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FACTOR SHARES AND THE CHIPMAN CONDITION

Tapan Mitra and Ronald W. Jones

Introduction

Counterexamples often prove to be daunting obstacles to further research. Such was the case for two independent enquiries a quarter of a century ago as to the impact of price changes in commodity markets on the distribution of income. In the October 1969 issue of the *International Economic Review*, John Chipman provided a condition on distributive factor shares which, if satisfied for an economy with three factors of production engaged in producing in non-joint fashion three distinct commodities, sufficed to guarantee that any single commodity price rise would cause the return to the productive factor uniquely assigned to that sector to increase by relatively more than the commodity price. This he dubbed the weak form of the Stolper–Samuelson Theorem. However, Chipman also provided a counterexample to show that his condition would not be sufficient if the number of factors and goods were to equal four (Chipman 1969). A scant seven pages further on in this issue of the *Review* Murray Kemp and Leon Wegge (1969) also exhibited a counterexample for the case $n = 4$ to the proposition that their condition on ratios of factor shares would suffice to guarantee the strong form of the Stolper–Samuelson condition – a form in which a price rise causes all factors save the "intensive" one assigned to that sector to suffer losses in their returns.

In reacting to counterexamples it is sometimes appropriate to abandon the approach taken in the initial conjecture. However, there are occasions in which the criteria originally suggested prove to be useful, but need somewhat to be strengthened to yield the sought-for results. This, we would argue, is indeed the case for these two propositions. In Jones, Marjit and Mitra (1993) a condition sufficient to establish the strong Stolper–Samuelson (1941) result was derived, building upon the earlier criterion of Kemp and Wegge. In this chapter we establish that the original Chipman criterion can also be strengthened to yield conditions which suffice to guarantee the weak form of the Stolper–Samuelson result. Indeed, we go further in establishing conditions which ensure a "strong version" of the weak form: The factor identified with a commodity will have its return increased by relatively more than any other factor if the price of the commodity rises.

The following section of this chapter presents more formally the original criteria suggested in these two papers, as well as the statement of the revised Kemp–Wegge condition. In the next section we prove the sufficiency of a revised form of the Chipman condition. As the proof illustrates, even this revised form can be weakened somewhat. This alteration is noted in the fourth section in order to confirm that Chipman's original condition indeed is sufficient for the 3×3 case. That even this revised condition is not necessary if n equals 4 is illustrated in the fifth section for a special class of production structures in which each industry displays a distribution of factor shares (a share rib) that has the same shape as every other industry (with an appropriate permutation of factors). Finally we argue in the concluding section that the form of the Stolper–Samuelson Theorem with which Chipman was particularly concerned (the "weak" form) is perhaps of more interest and relevance than the strong form.

The two propositions

Both Chipman (1969) and Kemp and Wegge (1969) examine an economy producing n commodities in independent non-joint linearly homogeneous activities using n distinct primary inputs in competitive markets so that market prices reflect unit costs. They each were concerned to establish criteria defining what was meant by the assertion that every industry was associated with a unique factor of production which it used most intensively. Let θ_{ij} denote the distributive share of factor i in industry j. Kemp and Wegge made the strong assumption that industry i so intensively utilized factor i that the ratio of i's share in industry i to that of any other input in industry i (say θ_{ri}) would exceed the corresponding share ratio in any other industry (say j). Thus their factor intensity condition (FI) stated:

$$\text{(FI): } \frac{\theta_{ii}}{\theta_{ri}} > \frac{\theta_{ij}}{\theta_{rj}} \quad \forall i, r, j, \text{ with } i \neq r, j \tag{1}$$

Chipman's condition was less severe and, indeed, is implied by condition (1) (as proved by Kemp and Wegge). It only requires that factor i's share in industry i would exceed its share in any other industry. Call this the Factor Share condition (FS):

$$\text{(FS): } \theta_{ii} > \theta_{ij} \quad \forall i, j, \text{ with } i \neq j \tag{2}$$

