

# Of Numbers and Electrons

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## 1 The indispensability argument

‘Some theories that logically entail the existence of numbers are better, as candidate explanations of certain facts about the concrete world, than any theories that are consistent with the non-existence of numbers.’

Why should we think that theories like AS-IF are bad as explanations?

AS-IF As far as the concrete world is concerned, it is just as if  $T$ .

## 2 An argument by analogy

Since inference to the best explanation gives us good reason to believe quite a lot about the unobservable world, theories like *this* must be bad:

BAD As far as observable matters are concerned, it is just as if  $T$ .

Argument: BAD and AS-IF are alike in form, hence alike in being bad as explanations.

## 3 Responses to the challenge

(i) Field’s program.

(ii) My response. Theories like *this* are *not* bad, and can be used to explain whatever facts about the concrete world were supposed to be explained by  $T$ :

NEC Necessarily, if  $M$  and the concrete world is intrinsically just as it actually is, then  $T$ .

What’s the relevant difference between AS-IF and BAD, on the one hand, and NEC on the other? The former theories use possibility-operators, whereas the latter uses a necessity-operator.

## 4 The epistemological difference between $\exists$ and $\forall$

Some theories that look to be bad in just the same way as BAD, but use existential quantifiers rather than possibility-operators:

- (1) For some function  $e$  from objects to numbers,  $e$  satisfies such-and-such laws for electric charge.
- (2) For some function  $\mathbf{x}$  from spacetime points to  $\mathbf{R}^4$  which respects the topological facts,  $T(\mathbf{x})$  (cf. geometrical conventionalism).
- (3) There are some particles  $p_1$ , and some particles  $p_2 \dots$  and some particles  $p_n$ , such that there is such-and-such attractive force between each of  $p_1$  and each of  $p_2$ , and such-and-such repulsive force between each of  $p_1$  and each of  $p_3$ , and  $\dots$ .
- (4) There is some model  $M$  of  $T$  which accurately represents the facts about the spatiotemporal arrangement of the centers of mass of atoms.

By contrast, theories which use *universal* quantification to avoid positing unwanted fundamental structure seem perfectly fine:

- (5) For every function  $\mathbf{x}$  from spacetime points to  $\mathbf{R}^4$  that respects the underlying geometric facts,  $T(\mathbf{x})$ .

A crude model: the badness of a theory increases with the number of symbols it takes to express the theory, in a language with appropriately fundamental predicates. But the rate of increase is greater within formulae governed by existential quantifiers (in positive contexts, or by universal quantifiers in negative contexts).

## 5 The epistemological difference between $\diamond$ and $\square$

Even if we reject possible-worlds analyses of modal operators, we can recognise important logical analogies between necessity operators and universal quantifiers, and between possibility operators and existential quantifiers.

Since BAD is bad in just the same way as (1)–(4), its badness should be attributed to the distinctive role played in it by a possibility-operator (which is logically analogous to an existential quantifier), rather than to any feature it shares with NEC.

## 6 A cunning attempt to restore the analogy

Can scientific anti-realists find theories which stand to the unobservable as NEC stands to the abstract? Here’s a suggestion:

- (6) Necessarily, if the laws are as simple as they could be consistent with facts about observable matters,  $T$ .

But the notion of simplicity is (a) really quite complex, and (b) used here in a context governed by a possibility operator, where its complexity counts for more.

## 7 The conservativeness claim

To explain anything at all, NEC must be combined with a “conservativeness” claim:

It is possible that mathematical axioms  $M$  are true and the concrete world is intrinsically just as it actually is.

Won’t this be bad for the same reasons as AS-IF and BAD?

*Cautious response:*  $M$  is far simpler than the full theory  $T$ , so we’ve made progress.

*Ambitious response:* The conservativeness claim is a logical truth, and so doesn’t need to be counted as part of the total theory in calculating badness. It does not depend for its justification on its role in scientific explanations.