National Product, Income Accounts and Sustainable Development

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1. INTRODUCTION

In recent years, there has been a renewal of interest in the concept of Net National Product (NNP) as a measure of welfare. This interest has arisen from concerns regarding exploitation of exhaustible and renewable natural resources, and regarding environmental effects of development. These concerns are to a large extent, but not solely, within the context of sustainable development. A significant body of literature has developed which seeks to promote changes in the way NNP is measured in the National Income Accounts, specifically the 'net investment' component, to better reflect the presence and the importance, in economic activity, of resources, exhaustible and renewable, and of environmental factors. It also seeks to draw some interesting connections between the notion of sustainable development on the one hand, and NNP and the pattern of investment on the other.

It is a distinctive feature of the literature mentioned in the preceding paragraph, that it is very firmly grounded in theoretical developments on a basic conceptual issue of wider interest, namely, the possibility of a current income concept being an appropriate measure of welfare in a dynamic context. In this essay we seek to provide an exposition of some of the theoretical developments in this area, and in the above mentioned literature on National Income Accounts and sustainable development,

*Research on this paper was started while the first author was on sabbatic leave at Cornell University. The first author acknowledges partial support from a NSERC grant. We also acknowledge detailed comments and suggestions by Amitava Bose which led to substantial improvements in the paper. An earlier version of the paper was presented at McGill University. Thanks are due to the participants of the seminar for their helpful comments.

which explores the implications of these theoretical developments. Along the way, we provide some critical assessments, indicate some questions of interest that arise from these developments, and outline some recent work along these lines. There is no attempt to be exhaustive in scope or in detail. The ground covered, as well as the citations provided, simply reflect our particular interest in some of the theoretical subject matter in this area, arising in the course of our investigation of certain problems. We hope that this will provide an introduction to an area which we believe is interesting, and stimulate the reader's curiosity. In the remainder of this introduction we present a brief overview of the main issues we address in the sections to follow.

It is generally accepted that if we wish to measure the economic well-being of a nation (including its current and future generations) we must determine the present discounted value of current and future utilities. However, 'Net National Income', measured by adding the value of net current investment to current consumption, is often used as an indicator of a society's welfare. A central question then is whether a theoretical justification can be provided for this practice of including 'net current investment' in a measure of welfare.

Weitzman (1976) made the fundamental observation that in theory, current Net National Product, appropriately measured, provides a precise measure of the present discounted value of current and future utilities. The observation is striking in (at least) two respects. The first is that a *current income* concept should contain all the information regarding the well-being of a society's entire future. This, of course, ceases to be a surprise when we consider the fact that Weitzman makes his observation with respect to perfect foresight competitive paths. The second, and perhaps more intriguing one, is that while it is to be expected that investment today translates in *some way* to the generation of future consumption, and therefore, to future utilities, it is quite remarkable that the current value of net investment should turn out to be such an accurate proxy for the present discounted value of future utilities.

In order to state this connection properly, we need to conduct our discussion in more precise terms. Weitzman's observation is that for a *competitive path* $\langle k(t), p(t)\rangle$ under a constant interest regime r (see following section for explanations of the fairly standard notation and definitions), for every t,

$$u(t) + p(t)\,\dot{k}(t) = r\int_t^{\infty} e^{-r(s-t)}u(s)\,ds \qquad (1)$$

Let us refer to (1) as *Weitzman's Rule*, henceforth abbreviated as WR. Interpreting the lefthand side of (1) as the Net National Product (NNP) (for more on this interpretation see section 4), WR asserts that the NNP at time t is equal to the annuity equivalent of the present discounted value of utilities along the path. The second section provides the formal framework of the discussion, along with necessary defintions and notational details. The third section provides a detailed discussion of WR.

If the competitive path $\langle k(t), p(t) \rangle$ was, in fact, an *optimal* path, i.e. one which maximized:

$$\int_0^\infty e^{-rt}\, u(t)\, dt \tag{2}$$

over all paths from the same initial conditions, then WR is really an interpretation of Bellman's equation of optimality in dynamic programming; namely, for all t,

$$-V'(k(t))\, \dot{k}(t) = u(t) - rV(k(t)) \tag{3}$$

where V is the 'value function' associated with the problem of maximizing (2), assumed differentiable for convenience. The support of the value function, $V'(k(t))$, is the price vector $p(t)$ of the capital goods (for convex structures), and (3) readily yields (1). A detailed discussion of Bellman's equation (3) may be found in the third section.

But Weitzman (1976) appears to claim more, namely, that (1) holds for every *competitive* path. This turns out to be invalid in general: one can construct examples of competitive paths which violate WR. Perhaps more interesting is the observation that a competitive path satisfies WR if and only if it satisfies the *investment value transversality condition*:

$$\lim_{t \to \infty} e^{-rt} p(t)\, \dot{k}(t) = 0 \tag{4}$$

just as a competitive path is optimal if and only if it satisfies the *capital value transversality condition*:

$$\lim_{t \to \infty} e^{-rt} p(t)\, k(t) = 0 \tag{5}$$

It turns out that in the standard model of optimal intertemporal allocation, in which an exhaustible resource is a factor of production [see, e.g. Solow (1974), Hartwick (1977)], the investment value transversality condition is *always* satisfied for competitive paths (although the capital value transversality condition is not). Thus, in this framework, WR holds for every competitive path. These issues are discussed in the third section.

There is an important implication that can be drawn from WR for National Income accounting procedures. WR asserts that NNP equals the annuity equivalent of the present discounted sum of utilities. For this to be valid, the notion of capital and, therefore, of net investment has to be properly interpreted. The standard measure of net investment in the National Income Accounts, namely, the value of change in stocks of producible capital goods is too narrow and fails to capture welfare properly. The notion of capital must be interpreted very broadly and net investments should, in principle, include changes in any stock which contributes to production. In the fourth section we discuss in some detail the principle involved, in the context of a simple model of exhaustible resources.

In the fourth section we also critically survey the literature, focusing on the connection between WR and sustainable development. Here we note the most important conclusion to be drawn. The validity of WR for all competitive paths has an important implication for the discussion on investment policies ensuring *sustainable development*. Following Weitzman (1995), if we define a path of sustainable development to be one for which at each date the constant utility equivalent of the present discounted value of future utilities is at least as large as the current utility, then a competitive path generates a path of sustainable development if and only if the value of investment is never negative. This calls for an investment policy for sustainable development in which producible capital goods are augmented at a rate sufficient to offset the depletion of non-producible capital goods such as exhaustible natural resources, and produce a non-negative *aggregate* value of net investment.

Finally, in the fifth section, we make some peripheral remarks of interest, especially from the point of view of an agenda for future research.

2. PRELIMINARIES

2.1 The Framework

Consider a framework in which population and technology are unchanging, individuals are identical in all respects so that there is no harm in thinking in terms of a single representative person (and so, ignoring distribution considerations) at each date, and most importantly, that consumption level in any period can be represented by a single number. 'It might be calculated as an index number with given price weights, or as a multiple of some fixed basket of goods, or more generally, as any cardinal utility function'.[1]

Denote by $k_i \geq 0$, the stock of the ith capital good, $i = 1, \ldots, n$, and the vector $(k_1, \ldots, k_n)$ by k. Let $I = (I_1, \ldots, I_n)$ stand for the vector of quantities of the investment goods. Denote by a set T, the technologically feasible input output combinations. A typical point of T is a triplet $(U, I, -k)$, which is understood to mean that from capital input stocks[2] k, it is technologically feasible to obtain the flow of consumption U and the flow of investment I, which is net of depreciation of the existing stock of capital k. We may alternatively represent the technological possibilities by associating with each k, the combinations of consumption and net investment that is possible to realize with input stocks k. Denote this by a set $S(k)$, i.e.

$$S(k) = \{(U, I)/(U, I, -k) \in T\} \tag{6}$$

A *path*, from initial stocks k, is a triplet of functions of time, $U(t)$, $I(t)$ and $k(t)$, defined for $t \geq 0$, of consumption, investment and capital stocks, respectively, satisfying[3]:

$$\begin{gathered} (U(t), I(t)), \in S(k(t)) \text{ for each } t \\ \dot{k}(t) = I(t) \text{ for each } t \\ k(0) = \underline{k} \end{gathered} \tag{7}$$

We use the notation $\langle U(t), I(t), k(t) \rangle$ to denote a path.