Jones, Marjit, and Mitra (1993) established that a strengthened version of Kemp and Wegge's condition (1) (labeled the Strong Factor Intensity condition – SFI) would suffice to guarantee the strong form of the Stolper–Samuelson Theorem:

$$\text{(SFI)}: \left(\frac{\theta_{ii}}{\theta_{ri}} - \frac{\theta_{ij}}{\theta_{rj}}\right) > \sum_{s \neq i,j} \left| \frac{\theta_{si}}{\theta_{ri}} - \frac{\theta_{sj}}{\theta_{rj}} \right| \quad \forall i, r, j, \text{ with } i \neq r, j \text{ and } r \neq j \tag{3}$$

For $n \geq 4$ Kemp and Wegge illustrated in their counterexample that the positivity of the left-hand side of (3) would not suffice. Additionally, bounds must be put on the discrepancy among purely off-diagonal ratios of factor shares.[1] Thus *two* features of factor intensity comparisons emerge as important – each industry must use a unique factor most intensively, and there must not be too much variation among the shares of the unintensively-used factors in any industry compared with the extent of intensity of the most intensive factor. In this chapter we consider a strengthened form of Chipman's Factor Share condition (2) (which we label SFS, the Strong Factor Share condition):

$$\text{(SFS)}: (\theta_{ii} - \theta_{ij}) > \sum_{r \neq i,j} |\theta_{ri} - \theta_{rj}| \quad \forall i, j, \text{ with } i \neq j \tag{4}$$

As we prove now, this condition is sufficient to establish the weak form of the Stolper–Samuelson Theorem.

Implications of the strengthened Chipman condition

We start by assuming condition (4) and letting the price of some commodity, k, rise with all other commodity prices constant. Our object is to prove that $\hat{w}_k$ exceeds $\hat{p}_k$, where a "^" over a variable indicates a relative change ($\hat{x} \equiv dx / x$). Let Q denote the set of all $i \neq k$, and given $\hat{p}_k > 0$ let s be the factor in Q which has experienced the greatest relative price rise and t be the factor in Q which has experienced the least relative price rise. Thus

$$\hat{w}_s \equiv \max_{i \in Q} \hat{w}_i \; ; \; \hat{w}_t \equiv \min_{i \in Q} \hat{w}_i$$

If we can prove that $\hat{w}_k$ exceeds $\hat{w}_s$, we are finished, for $\hat{p}_k$ must be a positive weighted average of all the $\hat{w}_j$, and therefore less than the largest of them. In proving this proposition we take a smaller step first – establishing that $\hat{w}_k$ must be larger than $\hat{w}_t$. That is, we start with the more modest aim of showing that when p_k rises, factor k's return is not lowered more than any other. Furthermore, just as set Q denotes all $i \neq k$, let set I be the set of all $i \neq k, s, t$.

Throughout our proofs we eschew formal matrix manipulations in favor of the heuristically more satisfying repeated use of the competitive profit equations of change whereby any relative commodity price change must equal the distributive share weighted average of all relative factor price changes. For industry k this is shown as

$$\theta_{kk}\hat{w}_k + \theta_{sk}\hat{w}_s + \theta_{tk}\hat{w}_t + \sum_{i \in I} \theta_{ik}\hat{w}_i = \hat{p}_k \tag{5}$$

Now subtract $\hat{w}_t$ (equal to $\Sigma_{i=1}^{n} \theta_{ik}\hat{w}_t$) from both sides:

$$\theta_{kk}(\hat{w}_k - \hat{w}_t) + \theta_{sk}(\hat{w}_s - \hat{w}_t) + \sum_{i \in I} \theta_{ik}(\hat{w}_i - \hat{w}_t) = \hat{p}_k - \hat{w}_t \tag{6}$$

In a representation similar to (5) we could also write the competitive profit equations of change for industry s – but the right-hand side would be zero since p_s is being held constant when p_k rises. Suppose we subtract $\hat{w}_t$ from each side of this equation (just as we did in going from (5) to (6)). That is,

$$\theta_{ks}(\hat{w}_k - \hat{w}_t) + \theta_{ss}(\hat{w}_s - \hat{w}_t) + \sum_{i \in I} \theta_{is}(\hat{w}_i - \hat{w}_t) = -\hat{w}_t \tag{7}$$

