) $$(ab\underset{w_1}{\Delta}r \wedge cd\underset{w_2}{\Delta}r \wedge w_1 \neq w_2) \supset \forall xy(xy\underset{w_1}{\Delta}r \equiv yx\underset{w_2}{\Delta}r)$$

Axioms needed to rule out cross-relational comparisons:

(A4) $$(ab\underset{w}{\Delta}r_1 \wedge cd\underset{w}{\Delta}r_2) \supset r_1 = r_2$$

(A5) $$ab\underset{w_1}{\Delta}r \supset \exists w_2(ba\underset{w_2}{\Delta}r)$$

I doubt many philosophers would be seriously tempted to recognise these "ways" as a new fundamental ontological category: they are just too similar to relations.[23] A more appealing

[23]In fact, ways as conceived in this system are *exactly* like non-symmetric relations, as conceived by the proponent of CONVERSES. The only difference is that we now add a new fundamental category of entities corresponding to pairs of converse relations, and reserve the title 'relation' for these new entities.

new fundamental ontological category is the category of *argument places*, considered not as numbers but as sui generis entities which can be associated with numbers only by some arbitrary convention.

System B: Argument places

Primitive Predicate:

'r holds between a and b with respect to the argument places α and β, respectively' ('$ab\underset{\alpha\beta}{\Delta}r$').

Definition of like-relatedness:

(B1) $$ab\underset{r}{\uparrow}cd =_{df} \exists\alpha\beta(ab\underset{\alpha\beta}{\Delta}r \wedge cd\underset{\alpha\beta}{\Delta}r)$$

Definition of bearing:

(B2) $$ab\Delta r =_{df} ab\underset{f(r)g(r)}{\Delta}r$$

where f and g are functions thought of as specifying a relation's "first" and "second" argument place, respectively.

Axioms needed to derive theory of like-relatedness:

(B3) $$ab\underset{\alpha\beta}{\Delta}r \equiv ba\underset{\beta\alpha}{\Delta}r$$

(B4) $$(ab\underset{\alpha\beta}{\Delta}r \wedge cd\underset{\gamma\delta}{\Delta}r) \supset ((\alpha = \gamma \wedge \beta = \delta) \vee (\alpha = \delta \wedge \beta = \gamma))$$

Axiom needed to rule out cross-relational comparisons:

(B5) $$(ab\underset{\alpha\beta}{\Delta}r_1 \wedge cd\underset{\alpha\gamma}{\Delta}r_2) \supset r_1 = r_2$$ [24]

A third ontological category which might be invoked in an analysis of '$\uparrow$' is the category of *states of affairs*, entities which many philosophers believe in for reasons having nothing to

[24] This is the principle 'distinct relations have distinct argument places', which I offered in section 4 as a metaphorical explanation of the incomparability of distinct relations. It works better as a metaphor: taken at face value (as a claim about fundamental entites) it has to be regarded as a brute necessity if it is necessary and a priori at all.

do with the problems of non-symmetric relations. The most straightforward way to invoke states of affairs in the analysis of like-relatedness is to take the notion of *entering into states of affairs in the same way* as primitive. Kit Fine (2000) defends something very close to this view.

System C: States of affairs

Primitive predicates:

'a stands to b in state of affairs s_1 in the same way that c stands to d in state of affairs s_2' ('$abs_1 \Uparrow cds_2$')

'x is a universal component of s' ('$x \mathsf{U} s$')

Definition of like-relatedness:

(C1) $$ab \underset{r}{\uparrow} cd =_{df} \exists s_1 s_2 (r \mathsf{U} s_1 \wedge r \mathsf{U} s_2 \wedge abs_1 \Uparrow cds_2)$$

Definition of bearing:

(C2) $$xy\Delta r =_{df} \exists s (r \mathsf{U} s \wedge xys \Uparrow f(r) g(r) h(r))$$

where h assigns each relation to a "paradigm state of affairs", and f and g label the components of that state of affairs as the "first" and "second", respectively.

Axioms needed to derive theory of like-relatedness:

(C3) $$abs_1 \Uparrow cds_2 \equiv cds_2 \Uparrow abs_1$$

(C4) $$abs_1 \Uparrow cds_2 \equiv bas_1 \Uparrow dcs_2$$ [25]

(C5) $$(abs_1 \Uparrow cds_2 \wedge cds_2 \Uparrow efs_3) \supset abs_1 \Uparrow efs_3$$

(C6) $$(r \mathsf{U} s_1 \wedge r \mathsf{U} s_2 \wedge abs_1 \Uparrow abs_1 \wedge cds_2 \Uparrow cds_2) \supset (abs_1 \Uparrow cds_2 \vee abs_1 \Uparrow dcs_2)$$

[25] C3 and C4 belong to the special category of *symmetries*, which as I argued in footnote 22 (p. 23 above), can properly be classified as logical truths, and so can be necessary and a priori even if '$\Uparrow$' is primitive.

