Are Quantities Qualitative?

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1. Qualitative versus haecceitistic properties

It would be nice to be able to provide an uncontroversial definition in other terms, or at least, introduce with uncontroversial examples. But I can't do either of these things, sorry.

- Least controversial examples of non-qualitative properties: *being identical to Obama*, *being in New York*…
- Candidates to be qualitative properties and relations: *being spherical, being blue*, *being metal*, *being wood, being a donkey, being a ship, being an earring, being a philosopher, love, identity, …*

2. It Ain't Necessarily Qualitative

Especially when we are dealing with "natural kind predicates", many of our general beliefs are open to empirical revision (cf. Putnam on 'All cats are animals'). This includes beliefs about qualitativeness.

Thought experiment: We find that sometimes a small spark is emitted when people touch each other. On further investigation it turns out that people can be grouped into a few non-overlapping sets, such that the spark is emitted when there is contact between members of different sets. We introduce predicates 'Type A person', 'Type B person' etc., which we can very reliably apply. Initially, it is natural to conjecture that there are some qualitative properties—rather natural ones—that these predicates express.

- But we might discover that what's really going on is that there are several gods, and each person is loved by exactly one god, and the gods have set things up so there's a spark when people loved by different gods touch.
  - Claim: if we discovered that, we should conclude that 'Type A', 'Type B' etc. had all along been expressing the haecceitistic properties *being loved by God A*, *being loved by God B*, etc.
- We might, alternatively, discover that what's really going on is that there is a certain natural relation R such that there's a spark whenever non-R-related people touch, and R just happens to behave as an equivalence relation on the people we have encountered so far. (Maybe we find this out by finding some new people on whom R doesn't behave as an equivalence relation.)
  - Claim: if we discovered that, we should conclude that 'Type A', 'Type B' etc. had all along been (a) quite vague, especially with regard to distant worlds, and (b) such that at least *some* of their precisifications were *massively haecceitistic* properties, e.g. of the

form 'Belonging to whichever maximal R-interrelated set contains more of the following people than any other such set: *a*, *b*, *c*….'.

- If we had got used to the predicates, we might have strong intuitions that conflict with these accounts of what it is to be Type A, Type B, etc.
  - We should not worry about fidelity to such intuitions when considering hypotheses about "the fundamental structure of the world".
  - Here's a Lewisian picture: a structure-hypothesis is something that just talks quantificationally about the pattern of perfectly natural properties and relations, e.g. 'There are exactly three perfectly natural properties, and no object instantiates all three of them'.
    - In evaluating such hypotheses we need only care about (i) simplicity, and (ii) fit with our evidence, narrowly conceived.
    - "Fidelity to intuition" is a big consideration only later, when we are thinking about the ramifications of our structure hypothesis for our beliefs expressed in other vocabulary.

3. Real world non-qualitativeness in folk biology

Take *being a donkey*. It's a live option that its haecceitistic in the following way: there are two organisms—Grandfather and Grandmother Donkey—such that to be a donkey is to be a descendent of them that meets certain further conditions.

- More plausibly: a larger collection of organisms, such that you have to be related in the right ways to *enough* of them…
- (I actually think that 'donkey' is pretty vague, in a way that probably infects 'Being a donkey is haecceitistic'. But let's let that pass.)

How does something like this get to be a live option?

- Folk biology tempts us to posit natural, qualitative "species-essences". But this expectation has not been borne out by scientific biology.

4. Some argument-schemas

To fix ideas, begin by instantiating F in the following schemas with is 'is a donkey' and R with 'shares a common ancestor'.

INTRA-WORLD SYMMETRY ARGUMENT

1. *SYMMETRY:* Every permutation that preserves all PNPs and their negations preserves all qualitative properties.
2. *SYMMETRIC POSSIBILITY:* Possibly: there is an F object x that is mapped by some PNP-preserving permutation to something that does not bear R to it.

3. *EXCLUSION*: Necessarily, any two F objects bear R to one another.
4. So F is haecceitistic.

INTER-WORLD SYMMETRY ARGUMENT

Definition: π is a *PNP-mapping* from w to w' iff its domain includes all the objects that exist at w, and its range includes all the objects that exist at w', and for every n-ary PNP r and objects $x_1 \ldots x_n$, $x_1 \ldots x_n$ instantiate r at w iff $\pi(x_1) \ldots \pi(x_n)$ instantiate r at w'.

1. *SUPERVENIENCE:* If π is a PNP-mapping from w to w, then for every qualitative property Q and object x, x is Q at w iff $\pi(x)$ is Q at w'.
2. *ROLE-SWAPPING:* There are possible worlds w and w', a PNP-mapping π from w to w ', and an object x such that x is F at both w and w' and $\pi(x)$ does not bear R to x at w'.
3. *EXCLUSION*: Necessarily, any two F objects bear R to one another.
4. So F is haecceitistic.