2.2 Competitive and Optimal Paths

We shall now elaborate what we mean by a time path of quantities and prices which evolves along an equilibrium of a competitive market economy, from an initial stock $\underline{k}$. It would be convenient, for what

[1]Weitzman (1976), pp. 156–7.

[2]Following usual convention we are using negative values for inputs.

[3]We are using conventional notation: $\dot{x}$ means time derivative. So if $x(t)$ is a vector valued function, i.e. $x(t) = [x_1(t), \ldots, x_n(t)]$, then $\dot{x}(t) = \frac{dx}{dt}(t) = \left(\frac{dx_1}{dt}, \ldots, \frac{dx_n}{dt}\right)$. If x is a vector $(x_1, \ldots, x_n)$ and $f(x)$ is a vector valued function defined from R^n to R^m, i.e. $f(x) = [f_1(x), \ldots, (f_m(x)]$ then $f'(x)$ is the '$m \times n$' matrix whose 'ijth' element is $\frac{\partial f_i}{\partial x_j}(x)$.

follows, to introduce the following notation and concepts. Let $p \geq 0$ denote the prices of the investment goods in terms of the consumption good U. Define a function $h(k, p)$ by:

$$h(k, p) = \underset{(U, I) \in S(k)}{\text{Max}} \quad [U + p \cdot I] \tag{8}$$

Here, $h(k, p)$ is the maximum value of output which can be achieved, given input stocks k, at prices p. Let $r > 0$ denote the market rate of interest.

A *competitive* path is a path $\langle U(t), I(t), k(t) \rangle$ with associated prices, denoted by a function of time, $\langle p(t) \rangle \geq 0$, satisfying the following two conditions:

$$U(t) + p(t)\, I(t) = h(k(t), p(t)) \text{ for each } t \tag{9}$$

and

$$\frac{dp_i}{dt}(t) = rp_i(t) - \frac{\partial h}{\partial k_i}(k(t), p(t)),\ i = 1, \ldots, n, \quad \text{for each } t \tag{10}$$

Here, $p(t)$ is the vector of current prices of the investment goods, in terms of the consumption good U, prevailing along an equilibrium path at each date t. Equation (9) says that, at each date t, value of output realized on the path, $U(t) + p(t)\, I(t)$, is the maximum over the set of feasible outputs $S[k(t)]$. Equation (10) says that asset markets are in equilibrium; i.e. no gains can be made by pure arbitrage. [See Dorfman, Samuelson and Solow (1958), Weitzman (1976).]

Use the notation $\langle U(t), I(t), k(t), p(t) \rangle$ to denote a competitive path with its associated prices. Along the competitive path at time t:

$$\text{NNP}(t) = U(t) + p(t)\, I(t) \tag{11}$$

Denoting the NNP at time t by $Y(t)$, rewrite (9) as:

$$Y(t) = h(k(t), p(t)) \text{ for each } t. \tag{12}$$

We will now relate all of the above, to standard formulations of Ramsey type optimal growth models [see Benveniste and Scheinkman (1979, 1982) for example]. For this purpose define the set F by:

$$F = \{(k, I) / \text{for some } U, (U, I, -k) \in T\} \tag{13}$$

F is the set of (k, I) pairs, such that it is feasible to attain the rate of net investment I (along with some feasible consumption goods output), given the capital inputs k. Define also a function u for each feasible (k, I) pair in F by:

$$u(k, I) = \text{Max } U \quad \text{s.t. } (U, I, -k) \in T \tag{14}$$

Thus, $u(k, I)$ is the maximum utility or consumption that can be obtained from the capital stock k given the output I of the investment goods. We may note, for later use, that if we consider the efficient frontier of $S(k)$, that is, the set $\bar{S}(k)$, defined by:

$$\bar{S}(k) = \{(U, I) \in S(k) \ / \text{ there is no other } (U', I') \text{ in } S(k) \text{ satisfying } (U', I') \geq (U, I)\} \tag{15}$$

then we have

$$u(k, I) = \{U/(U, I) \in \bar{S}(k)\} \tag{16}$$

REMARK 1: Given (k, p) maximizing $U + pI$, subject to $(U, I) \in S(k)$ is equivalent to maximizing $u(k, I) + pI$ with respect to variations in I such that $(k, I) \in F$. So

$$h(k, p) = \text{Max } [u(k, I) + p \cdot I] \quad \text{s.t. } (k, I) \in F \tag{17}$$

Writing $g(k, p)$ for the maximizing value of I in (17) as a function of (k, p), we obtain

$$h(k, p) = u(k, g(k, p)) + pg(k, p) \tag{18}$$

REMARK 2: Along a competitive path $\langle U(t), I(t), k(t), p(t)\rangle$, $U(t) = u((k(t), \dot{k}(t))$ for each t; we shall sometimes write $u(t)$ to mean $u(k(t), \dot{k}(t))$ $(= U(t))$ for each t.

In view of the remark above, we can conveniently describe a competitive path from $\underline{k}$ simply as a pair of time paths of capital stocks and prices $\langle k(t), p(t)\rangle$ satisfying: $(k(t), \dot{k}(t)) \in F$ for each t; $k(0) = \underline{k}$ and (19) and (20) below:

$$u(k(t), \dot{k}(t)) + p(t)\, \dot{k}(t) = h(k(t), p(t)) \ (= Y(t)) \text{ for each } t \tag{19}$$

$$\frac{dp_i}{dt}(t) = rp_i(t) - \frac{\partial h}{\partial k_i}(k(t), p(t)),\ i = 1, \ldots, n, \text{ for each } t \tag{20}$$

Consider now the following dynamic optimization problem[4].

$$\begin{aligned} &\text{Max} \int_0^\infty e^{-rt} u(k(t), I(t))\, dt \\ &\text{s.t. } (k(t), I(t)) \in S(k(t)) \text{ for each } t \\ &\dot{k}(t) = I(t) \text{ for each } t, \text{ and} \\ &k(0) = \underline{k} \end{aligned} \tag{21}$$

[4]We are taking for granted that the problem is well defined. Throughout this essay, we are presuming that regularity conditions, such as concavity, smoothness, boundedness, interiority conditions, which are necessary for the validity of arguments, are in place.

We shall say that a path from $\underline{k}$ is *optimal* if it is a solution to the above control problem.

For what follows, it is esential to note the distinction between a competitive path from $\underline{k}$ and an optimal path from $\underline{k}$. Under suitable assumptions, most important of which is convex structure on technology and utility, an optimal path is also competitive; that is, if a path $\langle k(t)\ I(t)\rangle$ from $\underline{k}$ is optimal then there is an associated time path of prices $\langle p(t)\rangle$ at which $\langle k(t), p(t)\rangle$ satisfies the competitive conditions (19) and (20) above. In other words, the competitive conditions are necessary conditions for optimality. In the literature of dynamic optimization, the necessary conditions for optimality, which correspond to the competitive conditions, are usually expressed somewhat differently and in terms of present value prices. In the appendix we formally show that these are equivalent to the competitive conditions.

While the competitive conditions are necessary, in general, they are not sufficient for optimality. From any given $\underline{k}$, typically, there will be a family of competitive paths, only one of which will be the optimal path. In addition to the competitive conditions, an optimal path must satisfy a 'transversality condition'. Usually this takes the form that, in the limit, the present value of future capital stock, i.e. $e^{-rt}p(t)\ k(t)$, goes to 0. For a competitive path, such a capital value transversality condition is both a necessary as well as a sufficient condition for it to be an optimal path.