Finally, subtract term-by-term (7) from (6) to yield (8):

$$(\theta_{kk} - \theta_{ks})(\hat{w}_k - \hat{w}_t) + (\theta_{sk} - \theta_{ss})(\hat{w}_s - \hat{w}_t) + \sum_{i \in I} (\theta_{ik} - \theta_{is})(\hat{w}_i - \hat{w}_t) = \hat{p}_k \tag{8}$$

The next step in the proof involves setting upper bounds on $\Sigma_{i \in I}(\theta_{ik} - \theta_{is})(\hat{w}_i - \hat{w}_t)$. Since factor t fares worse by the rise in p_k than any other $i \in Q$, each $(\hat{w}_i - \hat{w}_t)$ is non-negative. Therefore,

$$\sum_{i \in I} (\theta_{ik} - \theta_{is})(\hat{w}_i - \hat{w}_t) \leq \sum_{i \in I} |\theta_{is} - \theta_{ik}|(\hat{w}_i - \hat{w}_t) \tag{9}$$

Since $\hat{w}_s$ is the $\max_i \hat{w}_{i \in Q}$, this latter term is $\leq$

$$\{\sum_{i \in I} |\theta_{is} - \theta_{ik}|\} \cdot (\hat{w}_s - \hat{w}_t) \tag{10}$$

which, by the Strong Factor Share condition (4) is less than or equal to $(\theta_{ss} - \theta_{sk})(\hat{w}_s - \hat{w}_t)$. If the resulting inequality,

$$\sum_{i \in I} (\theta_{ik} - \theta_{is})(\hat{w}_i - \hat{w}_t) \leq (\theta_{ss} - \theta_{sk})(\hat{w}_s - \hat{w}_t) \tag{11}$$

is inserted into (8), it follows that

$$(\theta_{kk} - \theta_{ks})(\hat{w}_k - \hat{w}_t) \geq \hat{p}_k > 0 \tag{12}$$

so that (since θ_{kk} exceeds θ_{ks} by the FS condition (2)), $\hat{w}_k$ must at least exceed $\hat{w}_t$.

To proceed, we now show that $\hat{w}_k$ is greater than the largest $\hat{w}_i$ (for $i \in Q$), i.e. $\hat{w}_s$. To do so, follow the procedure used in establishing (6) and (8), except consider the competitive profit equations of change for industries t and s (instead of k and s), and subtract $\hat{w}_k$ from both sides (instead of $\hat{w}_t$). This yields:

$$\theta_{tt}(\hat{w}_t - \hat{w}_k) + \theta_{st}(\hat{w}_s - \hat{w}_k) + \sum_{i \in I} \theta_{it}(\hat{w}_i - \hat{w}_k) = -\hat{w}_k$$

$$\theta_{ts}(\hat{w}_t - \hat{w}_k) + \theta_{ss}(\hat{w}_s - \hat{w}_k) + \sum_{i \in I} \theta_{is}(\hat{w}_i - \hat{w}_k) = -\hat{w}_k$$

Subtract the second equation from the first to get:

$$(\theta_{ss} - \theta_{st})(\hat{w}_s - \hat{w}_k) + (\theta_{tt} - \theta_{ts})(\hat{w}_k - \hat{w}_t) + \sum_{i \in I} (\theta_{is} - \theta_{it})(\hat{w}_i - \hat{w}_k) = 0 \tag{13}$$

At this stage of the argument nothing has been established about the ranking of any $\hat{w}_i$ ($i \in I$) with $\hat{w}_k$, prompting us to split set I ($i \neq s, t, k$) into two components. Thus let

$$I_1 \equiv \{i \in I \mid (\hat{w}_i - \hat{w}_k) > 0\} \; ; \; I_2 \equiv \{i \in I \mid (\hat{w}_i - \hat{w}_k) \leq 0\} \tag{14}$$

Rewrite (13) to separate I_1 from I_2:

$$\left\{(\theta_{ss} - \theta_{st})(\hat{w}_s - \hat{w}_k) - \sum_{i \in I_1} (\theta_{it} - \theta_{is})(\hat{w}_i - \hat{w}_k)\right\} + \left\{(\theta_{tt} - \theta_{ts})(\hat{w}_k - \hat{w}_t) - \sum_{i \in I_2} (\theta_{it} - \theta_{is})(\hat{w}_i - \hat{w}_k)\right\} = 0 \tag{15}$$

Once again we use SFS to work on the terms behind the summation signs. We shall proceed by contradiction, assuming $\hat{w}_s \geq \hat{w}_k$ and showing that equation (15) must be violated in such a case.