Axioms needed to rule out cross-relational comparisons:

(C7) $(r\mathsf{U}s_1 \wedge abs_1 \Uparrow cds_2) \supset r\mathsf{U}s_2$

(C8) $(r_1\mathsf{U}s \wedge r_2\mathsf{U}s) \supset r_1 = r_2$

If we accept more than one of the three new ontological categories we have considered, still further systems of primitive predicates suggest themselves. If we were willing to accept ways of being related alongside states of affairs, we might take 'w is a way in which a stands to b in s' as primitive; if we were willing to accept argument places alongside states of affairs, we might take 'a and b occupy the argument places α and β of s, respectively' as primitive. These two systems add nothing of interest to systems A and B. But we get a more interesting theory of argument places and states of affairs if we don't take 'a and b occupy the argument places α and β of s, respectively' as primitive, but analyse it as 'a occupies argument place α of s, and b occupies argument place β of s'.

System D: States of affairs and argument places

Primitive predicates:

'a occupies argument place α of state of affairs s' ('$x\underset{\alpha}{\mathsf{C}}s$')

'x is a universal component of s' ('$x\mathsf{U}s$')

Definition of like-relatedness:

(D1) $ab\underset{\alpha\beta}{\Delta}r =_{df} \exists s(r\mathsf{U}s \wedge a\underset{\alpha}{\mathsf{C}}s \wedge b\underset{\beta}{\mathsf{C}}s \wedge \forall x(\exists y(x\underset{y}{\mathsf{C}}s) \supset (x = a \vee x = b))$

(B1) $ab\underset{r}{\uparrow}cd =_{df} \exists\alpha\beta(ab\underset{\alpha\beta}{\Delta}r \wedge cd\underset{\alpha\beta}{\Delta}r)$

Definition of bearing:

As in system B.

Axioms needed to derive theory of like-relatedness:

(D2) $$(r\mathsf{U}s_1 \wedge r\mathsf{U}s_2 \wedge x\underset{\alpha}{\mathsf{C}}s_1) \supset \exists y(y\underset{\alpha}{\mathsf{C}}s_2)$$

(D3) $$(x\underset{\alpha}{\mathsf{C}}s \wedge y\underset{\alpha}{\mathsf{C}}s) \supset x = y$$ [26]

Axioms needed to rule out cross-relational comparisons:

(D4) $$(r\mathsf{U}s_1 \wedge a\underset{\alpha}{\mathsf{C}}s_1 \wedge b\underset{\alpha}{\mathsf{C}}s_2) \supset r\mathsf{U}s_2$$

(C8) $$(r_1\mathsf{U}s \wedge r_2\mathsf{U}s) \supset r_1 = r_2$$

If we were prepared to embrace the thesis (defended in Armstrong 1978, pp. 91–93) that all relations are necessarily *irreflexive*—i.e. that things never bear relations to themselves—we could avoid the ontological commitment to argument places incurred by system D by regarding argument places as “abstractions” from states of affairs and their particular components. The idea is to take the formula ‘the argument place which a occupies in s_2 = the argument place which b occupies in s_2’ not, at face value, as involving genuine quantification over argument places, but as primitive.

System E: States of affairs without argument places

Primitive predicates:

‘a is a component of s_1 in the same way that b is a component of s_2’ (‘$as_1\mathsf{A}bs_2$’)

‘x is a universal component of s’ (‘$x\mathsf{U}s$’)

[26] D3 entails that even the holding of a *symmetric* relation between a and b must involve two states of affairs, differing in the argument places occupied by a and b. It would perhaps be more intuitive to posit just one state of affairs in this situation, which has both a and b in the same argument-place (see Fine 2000, p. 31). In fact, D3 is a stronger axiom than we really need. Its work could be done instead by the weaker (but non-first-order) principle that an argument place is always occupied by the same number of entities in every state of affairs in which it is occupied at all.

Definition of like-relatedness:

(E1) $$abs_1 \Uparrow cds_2 =_{df} as_1 \mathsf{A} cs_2 \wedge bs_1 \mathsf{A} ds_2 \wedge \forall x(xs_1 \mathsf{A} xs_1 \supset x = a \vee x = b) \wedge \forall x(xs_2 \mathsf{A} xs_2 \supset x = c \vee x = d)$$

(C1) $$ab \underset{r}{\uparrow} cd =_{df} \exists s_1 s_2 (r \mathsf{U} s_1 \wedge r \mathsf{U} s_2 \wedge abs_1 \Uparrow cds_2)$$

Definition of bearing:

As in System C.