3. NET NATIONAL PRODUCT AND WELFARE

3.1 WEITZMAN'S RULE

The concept of Net National Product (NNP) plays an important role in macroeconomics. It serves as a summary measure of the level of economic activity and is valuable on account of its close link to the level of employment. It also serves as a summary measure of the economic well being of a nation. We are interested here in the latter aspect only. Clearly, there are deficiencies in NNP as a measure of economic welfare. The size of the population which consumes the product should count and more appropriate would be NNP per capita. Inequalities in the distribution of the product over the population should also be relevant. NNP, as measured in National Income Accounts, is the sum of consumption and net investment. As is well known from the static theory of index numbers, there are problems in aggregating a heterogeneous bundle of consumption goods into a single measure of aggregate consumption. There are problems in comparison of aggregate measures of income or consumption across populations with different taste and environments. These are matters which we will not address, although they are clearly important. Instead, following Weitzman (1976), we will focus on one aspect, namely the net investment component of NNP and its role in a measure of welfare in a dynamic context.

Even if we accept the standpoint that it is consumption alone that promotes welfare and that the ultimate aim of economic activity is to provide consumption goods, the role of net capital formation in a welfare measure is unclear and indirect, at best. Intuitively, we understand that net capital formation now affects future productive potential, and hence, the consumption potential, in the future. If welfare is understood to depend on the time path of current and future consumption then, perhaps, net investment may be regarded as a proxy for the effect of this investment on welfare through its effect on consumption in the future. But it is not quite clear whether there is an exact relationship between the two, even within the confines of a drastically stripped down scenario, where it is possible to focus on this aspect of the problem alone. As Samuelson (1961) argues, a rigorous and meaningful welfare measure is a 'wealth like magnitude' such as the present discounted value of future consumption. What connection does this have, if any, with current income concepts? Weitzman sought to establish that these may be viewed as simply two sides of the same coin.

Consider the welfare achieved along a competitive path from time t onwards, using t as the origin; i.e. the discounted sum of utilities discounted to time t,

$$\int_t^{\infty} e^{-r(s-t)} u(s)\, ds \tag{22}$$

We may define a *constant utility equivalent* of this, namely, a hypothetical constant $\overline{u}$ such that if utility achieved at each date t were equal to $\overline{u}$ then the discounted sum of utility achieved would be the same as in (22).[5] So $\overline{u}$ is defined by

[5]This is similar to the device used by Milton Friedman (1957) in defining 'permanent income' in his theory of the consumption function. Of course, the horizon in his case is the finite lifetime of the individual whose consumption behaviour is being studied.

$$\int_t^\infty e^{-r(s-t)}\,\bar{u}\,ds = \int_t^\infty e^{-r(s-t)}\,u(s)\,ds \tag{23}$$

Nothing that $\bar{u}$ is a constant, we may rewrite this as:

$$\frac{\bar{u}}{r} = \int_t^\infty e^{-r(s-t)}\,u(s)\,ds \tag{24}$$

or as

$$\bar{u} = r\int_t^\infty e^{-r(s-t)}\,u(s)\,ds \tag{25}$$

Weitzman's main proposition seeks to establish that the NNP at time t, $Y(t)$, is this annuity equivalent $\bar{u}$ of the welfare along the path; that is,

$$Y(t) = r\int_t^\infty e^{-r(s-t)}\,u(s)\,ds \text{ for each } t \tag{26}$$

We refer to (26) above as *Weitzman's Rule* or WR for short. In what follows below, we shall follow through Weitzman's line of reasoning to establish (26).

Recall from the second section that $h(k, p) = u(k, g(k, p)) + pg(k, p)$ where $g(k, p)$ is the maximizing value of I in (17). Suppressing the points of evaluation of the functions involved below to save on notation,

$$\frac{\partial h}{\partial p_j}(\bullet) = \sum_i \frac{\partial u}{\partial I_i}(\bullet) \qquad \frac{\partial g_i}{\partial p_j}(\bullet) + I_j(\bullet) + \sum_i p_i \frac{\partial g_i}{\partial p_j}(\bullet)$$

Using the fact that at the optimal choice of I (assuming an interior solution),

$$\frac{\partial u}{\partial I_i}(\bullet) + p_i = 0 \tag{27}$$

it follows that

$$\frac{\partial h}{\partial p_j}(k, p) = g_j(k, p) \tag{28}$$

Along a competitive path $\langle I(t), k(t), p(t)\rangle$ from $\underline{k}$, the rate of change of NNP, $Y(t) = h(k(t), p(t))$, over time is:

$$\frac{dY}{dt}(t) = \sum_i \frac{\partial h}{\partial k_i}(k(t), p(t)) \bullet I_i(t) + \sum_i \frac{\partial h}{\partial p_i}(k(t), p(t)) \bullet \frac{dp_i}{dt}(t)$$

By (17) and (19), $I(t) = g(k(t), p(t))$. Using (20) and (28), the second term in the equation above becomes

$$r \Sigma I_i(t) \bullet p_i(t) - \Sigma \frac{\partial h}{\partial k_i}(k(t), p(t)) \bullet I_i(t)$$

So, we obtain

$$\frac{dY}{dt}(t) = r \sum_i p_i(t)\, I_i(t) = r[Y(t) - u(k(t), \dot{k}(t))] \tag{29}$$

Clearly, (26) is a solution to this differential equation. [It is worth noting that Weitzman does not address the question of whether (26) is the *only* solution of (29), an issue we will consider in the subsection following the next one.]

3.2. Bellman's Equation and Weitzman's Rule for Optimal Paths

For *optimal* paths, WR is closely connected to the Bellman Equation in Dynamic Programming [see for instance Intriligator (1971), Sorger (1992)] derived from the Principle of Optimality. Some authors [see Hartwick (1994), Kemp and Long (1982) for instance] take this approach to establish that WR holds for *optimal* paths. We sketch this line of argument below. [For alternate derivations of this result, see Asheim (1994), Lozada (1995).]

Consider an optimal path $\langle I^*(t), k^*(t)\rangle$ from $\underline{k}$. Let $p^*(t)$ be the supporting prices at which the path is competitive. Denote the NNP at each t along the path by $Y^*(t)$. The Principle of Optimality tells us that the segment of the path from time t onwards, that is, the functions $I^*(\tau)$ and $k^*(\tau)$ for $\tau \geq t$, constitutes the optimal path, for Problem 1, from initial stock $k = k^*(t)$.

For $k \geq 0$, $t \geq 0$, define the value function $V(k, t)$ by

$$V(k, t) = \text{Max} \int_t^{\infty} e^{-rs}\, u(k(s), I(s))\, ds \tag{30}$$

$$\text{s.t. } (k(s), I(s)) \in F \text{ and } I(s) = \dot{k}(s) \text{ for each } s \geq 0, \text{ and } k(t) = k.$$

It is clear that

$$V(k, t) = e^{-rt}\, V(k, 0) \tag{31}$$

The Bellman Equation for the control problem (21) is

$$-\frac{\partial V}{\partial t}(k^*(t), t) = \underset{I \text{ s.t. } (k^*(t), I) \in F}{\text{Max}} \left[e^{-rt}\, u(k^*(t), I) + \frac{\partial V}{\partial k}(k^*(t), t) \bullet I \right] \tag{32}$$

It is also known [see Benveniste and Scheinkman (1979, Corollary 1)] that under suitable assumptions

$$\frac{\partial V}{\partial k}(k', 0) = -\frac{\partial u}{\partial I}(k', I') \qquad (33)$$

where I' is the level of the control I at $t = 0$ along the optimal path from initial stocks k'.