MODAL SYMMETRY ARGUMENT

This is an effort to capture the idea of the previous argument just using modal operators rather than bringing in talk of possible worlds and things having properties 'at' worlds. This is relevant in the present context since some authors [such as Dasgupta] suggest accounts that disrupt the standard equivalences between modal claims and world-theoretic claims.

1. *SUPERVENIENCE*:* If f maps every PNP to its extension, and an object x has a qualitative property Q, then for any permutation π of all the objects there are: necessarily, if the same things exist and $\pi \circ f$ maps every PNP to its extension, then $\pi(x)$ has Q.
2. *ROLE-SWAPPING*:* Possibly: some f maps every PNP to its extension, and there is an F object x and permutation π such that possibly (the same things exist, and $\pi \circ$f maps every PNP to its extension, and x is F, and π(x) does not bear R to x).
3. *EXCLUSION*: Necessarily, any two F objects bear R to one another.
4. So F is haecceitistic.

- Different interpretations of the 'necessarily' and 'possibly'—metaphysical; nomic; inner-sphere; "so long as nothing too weird is going on…".
- Worries about SYMMETRY and SUPERVENIENCE—maybe *being a severe recession* and *being a mild recession* are qualitative, even though recessions don't instantiate any PNPs?
  - Possible responses: deny that there are recessions; restrict to fundamental objects; expand list of PNPs to include appropriate 'construction' relations; replace 'PNP' throughout with 'PNP or construction relation'.

5. Instantiating the schemas: leftness and rightness

Take F to be *leftness* understood as a property of hand-shaped objects—HSOs—where 'hand-shaped object' abbreviates a distance-theoretic definition.[1]
Take R to be *congruence* (same-handedness), defined in distance-theoretic terms. (Such a definition is possible but a bit complex, so I won't try to write it down.)

Ways of accepting the conclusion that leftness is haecceitistic

(i) The implausible way: to be a left HSO is to be congruent to a certain *hand* ('Ol' Brother Left Hand'). Problem: any given left hand could have not been hand-shaped, and could (arguably) have been right instead of left.

(ii) Kant's way: to be a left HSO is to be congruent to a certain *spatial region*. Overcome the problems with (i) by embracing *geometric essentialism*: any two congruent spatial regions are necessarily congruent (if they exist?).

(iii)The messy way: In a possible world where most objects actually on Earth exist and instantiate a broadly similar pattern of PNPs, the left HSOs are the ones in whichever congruence-class contains most of the actual left HSOs. As we go to more distant possible worlds, vagueness becomes extreme.

Why believe SYMMETRIC POSSIBILITY and ROLE-SWAPPING in these instantiations?

- We could derive *SYMMETRIC POSSIBILITY* from the following claims:

  WEAK SYMMETRIC POSSIBILITY ('Anti-chiralism'): Possibly, there is a PNP-preserving permutation that interchanges some incongruent HSOs.

  EXHAUSTIVENESS: Of any two incongruent HSOs, exactly one is left.

- *WEAK SYMMETRIC POSSIBILITY* can in turn be argued for on the basis of: (i) any one of very many attractively simple structure-hypotheses (or a disjunction of them), plus (ii) an account of what the necessary and sufficient conditions for *incongruence* are if that structure-hypothesis is true, plus (iii) an account of the modal profiles of perfectly natural properties that guarantees the existence of appropriately symmetric possibilities. Call the structure-hypotheses that would sustain such an argument 'non-chiral'.
- *EXHAUSTIVENESS* is only moderately plausible. But if we doubt it, we could fall back on the modal judgment that the leftness of certain *particular* HSOs is consistent with the existence of a PNP-preserving permutation of mapping those HSOs onto things incongruent with them. And this is much more plausible: we could exist in a mirror-symmetric universe and still be much as we actually are, including having left hands.
- We could argue for *ROLE-SWAPPING* as follows:

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[1] Any "chiral" property or relation would do here: e.g. the four-place relation "x_1, x_2, and x_3 are in clockwise order as seen from x_4" among points (or among lines that all meet at a single point, or among vectors at a point).

Possibly, there is an ordinary left hand x, and a permutation that interchanges x with some incongruent HSO that has a *locally similar* PNP-profile.

If x is an ordinary left hand, then x could instantiate any possibly-instantiated PNP-profile *locally similar* to the one x actually instantiates, while still being a left hand.

Strategies for resisting the arguments

* Deny *SYMMETRY* and *SUPERVENIENCE*.
* Posit "chiral" PNPs that enter into the analysis of leftness. Several options:
  a) It's *metaphysically impossible* for a permutation that exchanges incongruent HSOs to preserve these PNPs.
  b) This is metaphysically possible (though maybe nomically impossible); but *SYMMETRIC POSSIBILITY* is still false because the HSOs in a world like that would be neither left nor right. (So we must give up *EXHAUSTIVENESS*.)
  c) Give up *EXCLUSION* (interpreted as involving metaphysical possibility).
* Keep a non-chiral structure-hypothesis. Say that leftness supervenes in some messy but qualitative way. Give up *EXHAUSTIVENESS*, and maintain that in mirror symmetric worlds there are no left or right HSOs.