Axioms needed to derive theory of like-relatedness:

(E2) $$as_1 \mathsf{A} bs_2 \equiv bs_2 \mathsf{A} as_1$$[27]

(E3) $$(as_1 \mathsf{A} bs_2 \wedge bs_2 \mathsf{A} cs_3) \supset as_1 \mathsf{A} cs_3$$

(E4) $$(r \mathsf{U} s_1 \wedge r \mathsf{U} s_2 \wedge as_1 \mathsf{A} as_1) \supset \exists x (as_1 \mathsf{A} xs_2)$$

(E5) $$as \mathsf{A} bs \supset a = b$$[28]

Axioms needed to rule out cross-relational comparisons:

(E6) $$(r \mathsf{U} s_1 \wedge as_1 \mathsf{A} bs_2) \supset r \mathsf{U} s_2$$

(C8) $$(r_1 \mathsf{U} s \wedge r_2 \mathsf{U} s) \supset r_1 = r_2$$

Before we leave this survey of systems, it is worth noting an interesting feature which distinguishes systems D and E from all of the other systems of primitive predicates which we have considered. The other candidate primitive predicates are all specifically adapted for talking about *binary* relations. To accommodate relations of arbitrary degree within any of these systems, we would have to add infinitely many new primitive predicates, taking ever larger numbers of arguments, and governed by ever more complex systems of brute

[27] For the special status of this axiom, see footnote 25 above (p. 28).

[28] As in system D, if we wanted to allow that the holding of a symmetric relation between two things can involve just one state of affairs, we could replace E5 with the weaker principle that whenever $as_1 \mathsf{A} bs_2$, the xs such that $xs_1 \mathsf{A} bs_2$ are equinumerous with the ys such that $as_1 \mathsf{A} ys_2$.

necessities. By contrast, systems D and E are both already equipped to deal with relations of arbitrary degree. Their analyses of '$ab \underset{r}{\uparrow} cd$', can both be generalised straightforwardly to yield analyses of '$a_1 \dots a_n \underset{r}{\uparrow} b_1 \dots b_n$', for any n. Moreover, given these generalised analyses, the axioms already listed suffice to entail the generalised theory of like-relatedness (i.e. to entail $\textsc{Paradigms}_n$ for every n), and to rule out spurious cross-relational comparisons. Thus, these two systems are more *economical* in their primitive predicates, and in their need to posit brute necessities, than the other theories of non-symmetric relations which we have considered. But if the reasons for ruling out brute necessities which I gave in section 3 are good, they show that there are *no* brute necessities, not just that we should accept as few of them as possible.[29]

8 Doing without non-symmetric relations

We have been searching for a system of predicates with three features:

1. They do not allow for any credible analyses of the sentences 'r_1 and r_2 hold between a and b in the same direction' and 'r_1 and r_2 hold between a and b in opposite directions', on which both of these claims are consistent.

2. They allow for a credible analysis of the notion of like-relatedness, on which the theory of like-relatedness is analytic.

3. They allow for a credible analysis of 'All relations are symmetric'—or equivalently, '$\forall xyr(xy \underset{r}{\uparrow} xy \supset xy \underset{r}{\uparrow} yx)$'—on which this claim is not analytic.

[29] In section 10, I will take up the question what we should believe about non-symmetric relations if we replace the outright rejection of brute necessities with this weaker methodological principle.

'Bears' lacks feature 1, as does any system of primitive predicates in which 'bears' has a plausible and non-arbitrary analysis. '$\uparrow$' lacks feature 2. The systems considered in section 7 lack both of these features. On the basis of this survey of candidates, I conclude that no system of predicates has all three features. I don't know how to *prove* this—as currently stated, indeed, it is too vague to admit of proof. The best I can do is to challenge those who doubt the conclusion to produce a counterexample.

But, as I argued in sections 3 and 6, the true system of primitive predicates, whatever it is, must have features 1 and 2. Hence, the true system of primitive predicates lacks feature 3. Either 'bears' is semantically defective, or it has an analysis on which 'x bears r to y' is equivalent to 'y bears r to x'; in either case, 'all relations are symmetric' is metaphysically analytic.

I am in fact inclined to favour the Nominalist view that 'bears' is semantically defective. Nevertheless, even for me it is worth seeing how the less radical view that there are relations all of which are necessarily symmetric could be a defensible one. For I suspect that considerations very similar to those I have discussed should lead a Nominalist to embrace a Nominalistic "paraphrase" of the claim that all relations are symmetric. Even if there are *strictly speaking* no relations at all, even *loosely* speaking—speaking "according to the fiction of universals"—all relations are symmetric.

To help ourselves get used to the conclusion that all relations must be symmetric, we should consider various ways for a theory containing non-symmetric polyadic predicates to be true despite the non-existence of non-symmetric relations. For of course the fact that a non-symmetric predicate can't be meaningful by "standing for" a certain relation doesn't entail that non-symmetric predicates can't be meaningful at all, and doesn't entail that they can't be used to state true theories.

It's not hard to show that every theory using non-symmetric predicates can be modelled in a world of symmetric relations. Imagine a world where there is a property of "ghostliness", and a symmetric binary relation of "shadowing" which places the non-ghostly things in one-to-one correspondence with the ghostly things. At such a world, any consistent theory with only two-place predicates could be true, on an interpretation on which all quantifiers are restricted to non-ghostly entities, every one-place predicate is associated with a property of non-ghostly entities, and every two-place predicate F is associated with a symmetric relation r_F holding between non-ghostly and ghostly entities, with 'Fab' being analysed as 'r_F holds between a and b's shadow'. By postulating more than one ghostly realm, we can extend this method to interpret theories with non-symmetric predicates taking more than two arguments.