From (31), we have that

$$\frac{\partial V}{\partial k}(k^*(t), t) = e^{-rt}\frac{\partial V}{\partial k}(k^*(t), 0)$$

$$= e^{-rt}\frac{\partial u}{\partial I}(k^*(t), I^*(t))$$

[using (33) and the Principle of Optimality]

$$= e^{-rt}p^*(t) \quad \text{[using (27)]}.$$

So the right hand side expression in (32) is $e^{-rt}Y^*(t)$. And, the left-hand side expression in (32) is, using (31),

$$-\frac{\partial V}{\partial t}(k^*(t), t) = rV(k^*(t), t) = r\int_t^\infty e^{-rs}u(k^*(s), I^*(s))\, ds \qquad (34)$$

Thus, WR holds for optimal paths.

3.3 Weitzman's Rule for Competitive Paths and a Transversality Condition

While it is a point of interest that WR is valid for optimal paths, it would be of far more interest if indeed it were valid for any competitive path. In fact, Weitzman's discussion of the problem, providing a motivation, seems to be along precisely such lines. Given an infinite horizon path satisfying competitive equilibrium conditions in a perfect foresight framework, the current prices and quantities carry information about the future prices and quantities. Since current net investment affects future consumption possibilities, intuitively there are grounds to explore if there is a clear cut relation between current NNP and the flow of consumption along the path. If a rule like (26) could be established for any competitive path, that would be of considerable interest. We have explored this issue elsewhere [see Dasgupta and Mitra (1999b)] and summarize the salient points below, eschewing all technical details and proofs.

It turns out that, in general, WR does not hold for every competitive path. We construct an example of a one sector neoclassical growth model of the Cass-Koopmans type [see Cass (1965) and Koopmans (1965)] with a modified golden rule and a maximum sustainable stock, where a competitive path violates WR. The competitive path chosen for the purpose of the example is one where the capital stock converges to the maximum sustainable stock. (The path is, of course, also non-optimal.)

This leads one to try to identify the competitive paths which do satisfy WR. We show that these paths can be characterized by a transversality condition involving the present value of net *investment*. Specifically, a competitive path $\langle I^*(t), k^*(t), p^*(t)\rangle$ satisfies WR if and only if

$$e^{-rt}\, p^*(t)\, k^*(t) \to 0 \quad \text{as } t \to \infty \tag{35}$$

Note that the condition is apparently different from the usual transversality condition characterizing optimal paths, namely the present value of *capital* $e^{-rt}\, p^*(t)\, k^*(t)^*(t)$, goes to zero as $t \to \infty$.

The example and the characterization result prompts one to examine closely Weitzman's line of reasoning leading to (26), which we had sketched earlier in this section.

Suppose $\langle k^*(t), I^*(t), p^*(t)\rangle$ is a competitive path from $\underline{k}$. Write $u^*(t)$ for $u[k^*(t), I^*(t)]$. Consider the differential equation.

$$\frac{d}{dt} F(t) = r[F(t) - u^*(t)] \tag{36}$$

A solution to this equation is a function of time $F\colon (0, \infty) \to R$ satisfying (36) at each $t \geq 0$. In general there is a *family* of such solutions, any two of which differ by a constant of integration. The arguments leading up to (29), starting from (19) and (20), establish that the function $Y^*(t)$, the time path of NNP associated with the given competitive path, is indeed a solution of (36). Define now a function $b(t)$ by

$$b(t) = \int_t^\infty e^{-r(s-t)}\, u^*(s)\, ds$$

It may be verified by direct calculation that $b(t)$ satisfies the differential equation (36). The two functions $Y^*(t)$ and $b(t)$, therefore differ by a constant. WR is valid if this constant is, in fact, zero.

In this connection it is worth noting that the arguments until this point make use of only the competitive conditions (19) and (20). As is well known, typically, in optimization problems like (21), there is more than one competitive path from each initial stock. Usually an optimal path is characterized by the competitive conditions, together with a transversality condition like

$$e^{-rt}\, p^*(t)\, k^*(t) \to 0 \quad \text{as } t \to \infty \tag{37}$$

[See Weitzman (1973) for a discrete time formulation of the problem and Benveniste and Scheinkman (1982) for a continuous time treatment.] Of course Weitzman (1976) recognizes this, but he does not clarify the role of optimality, or anything like the transversality condition (37), in deducing (26).

Finally, we show that in a specialized version of Weitzman's framework, namely, one where there is a produced good which is used both as capital input as well as consumption good, and an exhaustible resource which is 'important' in production, in the sense that its share in total output (the marginal product of the resource times its quantity divided by the output) is bounded below by some positive constant (a property of Cobb-Douglas production functions, for instance), all competitive paths satisfy WR. This is shown by verifying that the present value of net investment does go to zero along any competitive path, that is, (35) is satisfied. Furthermore, competitive paths in such a model are not necessarily optimal. This last point is noteworthy, since it can be shown for a class of multi-sector models of optimal growth, satisfying certain technological restrictions, that every competitive path is optimal [see Dasgupta and Mitra (1999a)]. In such a situation, competitive paths do indeed satisfy WR, but only 'trivially' so, by virtue of being an optimal path as well.

The results mentioned above establish that the transversality condition in terms of present value of net investment is different from the usual one involving present value of capital. Along a competitive path, the former does not imply the latter. To put it differently, within the class of competitive paths, WR is of wider validity than optimality. Along the way we also establish that in the context of models involving exhaustible resources, of a kind which is generally prevalent in the literature, under mild conditions, competitive paths do satisfy WR. Hence, the conclusions reached from it are of much wider validity than that recognized in the literature.

4. NATIONAL INCOME ACCOUNTS AND SUSTAINABLE DEVELOPMENT

4.1 Interpretation of Net National Product

Before proceeding with the discussion in this section it will be useful to briefly remark on the inclusion of a cardinal utility function in NNP as defined in (11) or (19). In National Income and Expenditure accounts, NNP is an index constructed from a heterogeneous bundle of consumption and investment good outputs evaluated at constant base year prices (alternatively, evaluated at current prices and deflated by a suitable price index). As is well known from the static theory of index numbers, there are problems with using such an index number as a welfare measure. Conceptually speaking, with more than one kind of consumption good, explicit consideration of the underlying fundamental notion of welfare, namely, a utility function defined over the heterogeneous bundle of consumption goods, becomes unavoidable. Furthermore, to be able to discuss welfare in a dynamic context, keeping separate the notion of current welfare from future welfare, it is simplest to have a cardinal utility function defined over the consumption in each period, the discounted sum of which is the accepted fundamental notion of welfare achieved along a time path of consumption. From the standpoint of theoretical consideration of NNP as a welfare measure, in contrast to its role as an aggregative measure of output which is useful because of its relationship to aggregate employment, the definition adopted in (11) or (19) is the appropriate one.

Having made the distinction between conventional measures of NNP and the notion of NNP appearing in WR, it is worth pointing out, once again, that the focus of attention in WR, and its applications to National Income accounting procedures and sustainable development, is on the welfare interpretation of the net investment component of NNP in a dynamic context. From this viewpoint, the reader may find it helpful to keep in mind a model with a single consumption good, such as Uzawa's two sector model,[6] obviating the need to distinguish between

[6]See Uzawa (1964). Instead of a two sector model one may have in mind a multi sector model so long as there is only one consumption good whose output, expressed as a product transformation function of the outputs of the many investment goods, given the stock of capital, is a non-linear function. The non-linearity is to avoid well known problems associated with optimal control

consumption and utility and, therefore, between the customary definition of NNP and that employed in WR. In such a framework prices of the investment goods are expressed in units of the consumption good and NNP is the maximum output produced (measured in units of the consumption good).

We may end this discussion by noting that the remarks above are relevant solely from the standpoint of suitable interpretation of the model and the results. From the logical standpoint there is no need for any special restriction of the kind mentioned above. The formal results are valid for standard models of capital accumulation such as Problem 1, which includes, for example, the Cass–Koopmans model.