Examine, first, $\sum_{i \in I_1} (\theta_{it} - \theta_{is})(\hat{w}_i - \hat{w}_k)$. This must be less than or equal to $\sum_{i \in I_1} \mid \theta_{it} - \theta_{is} \mid (\hat{w}_i - \hat{w}_k)$ since $\hat{w}_i$ exceeds $\hat{w}_k$ in I_1. Since $\hat{w}_s$ is maximal in Q, this sum in turn is less than or equal to

$$\{\sum_{i \in I_1} \mid \theta_{it} - \theta_{is} \mid\} (\hat{w}_s - \hat{w}_k) \leq \{\sum_{i \in I} \mid \theta_{it} - \theta_{is} \mid \} (\hat{w}_s - \hat{w}_k)$$

which, by the SFS condition (4), must fall short of $(\theta_{ss} - \theta_{st})(\hat{w}_s - \hat{w}_k)$. Therefore the first bracketed term in (15) must be non-negative.

Next, consider $\sum_{i \in I_2} (\theta_{it} - \theta_{is})(\hat{w}_i - \hat{w}_k)$. Since in I_2, $\hat{w}_i$ is less than or equal to $\hat{w}_k$, this sum is less than or equal to:

$$\sum_{i \in I_2} \mid \theta_{it} - \theta_{is} \mid (\hat{w}_k - \hat{w}_i) \leq \{ \sum_{i \in I_2} \mid \theta_{it} - \theta_{is} \mid \} (\hat{w}_k - \hat{w}_t)$$

since $\hat{w}_t$ is the smallest element. This expression in turn is less than or equal to $\{\sum_{i \in I} \mid \theta_{it} - \theta_{is} \mid \} (\hat{w}_k - \hat{w}_t)$ which, by the SFS condition, is less than $(\theta_{tt} - \theta_{ts})(\hat{w}_k - \hat{w}_t)$. This implies that the second bracketed expression in (15) is strictly positive, which establishes the contradiction. Therefore $\hat{w}_k$ must exceed $\hat{w}_s$, and perforce must be larger than $\hat{p}_k$.

The strengthened Chipman condition (4) suffices to establish the weak version of the Stolper–Samuelson Theorem for any value of n even though the original FS condition (2) proved insufficient for $n \geq 4$. Furthermore, we have proved an extra result not stated in the original weak form of the Stolper–Samuelson Theorem: Not only does the kth factor gain in real terms ($\hat{w}_k > \hat{p}_k$), but it gains more than any other factor ($\hat{w}_k > \hat{w}_s$) even though some other factor returns might rise.

The Chipman condition in the 3 × 3 case

A consideration of the proof we just concluded reveals that the Strong Factor Share Condition (4) can be weakened. Condition (4) was invoked to compare terms like $(\theta_{ss} - \theta_{sk})$ with $\sum_{i \in I} | \theta_{is} - \theta_{ik} |$ or $(\theta_{ss} - \theta_{st})$ with $\sum_{i \in I} | \theta_{is} - \theta_{it} |$. But the set I contains only $(n - 3)$ terms, whereas condition (4) involves $(n - 2)$ terms. Clearly if (4) is satisfied, so also will it be if any of the absolute values on the right-hand side is deleted. Thus a weaker form of (4) could be stated as

$$\text{(SFS}'\text{)}: (\theta_{ii} - \theta_{ij}) > \sum_{r \neq i,j,k} | \theta_{ri} - \theta_{rj} | \ \forall i, j, \text{ with } i \neq j \text{ and } k \neq i, j \qquad (4')$$

The somewhat more awkward statement (4′) is relevant in confirming Chipman's observation that the dominance of factor *i*'s share in industry *i* over *i*'s share in any other sector is sufficient if $n = 3$ to establish the weak form of the Stolper–Samuelson Theorem. Such confirmation is immediate, for if $n = 3$ the right-hand side of (4′) vanishes since it has no elements.