Of course—and fortunately for common sense—this isn't the only sort of interpretation on which a theory containing non-symmetric polyadic predicates can be true. Many such theories can be interpreted without imposing any restriction on the quantifiers. For example, the predicate 'is part of' which occurs in the theory known as 'mereology' or 'the calculus of individuals' can be analysed in terms of a symmetric "overlap" relation: x is part of y iff whatever overlaps x overlaps y (see Goodman 1951, pp. 42–51). In other cases, we do need to posit some new entities, but they are much more familiar and easier to believe in than the realm of "ghostly" entities. For example, the three-place non-symmetric predicate 'between' which might feature in a formalisation of Euclidean geometry construed as a theory about points of space might be analysed in terms of a binary symmetric "overlap" relation whose relata include line segments as well as points: 'x is between y and z' is taken to mean 'every line segment that overlaps both y and z overlaps x.'

Mathematical platonists will be glad to learn that set theory is one of the theories that can be interpreted in a world of symmetric relations without commitment to any new entities. According to orthodox (well-founded) set theory, there are some special sets called *ranks*, such that whenever x is a member of y, there is some rank that contains x and not y, but no rank that contains y and not x. Allen Hazen (MS) has shown how this fact can be used to analyse membership in terms of two symmetric two-place predicates. Say that two things *overlap set-theoretically* iff one of them is a member of the other. Say that two things *are of the same rank* iff they are members of exactly the same ranks. It follows from orthodox set theory that a is a member of b iff a overlaps b and there is something of the same rank as b which overlaps everything of the same rank as a.[30] To account for the truth of set theory in a world of symmetric relations, we can adopt this biconditional as an analysis of 'is a member of' in terms of 'overlaps' and 'is of the same rank of', and posit a symmetric binary relation corresponding to each of these two predicates.[31]

Do we *ever* need to introduce new entities in order to interpret a theory containing non-symmetric predicates? It might be suggested that the answer to this question is 'no'. For couldn't we interpret any such theory by associating each non-symmetric two-place predicate

[30]If $a \in b$, then a overlaps b, and the *rank* which is of the same rank as b has everything of lower rank, and hence everything of the same rank as a, as a member. If $a \notin b$, then either a doesn't overlap b, or $b \in a$. But if $b \in a$, there is nothing of the same rank as b which is a member of everything of the same rank as a, and so there is nothing of the same rank as b which overlaps everything of the same rank as a. (Actually, this last claim is true only if there is at least one urelement; otherwise the case where b is the null set and a is its unit set is an exception. Hazen (MS) states a more complicated definition which can accommodate this case.)

[31]Since there are any number of different ways to define the notion of a *sequence* in set-theoretic terms, this interpretation of set theory gives us another perfectly general strategy for interpreting theories involving non-symmetric predicates: we simply associate each non-symmetric predicate F with a property p_F, analysing '$Fx_1 \ldots x_n$' as '$\langle x_1, \ldots, x_n \rangle$ instantiates p_F'.

F with a *ternary* symmetric relation r_F, analysing 'Fxy' as '$xyy\Delta r_F$? Likewise, we could analyse a three-place predicate '$Gxyz$' as '$xyyzzz\Delta r_G$', and so on.

To me, this looks like cheating: it is consistent with the letter of the thesis that all relations must be symmetric, but not with its spirit. What difference does it make whether we think of an assignment of numbers to the relata of a relation as measuring the order of the relata, or as measuring how many times over each of them occurs as a relatum? The idea that there could be non-arbitrary sense to be made of the question which assignment of numbers is the correct one seems to face more or less the same problems either way. Just like the disputes which I considered in section 4, the dispute between two metaphysicians who agree about everything except that whenever metaphysician A says '$xyy\Delta r$', metaphysician B says '$xxy\Delta r$', strikes me as a spurious one. If this analogy holds, then just as we have concluded that '$xy\Delta r$' and '$yx\Delta r$' are metaphysically analytically equivalent, we should also conclude that '$xyy\Delta r$' and '$xxy\Delta r$' are metaphysically analytically equivalent.[32] Thus, the suggested strategy for interpreting arbitrary theories containing non-symmetric predicates is a failure. If we want to find ontologically conservative interpretations of such theories, we must seek them case by case.

[32]This would be true, for example, if the simple theory of states of affairs which I described in section 2 were correct. For if '$xyz\Delta r$' is analysed as as 'there is a state of affairs with x, y and z as its only particular components, and r as its universal component', '$xyy\Delta r$' and '$xxy\Delta r$' are indeed equivalent.