4.2 PROPOSALS TO REVISE NATIONAL INCOME ACCOUNTS

We shall now consider some of the implications regarding National Income accounting procedures that have been drawn from WR. The principal point made appears to be as follows. For optimal programmes, WR shows that NNP is a measure of welfare. However, note that the investment component is 'all inclusive'. The concept of the capital stock, in Weitzman's formulation, is very broad. Any factor which influences the production possibilities, the menu of possible consumption and investment, which changes over time, and whose level is a matter of economic choice, is included in this.[7] The net change in such a stock, valued at the appropriate shadow price of that stock, ought to be included in the investment component in order that NNP properly measures the welfare achieved along the path, namely the annuity

problems in which the objective function is linear in the controls. This last point has been noted in the literature; see for example Hartwick (1990), Kemp and Long (1982), Brekke (1994), Hulten and Lofgren (1992), and Cairns (1997).

[7]In this regard, Weitzman follows the modern reformulation of capital theory appearing in the seminal work of Malinvaud (1953). In Malinvaud's framework, 'capital goods at time t include everything that has been made in preceding periods and is transferred to period t for further use in production'. [Malinvaud (1953, p. 235)]. In footnote 2 on p. 235, Malinvaud elaborates on the definition. 'The distinction between natural resources and produced means of production is not important as far as past activity is concerned. The only condition we need to keep in mind is the following: the available natural resources during all future periods must be independent of any present or future economic decision'. This clearly classifies non-renewable resources like mineral deposits as capital goods (produced means of production).

equivalent of future discounted sum of utility flow. If mineral resources, whose stocks can only be used up but not augmented, are used in the production processes, then the value of the change in these stocks ought to be included. Similarly, value of the changes in the stocks of renewable resources, like aquatic and forestry resources, whose harvesting, together with the underlying biological conditions, have an impact on the stocks of these resources, should also be included. Production activities generate pollution, adding to the level of pollutants. On the other hand, the level of pollutants can affect the output, given resources used in production, as for instance in fisheries and forestry products. In principle, the value of changes in the stock of pollutants should be included.

Obviously, all these are easier said than done. Very often there are difficult issues of measurement of the appropriate variables, prices and quantities. But the point being made is that to the extent that these measurement issues can be circumvented by suitable proxy measures (and in this respect similar problems are found in practice in dealing with other things like changes in quality) the value of such changes should be included in NNP. It should be emphasized that this is not an ad hoc prescription for including new items in NNP but well grounded in a clear cut theoretical link between NNP and welfare in a dynamic context.

The case of exhaustible resources brings to the fore the nature of changes that are being suggested and is worth discussing in a little more detail as typical case in point.[8]

Consider then, a very specialized version of a model of optimal exploitation of an exhaustible resource, considered quite frequently in the literature [see Dasgupta and Heal (1974), Hartwick (1990), for more elaborate versions of such a model]. Denote the stock of the exhaustible resource at date t by $S(t)$, and the flow of the resource extracted at date t by $R(t)$. A single consumption good is produced using the exhaustible resource and fixed labour as the sole inputs. Ignoring labour, which is fixed, denote the production function for the consumption good by f. Consider the following optimization problem:

[8]For more detailed discussion of this and other examples, the reader may wish to consult Dasgupta (1990), Hartwick (1990), and Maler (1991). The issues have been discussed in the context of open economies by Asheim (1995) and Sefton and Weale (1994).

$$\text{Max} \int_0^\infty e^{-rt}\, c(t)\, dt$$

such that $\quad c(t) = f(R(t)) \quad$ for each t

$$\frac{d}{dt} S(t) = -R(t) \quad \text{for each } t$$

$$S(t), R(t) \geq 0 \quad \text{for each } t$$

and $\quad S(0) = \underline{S}$

To avoid problems associated with linearity of the utility function [here $u(S, \dot{S}) = c = f(-\dot{S}) = f(R)$] in the control variable $\dot{S} = -R$ assume that f is a standard neoclassical production function with strictly diminishing marginal product. An appropriate 'Inada condition' like $f'(R) \to \infty$ *as* $R \to 0$ would ensure an interior optimal path. Let $S^*(t)$ denote the stocks of the resource along the optimal path and $p^*(t)$ denote the associated prices supporting the optimal path. In this case $u(S, \dot{S}) + p\dot{S} = f(-\dot{S}) + p\dot{S}$ is maximized along the optimal path, for each $t \geq 0$, at $\dot{S} = \dot{S}^*(t)$. So $p^*(t) = f'[R^*(t)]$, the marginal product of the resource at time t, at the optimal quantity. This is the current price of one unit of the stock of the exhaustible resource in terms of units of the consumption good.

Under standard income accounting practices, the NNP at date t, measured from the expenditure side, equals $c^*(t) = f[R^*(t)]$. Alternatively, using the value added approach, value added in the consumption goods sector is $c^*(t) - p^*(t)\, R^*(t)$, while that in the production of the extracted resource is $p^*(t)\, R^*(t)$, providing a total of $c^*(t)$. This is to be contrasted with NNP in Weitzman's sense, which is

$$c^*(t) = p^*(t)\dot{S}^*(t) = c^*(t) - p^*(t)R^*(t) = f(R^*(t)) - f'(R^*(t))R^*(t).$$

Exhaustible resource stocks, like any other 'produced' capital stocks, are part of the productive stocks and the value of net investment in these (in this case a negative one) is also to be accounted for in this so-called 'Green' NNP. Since marginal product is decreasing, this NNP is positive but smaller than the conventional measure. [If, for example, $f(R) = R^\alpha$ where $0 < \alpha < 1$, then $f(R) - f'(R)\, R = (1 - \alpha) f(R)$, so that the new measure of NNP can be substantially smaller than the traditional measure of NNP if α is close to 1.]

It is worth noting that all this follows from the initial premise that the 'right' concept of NNP should reflect the discounted sum of future consumption and, as per WR, that is indeed the case. NNP is the annuity equivalent of the variable consumption stream over time under the discounted sum criterion. Within the limited range of possibilities in this model, consumption is decreasing over time. We can see this by using (20) of the competitive conditions which must be satisfied. This says that $\frac{d}{dt}p^*(t) = rp^*(t)$ since the stocks S do not affect the integrand. So current prices $p^*(t)$ are growing at the rate $r > 0$ and since $p^*(t) = f'(R^*(t))$, $R^*(t)$ is decreasing over time. Thus, the annuity equivalent of the consumption stream from any time t onwards is always less than $c^*(t)$. This is what is being recognized by inclusion of the term $p^*(t)\,\dot{S}^*(t)$ above.[9]

4.3 Sustainable Devleopment

WR makes it very tempting to link NNP with some suitable notion of sustainable welfare. Various authors have examined this possibility [see Solow (1986), (1992), Hartwick (1994), Svensson (1986), Asheim (1994)]. Solow (1992), in particular, makes a very engaging heuristic case for the usefulness and possibilities of thinking along such lines, in relation to exploitation of non-renewable and renewable natural resources, and the natural environment, and seeks to link this with measurement issues in National Income Accounts, which we have sketched in the preceding sub-section.

The intuition behind seeking a connection between NNP and welfare, in the sense of the discounted sum of utilities, is that current net investment adds to future consumption potential. This could be rephrased, adopting a slightly different viewpoint. Gross Domestic Product is not suitable as an indicator of national well being, because, if two nations produce the same total final output for consumption, but one of them does so after making provisions to replace or repair worn out equipment used in production while the other does not, then we would say that the former nation is providing for its citizens better than the latter. The former can sustain this level of well being, while the latter

[9]If we incorporate extraction cost for the exhaustible resource, then the appropriate valuation per unit of the resource is its price net of cost of extraction at the margin [see Hartwick (1990)].

would be unable to do so, since it is running down its productive capacity. So, a depreciation charge needs to be deducted from GNP to arrive at a suitable measure of well being. Loosely speaking, the standpoint in this line of reasoning is that a proper notion of income, which is at one's disposal for consumption, is like an interest income on a fixed stock of capital. This is what can be consumed without depleting future earning and spending power.