The case of $n = 3$ can also be used to illustrate an asymmetry between Chipman's treatment of the weak Stolper–Samuelson Theorem and the Kemp–Wegge analysis of the strong form of the theorem. Kemp and Wegge show that the factor intensity condition (FI, (1)) is a *necessary* consequence if an economy throughout exhibits strong Stolper–Samuelson properties.[2] The analogous question can be raised: If an economy displays weak Stolper–Samuelson properties, is the factor share condition (FS), inequality (2), necessarily satisfied? The answer is in the negative as the following share matrix reveals:

$$\theta' = \begin{bmatrix} 0.31 & 0.3 & 0.39 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.315 & 0.385 \end{bmatrix}$$

Here, θ_{31}, with a value of 0.39, exceeds the θ_{33} value of 0.385. Computation of $(\theta')^{-1}$, however, reveals that diagonal elements all exceed unity. (The example clearly also shows that condition SFS – which is stronger than condition FS – is not necessary for an economy to display weak Stolper–Samuelson properties.)

If the weak form of the Stolper–Samuelson Theorem does not imply the Chipman FS condition, what can be said about factor shares if the *strong version* of the weak form is assumed to hold? Once again what is true for small values of n (≤ 3) fails to hold for $n \geq 4$. The following counter-example suffices: Let

$$\theta' = \begin{bmatrix} 0.28 & 0.24 & 0.24 & 0.24 \\ 0.24 & 0.28 & 0.24 & 0.24 \\ 0.22 & 0.23 & 0.28 & 0.27 \\ 0.24 & 0.25 & 0.25 & 0.26 \end{bmatrix}$$

The inverse of this 4×4 share matrix,

$$(\theta')^{-1} = \begin{bmatrix} 16.12 & -5.52 & -4.80 & -4.80 \\ -8.88 & 19.48 & -4.80 & -4.80 \\ 15.12 & 15.48 & 35.20 & -64.80 \\ -20.88 & -28.52 & -24.80 & 75.20 \end{bmatrix}$$

satisfies the strict version of the weak form of the Stolper–Samuelson Theorem, but the original share matrix violates the FS condition. However, it can be shown that for $n = 3$ the FS condition follows from the assumption that the strict version is satisfied.[3]

An example when $n = 4$

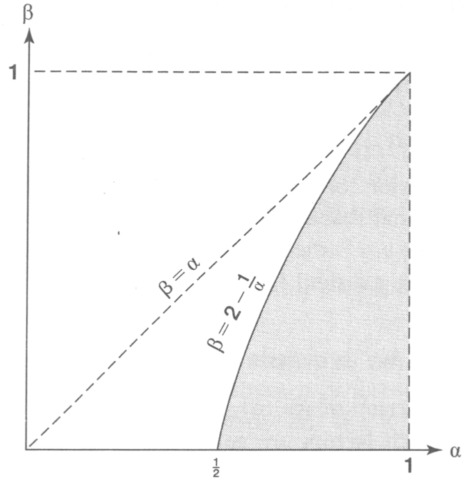
A special class of production relationships imposes a kind of strict symmetry among sectors, in which factors are numbered such that the array of distributive shares in the first industry is non-increasing and the shape of this "share rib" is replicated for all other sectors with a permutation such that θ_{ii} always is the dominant share in industry i. (In matrix form this is known as a circulant matrix.) In particular suppose the share rib for sector 1 is composed of:

$$\frac{1}{\Delta}\{1, \alpha, \alpha\beta, \alpha\beta^2\}$$

where Δ is defined as the sum of the elements in brackets, thus ensuring that distributive shares add to unity. (For example, θ_{31} is $\alpha\beta/\Delta$.) The terms α and β are assumed to be positive fractions. These parameter restrictions imply that the factor share condition (2) is satisfied.

Elsewhere we have examined in detail the properties of an $n \times n$ economy displaying share ribs of this type.[4] There we show that the weak form of the Stolper–Samuelson Theorem is always satisfied for restrictions on α and β noted above. Here we illustrate that the Strong Factor Share condition, even in form (4′), is not always satisfied, so that the condition is not necessary, even when condition FS is satisfied.