In fact, '$xyy\Delta r$' is equivalent on this analysis to '$xy\Delta r$'. Likewise, 'x bears r to x' is equivalent to 'x instantiates r'. A relation that things sometimes bear to themselves is really a *multigrade* universal: an entity that is sometimes instantiated like a property, and sometimes borne like an irreflexive relation. If we really wanted to block this conclusion, we could do so by tinkering with the analyses of 'instantiates' and 'bears'; for example, 'x instantiates y' could be analysed as 'x is the unique particular component of some state of affairs with y as its unique universal component, and no state of affairs with y as its universal component has more than one particular component'. But I'm not sure that the conclusion is odd enough to be worth blocking in this way.

9 The intuition that non-symmetric relations are possible

I have not been shy of appeals to modal intuition in this paper: in fact, my method of argument relies essentially upon them. Thus, I should say something about the widely shared intuition that non-symmetric relations *are* possible. Unsurprisingly, I think that the force of this intuition is defeated by the argument I have presented. But this is not just a clash of equally-matched intuitions: there are good reasons, independent of this argument, why we should be extremely tentative in our intuition that non-symmetric relations are possible.

First, let us consider a more general principle from which the possibility of non-symmetric relations might be derived. Say that a theory is *fundamental* iff it is true under an interpretation on which each of its predicates is interpreted as standing in the most direct possible way for some universal. Someone might hold that it is possible for *any* consistent theory to be fundamental. If this is correct, then in particular, it must be possible for consistent theories that include non-symmetric predicates to be fundamental, and so it must be possible for there to be non-symmetric relations.

In fact, however, no believer in universals should deny that there are some limits on the forms fundamental theories can take. Although logicians have generally been content to study languages whose atomic formulae consist of predicates together with ordered sequences of terms, one can imagine languages which attach semantic significance to quite different sorts of facts about the arrangement of the words that make up an atomic formula. For example, we can imagine a predicate that takes, not a straightforward list of arguments, but a list of arguments embellished with an equal number of left and right parentheses: if F is such a predicate, 'Fa', '$Fa(bc)$', '$F(ab)c$', '$F(a)(b(c))$', '$Fa(bc)(d(ef))$' are all well-formed and logically independent formulae.

Surely it makes no sense to suppose that a predicate of this exotic sort could "stand in the most direct possible way" for a universal u_F. What simple fact involving just a, b, c and u_F could make the difference between a situation in which '$Fa(bc)$' was true and one in which '$F(ab)c$' was true?[33] The true story about the interpretation of 'F', whatever it is, must be fairly complicated: we cannot simply say that F stands for a certain relation and leave it at that. For example, the analysis of F might involve set theory: we could associate F with a property p_F of sets, contextually analysing each formula '$F\varphi$' as 'φ' instantiates p_F', where φ' is a singular term derived from the parenthesis-embellished list φ by replacing all left and right parentheses with left and right curly braces, and putting another pair of curly braces around the whole expression.

Sentences of our language consist of words spread out along one dimension of time or space. So non-symmetric predicates seem to us to be the most natural thing in the world, whereas the parenthesis-embellished predicate F seems strange and unusual. But this fact is much too parochial to warrant us in continuing to insist that it must be possible for non-symmetric predicates, unlike parenthesis-embellished predicates, to occur in fundamental theories. There could be a race of beings whose sentences took an entirely different form. For example, they could build up their sentences by collecting words in bags of different colours, collecting these bags inside other bags, and so on.[34] To the "speakers" of such a language, non-symmetric predicates might seem just as exotic as parenthesis-embellished predicates

[33] If we spoke a language with lots of these "parenthesis-embellished" predicates, we might find it natural to introduce a parenthesis-embellished version of the non-symmetric predicate 'Δ'; so we could answer the question posed in the text by saying that in the first case, $a(bc)\Delta u_F$, while in the second case, $(ab)c\Delta u_F$. My point is that we surely do not want to accept that this new parenthesis-embellished predicate as primitive, or as definable in any very natural and simple way in terms of primitives.

[34] Cf. Williamson 1985, p. 259.

seem to us, while the equivalents of parenthesis-embellished predicates might strike them as utterly natural. If they were given to theorising about universals, they might find it just as intuitive that there could be universals corresponding directly to parenthesis-embellished predicates as we find it intuitive that there could be non-symmetric relations.

Can any other basis be found for a principled distinction between non-symmetric predicates and parenthesis-embellished predicates? It might be suggested that the crucial difference is something about the predicates' role in *scientific explanation*: non-symmetric predicates can be explanatorily essential in a way that parenthesis-embellished predicates cannot. But this is false: we can imagine situations in which parenthesis-embellished predicates are just as useful in scientific explanation as non-symmetric predicates could ever be. Let's begin by recursively defining the predicate 'forms a nested circle around point p' as follows: Every material object forms a nested circle around its centre of gravity. A set forms a nested circle around p iff (i) each of its members forms a nested circle around some point, and (ii) all of these points are at the same distance from p (see figure 1). Now, suppose we discover some special new particles. As we move them around, we notice that certain arrangements lead to the emission of a distinctive kind of radiation from nearby points. Eventually we find a pattern, which we describe as follows: there are some special sets, built up from these particles, such that the distinctive radiation is emitted from a point iff one of those sets forms a nested circle around it. Moreover, we find absolutely no way to predict whether a given set has this feature given any other facts about its members and their relations. So we end up introducing a new basic physical law, involving a new monadic predicate F of sets: whenever an F set forms a nested circle around a point p, radiation is emitted at p.