Now, if we also adopt the standpoint that what is meant by sustainable development is not the preservation of specific things—natural resources, environmental riches, etc., but rather, a policy under which future generations have the possibility of ensuring for themselves and their successors at least the same level of well being as the current generation, then it seems natural to suggest that a policy of sustainable development should be one which adequately provides for depreciation of capital in a broad sense. If non-renewable mineral resource stocks are depleted to enable current productive activities to be carried out, then this 'depreciation' of stocks should be made up by the only possible way, namely, through 'appropriate' investment in produced capital goods which can substitute for the resource in production and sustain the same level of output. This, of course, presumes the possibility of such substitution between resource and produced capital. Solow (1974) analyzed a model with a produced good which is used for consumption as well as for production, together with labour and an exhaustible resource. Restricting himself to a Cobb–Douglas production function, he showed that if the share of capital exceeds the share of resource, then it is possible to maintain a minimum positive level of consumption.[10] Granted this, the interesting question is what is the *appropriate* level of investment. It is here that a connection is being made with suggested procedures for adjusting NNP to include the value of the net disinvestment of resource stocks.

The claim is that:

(a) the appropriate net investment in produced capital goods should be the value of the resource stocks which are used up; that is, the so-called *Hotelling rents* from the resource used.

This appears to be linked to two other claims:

(b) properly defined and properly calculated, this year's NNP can be regarded as this year's interest on society's total stock of capital interpreted in the broadest sense.

[10]Cass and Mitra (1991) have analyzed this in detail in a general framework.

(c) NNP indicates the largest consumption level that can be allowed this year if future consumption is never to be allowed to decrease.

Claims (b) and (c) together can be interpreted as saying that if the goal of investment policy is not to erode sustainable income then the aggregate capital stock must be maintained intact and this, presumably, leads to (a).

Below, we first sketch some arguments that have appeared in the literature which purport to provide a more precise form of these claims. Consider the framework of discussion of WR in the previous section. If a path $\langle U^*(t), I^*(t), k^*(t)\rangle$ from $\underline{k}$ is optimal then, evaluating NNP using the supporting prices $p^*(t)$, WR is valid. Suppose now that

$$p^*(t)\,\dot{k}^*(t) = 0 \quad \text{for each } t \tag{38}$$

Note that $k^*(t)$ and $p^*(t)$ are the vectors of capital stocks and their prices. If the ith component of k is an exhaustible resource then $\dot{k}_i^*(t)$ is always non-positive and $-\dot{k}_i^*(t)$ is the flow of the resource used in production. The price of this resource is $p_i^*(t)$, the 'Hotelling rent'. Then (38) states the investment rule mentioned in claim (a) above. Hartwick (1977) first stated the rule in this general form. We shall, henceforth, refer to (38) as *Hartwick's Rule* or HR for short. HR implies that $Y^*(t) = u^*(t)$. Using this in WR, and taking the time derivative on both sides of (26) yields the conclusion that $u^*(t)$ is constant [see Solow (1986) for details].

It is worth noting that the conclusion that $u^*(t)$ is constant may be derived solely from the competitive conditions (19) and (20), and HR. If a path satisfies all three conditions then, as we have seen in the second section, the competitive conditions imply [see (29)]:

$$\dot{Y}^*(t) = r(Y^*(t) - u^*(t)) = rp^*(t)\,\dot{k}^*(t) \quad \text{for all } t \tag{39}$$

HR then implies $Y^*(t)$ is constant, Finally, HR says that $Y^*(t) = u^*(t)$. This shows that if a path is competitive and satisfies HR, then utility $u^*(t)$ is a constant, say U^*. If, in addition, the path is also optimal, then clearly, it is also a *maximin* path [see (Solow (1974)];[11] i.e. there is

[11] A path $\langle U'(t), k'(t), I'(t)\rangle$ from $\underline{k}$ is a maximin path if and only if for every feasible path $\langle U(t), k(t), I(t)\rangle$ from $\underline{k}$, $\operatorname{Inf}_t U'(t) \geq \operatorname{Inf}_t U(t)$.

no other path $\langle U(t), I(t), k(t)\rangle$ from the same initial stocks $\underline{k}$ with $\inf_t U(t) > u^*$. So, there is no other path from $\underline{k}$ along which a larger utility can be sustained. This summarizes the arguments behind claim (a).

Turning now to claim (b), consider again an optimal path $\langle k^*(t), I^*(t)\rangle$ with its price support $p^*(t)$. Denote by $V(k)$ the value function at time $t = 0$ [see (30)]. Denote by $V^*(t)$ the value $V[k^*(t)]$. WR says $Y^*(t) = rV^*(t)$ for each t. Examining the time derivative of $V^*(t)$ and using standard duality theory for infinite horizon optimal growth models [see Benveniste and Scheinkman (1979)] it may be concluded that [see Hartwick (1994) for details]

$$Y^*(t) = Y^*(0) + r\int_0^t p^*(s)\,\dot{k}^*(s)\,ds \text{ for each } t \qquad (40)$$

Once again, we may observe that this equation can be derived solely from the competitive conditions (19) and (20). This is because, as noted earlier, they imply (39). Taking definite integrals on both sides we obtain

$$Y^*(t_2) - Y^*(t_1) = r\int_{t_1}^{t_2} p^*(s)\,\dot{k}^*(s)\,ds \text{ for any } t_2 \geq t_1 \geq 0 \qquad (41)$$

This, essentially, seems to be the formal statement corresponding to claim (b). Hartwick (1994) interprets (40) as showing that the current NNP, in utils, is interest on past accumulations of investments valued at the current price. It seems more accurate to state, as Solow (1986) does, that the *increment* in NNP, from t_1 to t_2, is representable as interest[12] on the accumulation of value of investment over the period t_1 to t_2.

Turning our attention now to claim (c), this is, of course, true if a path from initial stocks $\underline{k}$ is *optimal and also maintains a constant consumption* (or utility). This is because, by WR and $u(t) = u^*$ for all t, it follows that NNP, $Y^*(t)$, equals u^*. Then u^* must be the maximum

[12]Some effort has also been made to proceed from (40) to a definition of a suitable stock concept, and to the interpretation that it is such a stock which is being maintained intact under HR, and that NNP is the interest on such a stock. For details the reader should consult Solow (1986) and Svensson (1986).

[13]By maximum sustainable consumption from k we mean a level u such that there is a path $\langle U(t), I(t), k(t)\rangle$, from k satisfying $U(t) = u$ for all t, and that there is no other path $\langle U'(t), I'(t), k'(t)\rangle$ from k satisfying $\inf_t U'(t) > u$.

sustainable consumption[13] from $k^*(t)$ at each t, because otherwise there is a feasible path $\langle U(\tau), I(\tau), k(\tau)\rangle$ starting from $k^*(t)$ at time t, which provides $U(\tau) > u^*$ for $\tau \geq t$, contrary to the optimality of the given path. We may note along the way that $Y^*(t)$ constant implies, by (39), that $p^*(t)\,\dot{k}^*(t) = 0$ for all t; that is, HR is satisfied.

Claim (c), however, appears to be a lot more general than this. It seems to suggest that NNP, $Y^*(t)$, along any optimal path, measures the maximum sustainable consumption from $k^*(t)$, for each t. This, however, will not be true in general.