A worry about higher-order contingentism

Many philosophers think that what haecceitistic properties there are is a contingent matter: if there had been different particulars, there would have been different haecceitistic properties. Given this picture, it should be possible for there to be incongruent HSOs even while all of the haecceitistic properties there *actually* are are coextensive with qualitative properties. This makes trouble for the combination of the view that leftness is haecceitistic with *EXHAUSTIVENSS* (Necessarily, exactly one of any two incongruent HSOs is left).

- Vagueness doesn't help!
- Insofar as the empirical case for a non-chiral structure-hypotheses holds up, higher-order contingentists should reject *EXHAUSTIVENESS*. But they can still endorse the fallback argument for *SYMMETRIC POSSIBILITY*.

6. Instantiating the schemas: positive and negative charge

Take F to be *having charge –1 Coulomb*.
Take R to be *having the same charge*.

- The main difference between this and the handedness instantiation is that certain particular non-chiral candidate packages of PNPs are historically influential, whereas it seems an open scientific question whether a PNP-preserving permutation could interchange oppositely charged objects.

- The answer to this question turns out to be intimately bound up with handedness considerations, because of the CPT theorem.

7. Determinate lengths

Take F to be *being one metre long*.
Take R to be *being equally long as*. (Or: *not being twice as long as*.)

Ways of accepting the conclusion that being one metre long *is haecceitistic*

(i) The implausible way: to be one metre long is to be the same length as S (the standard metre).

(ii) The "Kantian" way: to be one metre long is to b the same length as a certain particular region of *space*. Overcome problems with (i) by adopting geometric essentialism. This has not been as popular as the corresponding claim about chiral properties; but it seems just as good.

(iii)The messy way: as before.

- Argument for *SYMMETRIC POSSIBILITY*:

  WEAK SYMMETRIC POSSIBILITY ('Comparativism about distance') Possibly, there are objects that stand in all positive length-ratios, and there is a PNP-preserving permutation that maps everything with a length to something twice as long as itself. (Or more specifically: it is possible that this is the case when all there is is empty, flat space.)

  EXHAUSTIVENESS: Necessarily, in any collection of things that stand in all positive length-ratios, one of them is 1m long.

- What could we take the PNPs to be, if we want to reject *SYMMETRIC POSSIBILITY* and *ROLE-SWAPPING*? We seem faced with two bad choices. We could either claim that there are *continuum many* PNPs, or we could *arbitrarily privilege* a few of them.
  - Why not have continuum many PNPs? It looks hard to fit in to a good epistemological story about the role of simplicity in evaluating structure-hypotheses.
  - Do the laws mention them specifically? If so we have continuum many laws. Do the laws merely quantify over them? But then why not confine the PNPs to the vocabulary that's actually needed to state the laws?

8. Determinate masses

This is mostly the same, but the 'Kantian' option bifurcates:

- First version: there is a certain space of points, just as real and concrete as physical space, and a relation of 'occupation' that massive objects bear to those points, such that to have mass 1kg is just to occupy a particular one of these points. (Cf. Arntzenius and Dorr, 'Calculus as Geometry').

- Second version: the fundamental relations relevant to mass are 'mixed' relations between objects, points or regions of space, and instants or intervals of time, and to have mass 1kg is to bear these relations to some specific points, regions, instants or intervals. (Cf. Baker, 'Mixed Relations').

9. Determinate speeds

Take F to be *having a relative speed of 1 m/s* (a relation between two timelike paths that intersects at just one point).
Take R to be *having the same relative speed as* (a four-place relation between two paths intersecting at a point and two other paths intersecting at a perhaps different point). Or: *not having twice the relative speed of*.

- In this instantiation, *SYMMETRIC POSSIBILITY* and *ROLE-SWAPPING* are just not plausible, because of something we have learnt from relativity theory about the structure of spacetime.
- There are many candidate short lists of PNPs in terms of which we could provide plausible reductions of all of the continuum many possible relative speeds. E.G. it turns out that you can do the whole thing with the single symmetric binary relation *timelike-relatedness* among spacetime points. (Robb, Latzer)
- Note that such reductions will plausibly entail that it is *metaphysically necessary* that the value of c—the relative speed of any two paths one of which is timelike and the other lightlike—is what it is, in m/s. This is plausible.

10. The real world: natural units

Planck's observation: there are only two systems of units for distance, duration, mass and charge in which c (Einstein's constant) = G (Newton's constant) = $\hbar$ (the reduced Planck constant) = C (Coloumb's constant) = 1. They are the same except for a charge flip.

- It's not *obvious* that we should conclude that the values of these constants in ordinary units are metaphysically necessary. But it seems to me that this is attractive for the same reason as in the case of c.
- Even though many claims having to do with the combination of gravitation and quantum theory are speculative, I think it's safe to conclude from Planck's observation that *even if the list of instantiated PNPs is short*, our determinable mass, length, duration, and absolute-value-of-charge predicates express qualitative properties, and have pretty precise intensions across nomically possible worlds.