It comes naturally to us to put the new law in terms of sets. But this isn't inevitable: we might instead introduce a new parenthesis-embellished predicate, writing '$Fa(bc)(d(ef))$'

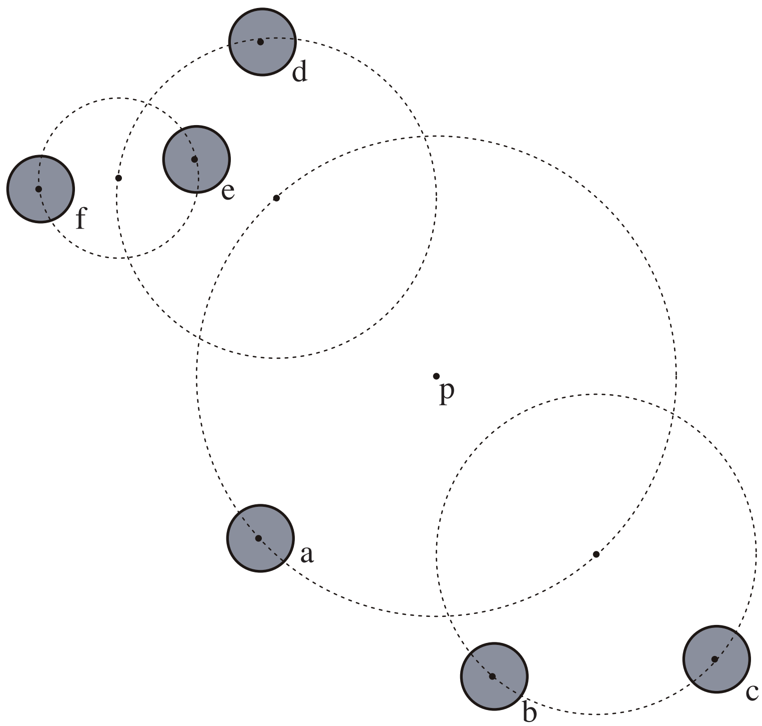

Figure 1: $\{a, \{b, c\}, \{d, \{e, f\}\}\}$ forms a nested circle around p

instead of '$F\{a, \{b, c\}, \{d, \{e, f\}\}\}$'. If we did this, the new parenthesis-embellished predicate would play just as essential an explanatory role in our theory as non-symmetric predicates could possibly play in any theory. The parenthesis-embellished predicate isn't absolutely indispensable, since it can be replaced with an ordinary predicate of sets. But of course non-symmetric predicates can also be dispensed with in favour of symmetric predicates applicable to sets; and as I've already shown (pp. 34–35 above), set theory itself can be developed without reliance on non-symmetric predicates. Thus, there is no principled distinction between the roles non-symmetric predicates and parenthesis-embellished predicates can play in scientific explanation: if the former are in fact more useful in science, this is just a contingent fact about the laws of the actual world.

Our intuition that non-symmetric relations are possible turns out to be a product of the linear structure of human language. If our languages had a different sort of structure, we would have had entirely different intuitions; and there is no clear sense in which our actual languages are objectively superior to those other possible languages. It seems to me that this consideration should greatly reduce our confidence in the intuition. Universals, if they exist, are *fundamental* entities, not mere projections onto the world of idiosyncratic facts about human language. We should, therefore, be very tentative in our intuitions about the possibility of universals corresponding to familiar sorts of predicates; we should always be sensitive to the possibility that these intuitions will be defeated by the 'rigorous programme for identifying and eliminating the illicit influence of linguistic structures on our theory of universals' which Williamson (1985, p. 262) rightly advocates, and which I have been attempting to carry out in this paper.

10 Economy

My argument so far has depended entirely on the premise that there are no brute necessities. In this final section, I will consider the question what those who deny this premise should think about the issue of non-symmetric relations.

If we cannot choose between different systems of primitive predicates by eliminating those which entail that there are brute necessities, how else are we make this choice? Do we suspend all judgment on the matter, or simply believe whatever we find most appealing? If we are to have any useful debate on the question, we need some objective basis for comparing systems. A natural suggestion is that *economy* should play this role. Ceteris paribus, we should believe the *simplest* system, where the simplicity of a system is determined by such factors as the number of primitive predicates it has, the number of arguments taken by those predicates, and the simplicity of its system of brute necessities (i.e. of the simplest set of axioms sufficient to entail all necessary and a priori truths involving only the primitive predicates of the system).