Consider the familiar one sector optimal growth model of the Cass–Koopmans type. Labour is assumed to be fixed and all magnitudes are in per capita form. The gross production function is denoted by f; then $y = f(k)$ is the output of the single good which can be achieved from input $k \geq 0$ (of the same good) and which is distributed over current consumption, c, and gross investment. It is assumed that f is a non-negative function, which is C^2 on the positive reals and strictly concave, with $f' > 0$ and $f'' < 0$ for $k > 0$; $f'(k) \to \infty$ as $k \to 0$ and $f'(k) \to 0$ as $k \to \infty$. Capital is assumed to depreciate at a constant exponential rate $\mu > 0$. Denote the output net of depreciation by $\phi(k) = f(k) - \mu k$. Let w denote a utility function defined over non-negative reals and C^2 on the positive reals, with $w'(c) > 0$ and $w''(c) < 0$ for $c > 0$, and $w'(c) \to \infty$ as $c \to 0$.

In the terminology adopted here, in this model,

$$S(k) = \{(U, I) \mid \text{there is } c \geq 0$$

such that $$c + I \leq \phi(k),\ I \geq -\mu k \text{ and } U = w(c)\}$$

$$F = \{(k, I)/{-\mu k} \leq I \leq \phi(k) \text{ and } k \geq 0\}$$

$$u(k, I) = w(c), \text{ where } c = \phi(k) - I, \text{ for } (k, I) \in F$$

It is known that there is a unique modified golden rule capital stock k^r, defined by $\phi'(k^r) = r$ which has the property that along the optimum path from k^r, denote it by $\langle k^r(t), I^r(t)\rangle$, $k^r(t) = k^r$ for all t, consumption $c^r(t) = \phi(k^r(t))$ is a constant, say c^r, and the associated prices $p^r(t)$ are also constant, say p^r. There is a unique optimum path, denote it by $\langle k^*(t), I^*(t)\rangle$, from any initial $\underline{k} \neq k^r$, with $\underline{k} > 0$, which is strictly increasing (decreasing) if $\underline{k} < k^r$ ($\underline{k} > k^r$), and along the optimal path $k^*(t) \to k^r$ as $t \to \infty$. It is straightforward to verify that if $\overline{k}$ denotes the golden rule stock [i.e. $\bar{k}$ is defined by $\phi'(\bar{k}) = 0$ and the initial stock $\underline{k} \le \bar{k}$ then $\phi(\underline{k})$ is the maximum sustainable consumption $\underline{k}$, that the unique path which attains this is the constant path $k(t) = \underline{k}$ for all t, and that this is an efficient path.[14]

Consider such a constant path $k(t) = \underline{k}$ for all t, with $0 < \underline{k} < k^r$. Then, the maximum sustainable consumption (utility) from $\underline{k}$ is $u\,\phi(\underline{k})$. Let $u^*(t)$ denote the utility at time t along the optimal path from $\underline{k}$. Then, we have

$$\int_0^{\infty} e^{-rt}\, u^*(t) > u(\phi(\underline{k}))/r$$

by uniqueness of the optimal path (which is distinct from the constant path, since $k^*(t)$ converges to k^r as $t \to \infty$. But, using WR for optimal paths,

$$\int_0^{\infty} e^{-rt}\, u^*(t) = Y^*(0)/r$$

where $Y^*(0)$ is the NNP on the optimal path at time $t = 0$. Thus, we must have

$$Y^*(0) > u(\phi(\underline{k}))$$

so that the NNP at time $t = 0$ does *not* measure the maximum sustainable consumption (utility) from $k^*(0)$.

The above discussion also serves to underscore the fact that the kind of cases to which claim (a) may be applicable is very limited in scope, far more so, than the kind of situations where one may sensibly speak of maximum sustainable consumption through efficient allocation of resources. If we consider the class of competitive paths in this model (from any initial stock $\underline{k} > 0$), which also satisfies HR, then by (39), along a competitive path $\langle k(t), p(t)\rangle$, it follows that, since $p(t)\,\dot{k}(t) = 0$, $Y(t)$ must be a constant and $Y(t) = u(t) = w(c(t))$. Thus, $c(t)$ is a constant. It may be verified that current prices $p(t) = w'(c(t))$. So $p(t)$ is a constant, and therefore, the present value price $e^{-rt}\,p(t) \to 0$ as $t \to \infty$. As is well known, this together with the competitive conditions, implies that the path is optimal. In such a framework then, the class of paths under consideration consists only of the unique stationary path

[14]A path is efficient if there is no other path, from the same initial conditions, which provides consumption at least as large as that along the given path at all dates and strictly larger at some date (see Cass (1972)).

from k^r. The class of paths in which the maximum sustainable consumption is the constant consumption along the path, and which is also efficient, is therefore much larger than that which can be characterized by the competitive conditions (19) and (20), together with HR. In the model considered above, there is a stationary optimal path, which is the *only* path where both these conditions are satisfied. In models including exhaustible resources, where typically there are no stationary optimal paths, even though there are efficient paths in which consumption is constant, it is unclear whether there are *any* paths in which the dual sets of conditions, (19) and (20), and (38) are satisfied.

An interesting variant of these questions ought to be mentioned. We could ask whether the value of aggregate net investment is an indicator of whether or not current consumption is below or above the maximum sustainable level of consumption from the current stock of capital. In the one sector model, for example, it is easy to see that it indeed is such an indicator along optimal paths from $\underline{k} < k^r$ (the modified golden rule). Because, as we noted above, when $\underline{k} < k^r$, the optimal path is monotone strictly increasing, and so, current consumption $c^*(t)$ is below the maximum sustainable level of consumption $\phi(k^*(t))$. At the same time, $p^*(t)\,\dot{k}^*(t) > 0$, since $k^*(t)$ is strictly increasing. The reverse is true when $\underline{k} > k^r$. The sign of $p(t)\dot{k}^*(t)$ is, therefore, an indicator of whether or not current consumption is below or above the maximum sustainable level from current stocks. Asheim (1994) considers this possibility in detail in the context of an optimum growth model including an exhaustible resource. Drawing on results from Dasgupta and Heal (1974, 1979), he shows that in general the value of net investment in capital stocks (including exhaustible resource stocks) is *not* an indicator of sustainability in this sense.

The discussion above indicates that when we consider in some detail the extent of validity and the richness of interpretation of the claims surrounding the links between NNP and sustainable development, the results are not very encouraging. We will conclude on a somewhat more positive note, making the following observation due to Weitzman (1995). If, by a path of sustainable development, we do not mean maintaining the maximum sustainable consumption level at each date, but rather the weaker, but sensible, requirement that welfare, as measured by the discounted sum of consumption criteria, of future generations is at least as much as the welfare of the current generation, then, for paths satisfying WR, the value of net investment is an indicator of whether or not the path considered is one of sustainable development. The welfare of future generations can be represented by the constant consumption equivalent of the future flow of consumption and this is measured by NNP. This is larger than current consumption if and only if the value of net investment is positive.

5. SOME REMARKS AND QUESTIONS OF INTEREST

In a discrete time model of one produced good and one exhaustible resource, Dasgupta and Mitra (1983) provide a complete characterization of constant consumption paths, which are also efficient (and hence, consumption in each period along such a path is the maximum sustainable consumption from the capital stocks in that period), in terms of supporting prices. The prices are present value prices, in the sense that the price of each good is expressed in units of the single consumption good at time *0*. These are prices associated with an efficient path [see Cass (1972) and Mitra (1978)]. At these prices, intertemporal present value profit maximization conditions are satisfied along an efficient path in each period. Compared to the kind of price support properties discussed in the second and third sections, the distinguishing features of such a characterization are:

(i) the absence of any discount factor for the consumption good; and

(ii) the presence of a price support property for an appropriate value function, in the tradition of such support properties familiar from the optimal growth literature [see Weitzman (1973)]. This value function shows the level of maximum sustainable consumption which is possible from initial capital stocks.

There is no restriction on the kind of paths to which the characterization applies, other than strict positivity of initial stocks. Condition (ii), especially, makes it clear that such prices and the support properties are to be interpreted as planner's prices and planner's rules.

