‘How Fine-grained is Reality?’

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1. To be F is to be G

Reflexivity: To be F is to be F.
Symmetry: If to be F is to be G, to be G is to be F.
Transitivity: If to be F is to be G and to be G is to be H, to be F is to be H.

2. Formalisation using a variable binder

$\mathrm{Vixen}(x) \doteqdot_x (\mathrm{Female}(x) \wedge \mathrm{Fox}(x))$

- Allowing $\doteqdot$ to bind multiple variables, or none.
- Tricky case: $\phi \doteqdot_{v_1 \ldots v_n} \psi$ when some of $v_1 \ldots v_n$ are not free in ϕ or ψ.

3. Formalisation using complex predicates

$\mathrm{Vixen} \equiv \lambda x[\mathrm{Female}(x) \wedge \mathrm{Fox}(x)]$

Translations: $\phi \doteqdot_{v_1 \ldots v_n} \psi$ becomes $\lambda v_1 \ldots v_n[\phi] \equiv \lambda v_1 \ldots v_n[\psi]$

$\mathrm{F} \equiv \mathrm{G}$ becomes $\mathrm{F}(v_1, \ldots, v_n) \doteqdot_{v_1 \ldots v_n} \mathrm{G}(v_1, \ldots, v_n)$

4. Transparency

Transparency for a binary sentential operator $\odot$ (such as $\wedge$).

My version: $((\phi_1 \doteqdot_{v_1 \ldots v_n} \phi_2) \wedge (\psi_1 \doteqdot_{v_1 \ldots v_n} \psi_2)) \rightarrow (\phi_1 \odot \psi_1) \doteqdot_{v_1 \ldots v_n} (\phi_2 \odot \psi_2)$

Transparency for $\doteqdot$ entails:

Dropping Variables: $(\phi \doteqdot_{v_1 \ldots v_{n+1}} \psi) \rightarrow (\phi \doteqdot_{v_1 \ldots v_n} \psi)$

But there is alternative “transparency” principle that does not entail Dropping Variables:
$((\mathrm{F}_1 \equiv \mathrm{F}_2) \wedge (\mathrm{G}_1 \equiv \mathrm{G}_2)) \rightarrow \lambda v_1 \ldots v_n(\mathrm{F}_1(v_1 \ldots v_n) \odot \mathrm{G}_1(v_1 \ldots v_n)) \equiv \lambda v_1 \ldots v_n(\mathrm{F}_2(v_1 \ldots v_n) \odot \mathrm{G}_2(v_1 \ldots v_n))$

5. Identifications and higher-order logic

Predicative type theory: e is a type. $\langle t_1, \ldots, t_n \rangle$ is a type for any types $t_1 \ldots t_n$ ($n \geq 0$). Each variable and constant has a type, shown as a superscript on first occurrence. When β, $\alpha_1, \ldots, \alpha_n$ are terms of types $\langle t_1, \ldots, t_n \rangle$, $t_1, \ldots, t_n$, $\beta(\alpha_1, \ldots, \alpha_n)$ is a formula (a term of type $\langle\rangle$).

COMP $\exists \zeta^{\langle t_1, \ldots, t_n \rangle}(\zeta(v_1{}^{t_1}, \ldots, v_n{}^{t_n}) \doteqdot_{v_1 \ldots v_n} \phi)$

COMP^{r} $\exists \zeta^{\langle t_1, \ldots, t_n \rangle}(\zeta(v_1{}^{t_1}, \ldots, v_n{}^{t_n}) \doteqdot_{v_1 \ldots v_n} \phi)$ where all of $v_1 \ldots v_n$ occur free in Φ

Indiscriminability $\forall \zeta^{\langle t_1, \ldots, t_n \rangle} \forall \chi^{\langle t_1, \ldots, t_n \rangle}(\zeta \equiv \chi \rightarrow \forall \xi^{\langle\langle t_1, \ldots, t_n \rangle\rangle}(\xi(\zeta) \leftrightarrow \xi(\chi)))$

6. Tautological Substitution

Tautological Substitution: $\phi \doteqdot_{v_1 \ldots v_n} \psi$ whenever $\phi \leftrightarrow \psi$ is a tautology (theorem of classical propositional logic).

- Given transparency for $\wedge$, $\vee$, $\neg$, this is equivalent to the combination of the principles in Table 1.
- Defining $\Box\phi$ as $\top \doteqdot \phi$.

7. Some weakenings of Tautological Substitution

8. Non-circularity

GRUEDEF	To be grue is to be either green and observed or blue and not observed.
GREENDEF	To be green is to be either grue and observed or bleen and not observed.

(1) x is perpendicular to $y \doteqdot_{x,y} x$ is perpendicular to y and not perpendicular to any line that intersects y.

$$\textbf{NC}_{\langle\rangle} \quad (\zeta^{\langle\langle\rangle\rangle}(p^{\langle\rangle}) \doteqdot p) \rightarrow (\zeta(q^{\langle\rangle}) \doteqdot_q q \vee \zeta(q) \doteqdot_q p)$$

To endorse $\text{NC}_{\langle\rangle}$, we must deny *Generation*, which says that all 'functional relations between propositions' correspond to genuine *operators*:

Generation $\quad \forall p^{\langle\rangle} \forall q^{\langle\rangle} \forall r^{\langle\rangle} (\zeta^{\langle\langle\rangle,\langle\rangle\rangle}(p,q) \wedge \zeta(p,r)) \rightarrow q \doteqdot r)) \rightarrow \exists \chi^{\langle\langle\rangle\rangle}(\forall p \forall q(\zeta(p,q) \leftrightarrow (q \doteqdot \chi(p))).$

9. Defining priority in terms of identifications

General strategy: define a 'weak priority' connective $\preccurlyeq$ taking arguments of various types. α is "strictly prior" to β iff $\alpha \preccurlyeq \beta$ and not $\beta \preccurlyeq \alpha$.