It seems to me that if this sort of economy matters at all, it counts strongly against all but three of the systems of primitive predicates which I have considered in this paper: systems D and E from section 7, and the simple theory of states of affairs which I described in section 2 as an example of a system which rules out non-symmetric relations. (Recall that this system has just two primitive predicates, 'is a particular component of' and 'is a universal component of'.) All the other systems I have considered have primitive predicates specially adapted for talking about *binary* relations. But surely it would be unacceptably arbitrary to rule out the possibility of relations of higher degree than two, just as it would be unacceptably arbitrary to rule out the possibility of relations of higher degree than seventeen. That there are no such arbitrary limitations on the space of possibilities is one of our firmest modal intuitions.[35] So our theory of binary relations must carry over, mutatis mutandis, to relations of higher degree. This means that if we adopt as primitive any predicates which are specifically adapted for talking about binary relations, we will end up having to recognise infinitely many primitive predicates, with ever-increasing numbers of argument places. And since each of these predicates will be governed by some brute necessities, the set of brute necessities will not even be finitely axiomatisable.[36]

[35]See Bricker 1991.

[36]Moreover, the more arguments a primitive predicate takes, the more complex the system of brute necessities involving that primitive predicate will be. We have already met one example of this: as n increases, the length of $\textsc{Paradigms}_n$ increases factorially. Similarly, if we take 'bears' as primitive and accept CONVERSES as a brute necessity, we will also have to accept as independent, axiomatic brute necessities all sentences of the form

$$\forall r \exists r' (x_1 \dots x_n \Delta r' \equiv x_{k_1} \dots x_{k_n} \Delta r)$$

Of the three systems that remain when the ones with infinitely many primitive predicates have been ruled out, the simple theory of states of affairs is clearly the most economical. Its primitive predicates take fewer arguments: it makes do with a two-place predicate ('x is a particular component of s') where they have, respectively, a three-place predicate ('x occupies argument place α of s') and a four-place predicate ('a is a component of s_1 in the same way that b is a component of s_2'). More importantly, it requires much less in the way of brute necessities. I don't think it lets us do without brute necessities altogether: for example, we will probably have to accept the claim that *no state of affairs has more than one universal component* (C8) as a brute necessity. But for any sentence which the simple theory of states of affairs forces us to accept as a brute necessity, there will be a corresponding brute necessity in each of the other two systems. By contrast, if we adopt the simple theory of states of affairs, we have no need for any brute necessities corresponding to the other axioms listed in section 7 (D2–D4 and E2–E6).

Each of these systems of primitive predicates admits of some further simplifications. For example, we could make the simple theory of states of affairs even simpler by making do with just one two-place primitive predicate, 'is a component of', analysing 'x is a universal component of y' as 'x is a component of y and x is a universal', and 'x is a particular component of y' as 'x is a component of y and x is not a universal'.[37] The predicate 'is a universal' which occurs in these analyses could be taken as primitive, or analysed somehow in terms of 'is a component of'. For example, following a suggestion of Russell's, we might

for each of the $n! - 1$ nontrivial permutations $k_1 \ldots k_n$ of the first n positive integers.

[37]These analyses rule out the possibility of *higher-order* universals, i.e. universals which are sometimes instantiated by other universals. If we were unhappy with this consequence, we might try analysing 'x is a universal component of y' as 'x is a component of y that is of higher order than any other component of y', taking 'x is of higher order than y' as a new primitive predicate.

analyse 'x is a universal' as 'all the states of affairs of which x is a component have the same number of components' (cf. Whitehead and Russell 1910, vol. I, pp. xix–xx).[38] Simplifications of a similar sort can be made in systems D and E.[39] But these simplifications do not affect the basic terms of the comparison: it is still true that if we want to allow for non-symmetric relations, we must do so at the cost of making our system of primitive predicates considerably less economical.

None of these systems is very economical with *ontological categories*, since they all require us to believe in states of affairs as well as particulars and universals. But there is another

[38]MacBride (1999) objects that this analysis incorrectly entails the impossibility of *multi-grade* relations. But this is hardly a fatal objection. Rather more counterintuitively, the analysis entails that every particular necessarily bears some relation to some other particular.

A different approach, not subject to these objections, is to analyse 'x is a universal' as 'x is a component of itself', or alternatively as 'x is not a component of itself'. But these proposals have an air of trickery that make them hard to take seriously. In part, this is because it seems to be much harder to accept 'everything that has components has exactly one component that is a component of itself' as a brute necessity than it is to accept 'everything that has components has exactly one universal component' as a brute necessity. The latter claim, at least, intuitively strikes us as necessary, while the former certainly doesn't.

In fact, no completion of the schema 'everything that has components has exactly one component that ... ' involving just the predicate 'is a component of' seems like a very good candidate to be a brute necessity. Thus, if we take 'is a component of' to be the only primitive predicate, we may well end up agreeing with Ramsey (1925) that we cannot know a priori that the components of states of affairs can be divided into two classes different enough from one another to be appropriately labelled as 'particulars' and 'universals', even if we are in principle prepared to posit brute necessities.