Solow (1974), working with a Cobb-Douglas production function, obtains closed form solution for constant consumption paths along which HR is satisfied. Hartwick (1977) extends this to general production functions and shows that paths which are price supported (by efficiency prices, no discount rate is involved) and which satisfy HR are constant consumption paths. They work in a continuous time framework and they assume efficiency of the path. Whether HR is a necessary condition for efficient constant consumption paths is a question they do not address.

In discrete time formulation, the rule that investment in produced capital equals the value of net depletion of exhaustible resource stocks, is not satisfied at any time along efficient constant consumption paths, regardless of which of the two sets of prices (beginning or end of period prices) are chosen for the valuation. However, there is an analogue of the rule in discrete time in the form of an inequality. In each period, investment in the produced capital good takes a value between an upper and a lower value, which are the values of the depletion in the resource stock during that period, evaluated according to the beginning and end of period prices, respectively. HR in its equality form may be said to hold asymptotically, since the difference between the upper and lower values in the inequality, asymptotically converges to zero [see Dasgupta and Mitra (1983) for details].

All this would suggest that a similar difference may be expected in the formulation of Weitzman's Rule, between continuous and discrete time frameworks, since the net investment component may be evaluated according to the end of period or beginning of period prices. Indeed, our analysis in a discrete time model shows that the constant consumption equivalent of an optimal path lies between the two values of NNP calculated according to the two sets of prices for the investment goods.

Throughout our discussion it was assumed that the interest rate is strictly positive. As is well known, when $r = 0$, the welfare function in *Problem 1* (see the second section) may be undefined. However, redefining the origin of measurement of utility to be the maximum sustainable utility (from arbitrary initial stocks), denote this by U^*, it may be possible to speak of optimal paths [see Gale (1967)]. Suppose that the control *Problem 1* is well defined after replacing $U(t)$ by $U(t) = U^*$ for each t, and that the turnpike theorem is valid, i.e. the capital stocks along the optimal path from $\underline{k}$, denoted by $\langle U^*(t), I^*(t), k^*(t) \rangle$, converge to k^*, the golden rule stocks from which there is a stationary path with utility equal to U^*. Along the optimal path, the Hamiltonian will be constant and equal to zero. Since the RHS of (26) is now zero, equation (26) is valid. Remembering that, in this formulation, the origin of measurement of utility is U^*, this says that the term $U^*(t) + p^*(t)\, I^*(t)$ is a constant, and equals U^* for all t, where $p^*(t)$ are the prices associated with the optimal path from $\underline{k}$. This seems to lead to an interesting observation, namely, that in a stationary framework, growth in NNP signals non-optimality and/or time preference.

We should note, however, that while WR is valid in the sense of equation (26), its main purpose, which is to show that NNP equals the constant utility equivalent $\overline{u}$, defined by (23), is not achieved here. Indeed, in general, $\overline{u}$ is not defined. This is because the LHS of (23) is finite only when $\overline{u}$ is zero. However, the RHS of (23) is well defined along an optimal path and, in general, will not be zero and will be changing over time. For example, in a one sector model, if the initial capital stock is below the level k^*, then utility and capital increase over time and the welfare along the optimal path, i.e. the RHS of (23), is increasing over time. Unlike the situation in the discounted case, this not reflected in the Hamiltonian, which, as we observed above, is a constant.

In brief then, the validity and interpretation of WR in the undiscounted case is unclear and seems to be an interesting avenue for further exploration.

Finally, throughout our discussion, the interest rate was assumed to be constant over time. Exploring the implications of allowing a variable rate of interest would be an interesting line of investigation. The interested reader may wish to consult Asheim (1997) and Long and Hartwick (1996) for some works which seek to incorporate a variable rate of interest.

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Appendix

Consider the dynamic optimization problem noted in section 2.2, namely,

PROBLEM 1: $$\text{Max} \int_0^\infty e^{-rt}\, u(k(t), I(t))\, dt$$

such that $$(k(t), I(t)) \in S(k(t)) \text{ for each } t$$

$$\dot{k}(t) = I(t) \text{ for each } t$$

and $$k(0) = \underline{k}$$

We shall verify that if a path from $\underline{k}$ is *optimal* then it satisfies the competitive conditions (19) and (20) of section 2.2. Define the associated Hamiltonian function $H(k, P, t)$ by:

$$H(k, P, t) = \text{Max } e^{-rt}\, u(k, I) + PI \tag{A.1}$$

such that $$(k, I) \in F$$

Given p, if we define P by: $P = e^{-rt}\, p$, and conversely, then, in view of (17), it is clear that

$$H(k, P, t) = e^{-rt}\, h(k, p) \tag{A.2}$$

It may then be verified that given a path $\langle U(t), I(t), k(t)\rangle$ from $\underline{k}$, the existence of a time path of prices $p(t)$ at which (19) and (20) are satisfied, is equivalent to the existence of a time path of prices $P(t)$ at which the path satisfies:

$$e^{-rt}\, u(k(t), \dot{k}(t)) + P(t)\, \dot{k}(t) = H(k(t), P(t), t) \text{ for each } t \tag{A.3}$$

and $$\dot{P}(t) = -\frac{\partial H}{\partial k}(k(t), P(t), t) \text{ for each } t \tag{A.4}$$

The prices $P(t)$ are present value prices, discounted from t, at the discount rate r, and are related to the current value prices $p(t)$ by:

$$P(t) = e^{-rt}\, p(t) \text{ for each } t \tag{A.5}$$

The left hand side expression in (A.3) is simply the NNP in present value form and the Hamiltonian is the maximum present value of output which can be produced, given the present value prices P and capital stock k. Equations (A.3) and (A.4) above are (19) and (20), respectively, in their present value form.

For given k, P, write the maximizing value of the control I, in (A.1), as $G(k, P)$. Define the (present value) pre-Hamiltonian function $J(k, P, I, t)$ by:

$$J(k, P, I, t) = e^{-rt}\, u(k, I) + PI \tag{A.6}$$

It is clear from the definitions that the pre-Hamiltonian is maximized at $I = G(k, P)$ for each k, P. So,

$$\frac{\partial J}{\partial I}(k, P, G(k, P), t) = 0 \quad \text{for each } k, P \text{ and } t$$

and
$$J(k, P, G(k, P), t) = H(k, P, t) \quad \text{for each } t \tag{A.7}$$

It follows that:

$$\begin{aligned}\frac{\partial H}{\partial P}(k, P, t) &= \frac{\partial J}{\partial P}(k, P, G(k, P), t) + \frac{\partial J}{\partial I}(k, P, G(k, P), t)\frac{\partial G}{\partial P}(k, P)\\ &= \frac{\partial J}{\partial P}(k, P, G(k, P), t) = G(k, P)\end{aligned} \tag{A.8}$$

and similarly that

$$\frac{\partial H}{\partial k}(k, P, t) = \frac{\partial J}{\partial k}(k, P, G(k, P), t)$$

So, (A.3) and (A.4) are equivalent to:

$$\dot{k}(t) = G(k(t), P(t)) = \frac{\partial H}{\partial P}(k, P, t) = \frac{\partial J}{\partial P}(k, P, G(k, P), t) \text{ for each } t \tag{A.9}$$

and
$$\dot{P}(t) = -\frac{\partial J}{\partial k}(k(t), P(t), G(k(t), P(t)), t) \quad \text{for each } t \tag{A.10}$$

The existence of a price path $P(t)$, at which (A.9) and (A.10) are satisfied, is a necessary condition for a path to be optimal.[15] So, the equations (19) and (20), defining a competitive path, are precisely the necessary conditions for a path to be an optimal solution to the control *Problem 1* above. This represents the 'maximum principle' of Pontryagin *et al.* (1962).

[15]See, for example, Benveniste and Scheinkman (1982). We should note, though, that their problem is in a more general setting since they do not assume differentiability of the utility function.