Consider the term $(\theta_{11} - \theta_{14})$ which, for this case,[5] is $(1 - \alpha)/\Delta$. Condition (4′) would require this to exceed, inter alia, the term $|\theta_{21} - \theta_{24}|$ or $(\alpha - \alpha\beta)/\Delta$. Such a condition reduces to requiring $\beta > 2 - (\frac{1}{\alpha})$. For values of α less than $\frac{1}{2}$ this is clearly satisfied, so suppose $\frac{1}{2} < \alpha < 1$. The accompanying diagram illustrates in the shaded area those values of β which do not satisfy the inequality required by (4′). The weak version of the Stolper–Samuelson Theorem is none the less valid (and condition FS is satisfied) so that condition (4′) (much less the SFS condition (4)) is not necessary.



Concluding remarks

In his 1969 article John Chipman was on the right track in suggesting that a comparison of a factor's distributive share in the industry in which it is uniquely associated with its share in any other industry is of relevance in establishing the weak form of the Stolper–Samuelson Theorem. But the dominance of θ_{ii} over all other θ_{ij} proved, as he recognized, not to suffice for economies with large numbers of goods and factors. In this chapter we have pursued the search for sufficient conditions. In our Strong Factor Share condition (4), or somewhat weaker version (4′), we display the requirement that the dominance of factor *i*'s share in industry *i* over its share anywhere else in the economy be large enough to exceed the aggregate absolute value of discrepancies of other factor shares in these two sectors. Thus if variation among shares of unintensive factors is sufficiently limited, the weak Stolper–Samuelson Theorem emerges. In our proof we established the extra property that a single price rise not only results in an increase in the real return to the associated intensively-used factor, but no other factor gains in percentage terms by more.

The weak form of the Stolper–Samuelson Theorem seems more relevant than the strong form from the point of view of the motivation of any particular factor to lobby for a commodity price change which will improve its real reward. Such a factor might either not care about what happens to other income earners or, indeed, may welcome the chance for disguise provided by the possibility that some other factors also gain. In the strong form of the Theorem there is only one winner who unambiguously and nakedly emerges from such a price change.

Notes

1 Note that the SFI condition (as well as the strong Stolper–Samuelson result) must be violated if there is any factor that is not used in positive amounts in every industry.

2 A proof of the necessity of (FI), making use of competitive profit equations of change, is found in Jones (1993).

3 Although a proof is not provided here, it makes use of the fact that the off-diagonal elements of the inverse of a share matrix can be written as the appropriate diagonal entry minus a term related to the FS condition, e.g. $(\theta_{33} - \theta_{32})$. It can be shown that a stochastic matrix with a "dominating" diagonal (i.e. $\theta_{ii} > \theta_{ij}$ along the ith row) has an inverse which is also stochastic and, as well, has a "dominating diagonal" in the 3×3 case.

4 See Jones and Mitra (1995).

5 The share rib for sector 4 is shown by $\frac{1}{\Delta}\{\alpha, \alpha\beta, \alpha\beta^2, 1\}$.

Bibliography

Chipman, John (1969) "Factor Price Equalization and the Stolper–Samuelson Theorem," *International Economic Review* (October): 399–406.

Jones, Ronald W. (1993): "Reflections on the Stolper–Samuelson Theorem," in H. Herberg and N. VanLong *Trade, Welfare and Economic Policies*, pp. 21–36. Michigan: University of Michigan Press.

Jones, Ronald, Marjit, Sugata and Mitra, Tapan (1993) "The Stolper–Samuelson Theorem: Links to Dominant Diagonals," in R. Becker et al. *General Equilibrium, Growth and Trade II*, pp. 429–444. New York: Academic Press.

Jones, Ronald and Mitra, Tapan (1995): "Share Ribs and Income Distribution," *Review of International Economics*: 36–52.

Kemp, Murray C. and Wegge, Leon (1969): "On the Relation Between Commodity Prices and Factor Rewards," *International Economic Review* (October): 407–413.

Stolper, Wolfgang and Samuelson, Paul (1941) "Protection and Real Wages," *Review of Economic Studies* (November): 58–73.