[39]Some examples: (i) We could simplify system E by analysing 'x is a universal' as 'whenever x enters into a state of affairs, it does so in the same way' ($\forall s_1 s_2 (x s_1 \mathsf{A} x s_1 \wedge x s_2 \mathsf{A} x s_2 \supset x s_1 \mathsf{A} x s_2)$). (ii) We could simplify system D by analysing 'x is a universal component of s' as '$x \underset{x}{\mathsf{C}} s$' or as '$x \underset{s}{\mathsf{C}} s$'. (iii) We could analyse the three-place primitive predicate of system D in terms of the two-place primitive predicates of the simple theory of states of affairs. For example, we could take argument places to be *properties* of the things that "occupy" them, analysing '$x \underset{\alpha}{\mathsf{C}} s$' as 'there is an s' with x as its sole particular component and α as its sole universal component, and s' is a particular component of s'.

highly economical system—to my mind, perhaps the most appealing of all those I have considered—which lets us avoid commitment to states of affairs. In this system there is just one primitive predicate, '...holds among ...', taking one singular and one plural term as arguments.[40] This system clearly rules out non-symmetric relations: 'x bears r to y' can only plausibly be analysed as 'r holds among x and y', which is of course logically equivalent to 'r holds among y and x'.[41] It's hard to say how this system should be compared with the others as regards the simplicity of its primitive predicates. Does a primitive predicate which takes one singular and one plural argument contribute more to the complexity of a system than a primitive predicate which takes two singular arguments, and if so, how much more?[42] But by doing away with the ontological category of states of affairs, this system clearly does better than any of the others as regards the simplicity of its system of brute necessities. To take just one example, we now need no counterpart of the principle that every state of affairs has just one universal component.[43]

[40]Many philosophers and linguists have endorsed some general strategy for "analysing away" plural expressions: if they are right, predicates taking plural arguments cannot be primitive. To accept such a predicate as primitive, one would have to adopt the view—defended by Boolos (1984), and more recently by Oliver and Smiley (2001)—that plural terms are perfectly clear as they are, and stand in no more need of reduction or explanation than singular terms.

[41]Armstrong (1997, p. 91) suggests this way of understanding what it is for something to bear a symmetric relation to something else; he attributes the idea to Lewis.

[42]On the numerical measure of complexity described by Goodman (1951, chapter 3), any system one of whose primitive predicates takes a plural argument would be *extremely* complex—more complex than any system whose predicates take only singular arguments. But this judgment strikes me as quite wrong. At least, it certainly seems much *easier to believe* that a predicate that takes one singular and one plural argument is primitive than that, say, a seventeen-place predicate is primitive.

[43]This system can be simplified still further in the same way as the simple theory of states of affairs. Instead of taking 'r holds among the xs' as primitive, we might analyse it as 'r is a universal, and r and the xs are bound together', taking the predicate 'are bound together', which takes just one plural term as its argument, as our new primitive. 'x is a universal' might be taken as primitive, or analysed in terms of 'are bound together'—perhaps, in the

Thus, even if I am wrong that there are no brute necessities, considerations of economy count strongly in favour of the thesis that all relations must be symmetric. And there may be other things to be said in favour of the thesis as well.[44] Of course, these advantages must be weighed against the disadvantage of conflicting with our intuition that non-symmetric relations are possible. I won't presume to say which way the balance should tilt. But I hope that my discussion has made it plausible that the disadvantage is not as weighty as it may initially have seemed.[45]

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style of Russell, as 'whenever the ys are bound together, and the zs are bound together, and x is among both the ys and the zs, the ys and the zs are equinumerous'.

[44]David Lewis (1986; 1992, p. 200) objects to the theory of states of affairs on the grounds that it violates the principle of *uniqueness of composition*, according to which it is impossible for two things to have exactly the same components. Certainly a believer in states of affairs who also believes in the possibility of relations which are neither symmetric nor anti-symmetric is under considerable pressure to deny this principle. If a bears a non-symmetric relation r to b and b bears r to a, there must be two states of affairs whose only components are a, b and r. But if we give up on non-symmetric relations, and adopt the simple theory of states of affairs or one of its simplifications, there is no motivation at all for positing such failures of uniqueness.

Lewis also objects that the believer in states of affairs must accept a "non-mereological mode of composition", i.e. a notion of "componenthood" distinct from the notion of "parthood" which Lewis takes to be governed by the axioms of Mereology. But this is not inevitable: if we wanted to, we could analyse 'x is a component of s' as 'x is simple, and s is a state of affairs, and x is part of s', taking 'is a state of affairs' as a new primitive predicate. Thus, even though (according to Mereology) any object and any universal have a fusion, for the object to count as *instantiating* the universal, the fusion of the two must be a *state of affairs*, and both must be simple. (The restriction to simple things is needed to avoid having to count fusions of components of a state of affairs as further components of the state of affairs. The consequence that only simple things can instantiate or be instantiated is indeed surprising, but far from being entirely incredible.)

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