First stab:	(type $\langle\rangle$)	$\alpha^{\langle\rangle} \preccurlyeq \beta^{\langle\rangle} =_{\text{df}} \exists \zeta^{\langle\langle\rangle\rangle}(\zeta(\alpha) \equiv \beta)$
	(more generally)	$\alpha^{t_0} \preccurlyeq \beta^{\langle t_1,\ldots,t_n\rangle} =_{\text{df}} \exists \zeta^{\langle t_0,\ldots,t_n\rangle}(\zeta[\alpha] \equiv \beta)$

where '$\zeta[\alpha]$' abbreviates '$\lambda v_1^{t_1},\ldots,v_n^{t_n}[\zeta(\alpha,v_1,\ldots,v_n)]$'.

Problem with first stab: COMP requires us to posit 'forgetful' operators that map everything onto some one thing, so $\alpha \preccurlyeq \beta$ will always be true!

Second stab: $\quad \alpha^{t_0} \preccurlyeq \beta^{\langle t_1,\ldots,t_n\rangle} =_{\text{df}} \exists \zeta^{\langle t_0,\ldots,t_n\rangle}(\zeta[\alpha] \equiv \beta \wedge \neg\text{Forgetful}(\zeta))$

Problem with second stab: The composition of two non-forgetful operators can be forgetful, so $\preccurlyeq$ so-defined is not transitive. *Forgetfulness* turns out to be inconsistent with COMP.[1]

- Possible solution: restrict COMP to COMP^{r} and stick with the first stab?
- Alternatively: somehow define 'hereditary non-forgetfulness', and use that instead of non-forgetfulness in the definition?

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[1] Let $\theta^{\langle\langle\rangle\rangle} \equiv \lambda p^{\langle\rangle}(Q^{\langle\rangle})$, $\zeta^{\langle\langle\langle\langle\rangle\rangle\rangle\rangle} \equiv \lambda \alpha^{\langle\langle\langle\rangle\rangle\rangle}(\alpha(\theta))$, and $\chi^{\langle\langle\rangle,\langle\langle\rangle\rangle\rangle} \equiv \lambda p^{\langle\rangle}\beta^{\langle\langle\rangle\rangle}(\beta(p))$. Note that although θ is forgetful, neither ζ nor χ is. But the result of composing ζ and χ is $\lambda p(\zeta[\chi[p]]) \equiv \lambda p(\chi[p](\theta)) \equiv \lambda p(\chi(p,\theta)) \equiv \lambda p(\theta(p)) \equiv \lambda p(Q) \equiv \theta$, which is forgetful.

	Angell	FDE	Parry	Fine	Me
Commutativity a. $(P \wedge Q) \doteq (Q \wedge P)$ b. $(P \vee Q) \doteq (Q \vee P)$	yes	yes	yes	yes	yes
Associativity a. $P \wedge (Q \wedge R) \doteq (P \wedge Q) \wedge R$ b. $P \vee (Q \vee R) \doteq (P \vee Q) \vee R$	yes	yes	yes	yes	?
Double negation $P \doteq \neg\neg P$	yes	yes	yes	*	yes
De Morgan a. $\neg(P \wedge Q) \doteq (\neg P \vee \neg Q)$ b. $\neg(P \vee Q) \doteq (\neg P \wedge \neg Q)$	yes	yes	yes	yes	yes
Distributivity a. $P \wedge (Q \vee R) \doteq (P \wedge Q) \vee (P \wedge R)$ b. $P \vee (Q \wedge R) \doteq (P \vee Q) \wedge (P \vee R)$	yes	yes	yes	**	no
Idempotence a. $P \simeq P \wedge P$ b. $P \simeq P \vee P$	yes	yes	yes	***	no
Weakening a. $P \doteq P \wedge (P \vee Q)$ b. $P \doteq P \vee (P \wedge Q)$	no	yes	no	no	no
Explosion a. $Q \wedge \neg Q \doteq P \wedge (Q \wedge \neg Q)$ b. $Q \vee \neg Q \doteq P \vee (Q \vee \neg Q)$	no	no	no	no	?
Parry 1 a. $(P \wedge Q) \doteq (P \wedge Q) \vee (Q \wedge \neg Q)$ b. $(P \vee Q) \doteq (P \vee Q) \wedge (Q \vee \neg Q)$	no	no	yes	no	no
Parry 2 a. $P \vee (P \wedge (Q \wedge \neg Q)) \doteq P \vee (Q \wedge \neg Q)$ b. $P \wedge (P \vee (Q \vee \neg Q)) \doteq P \wedge (Q \vee \neg Q)$	no	no	yes	no	?

Table 1: Weakenings of Tautological Substitution

*: *Double Negation* holds in Fine's theory of 'bilateral propositions', but fails in the theory of 'exclusionary negation'.

**: *Distributivity* (a) holds in all of Fine's systems, while *Distributivity* (b) holds only in the theories of 'regular' propositions.

***: *Idempotence* (a) holds in Fine's theories of regular and closed propositions.

Define $P \leq_c Q$ iff $Q \equiv P \wedge Q$, and $P \leq_d Q$ iff $P \equiv P \vee Q$.

Given *Commutativity*, $\leq_c$ and $\leq_d$ are both antisymmetric.

Given *Associativity*, and the transparency of $\wedge$ and $\vee$, $\leq_c$ and $\leq_d$ are both transitive.

Given *Weakening*, $\leq_c$ and $\leq_d$ are equivalent.