"Dipak Banerjee . . . will be remembered as one of the most famous teachers of economics at Presidency College.

". . . [He was] an extraordinarily erudite person in areas he developed a liking for. It turned out that he had decided chemistry was not going to be one of those areas."

Tapas Majumdar, *The Telegraph*, January 25, 2007.

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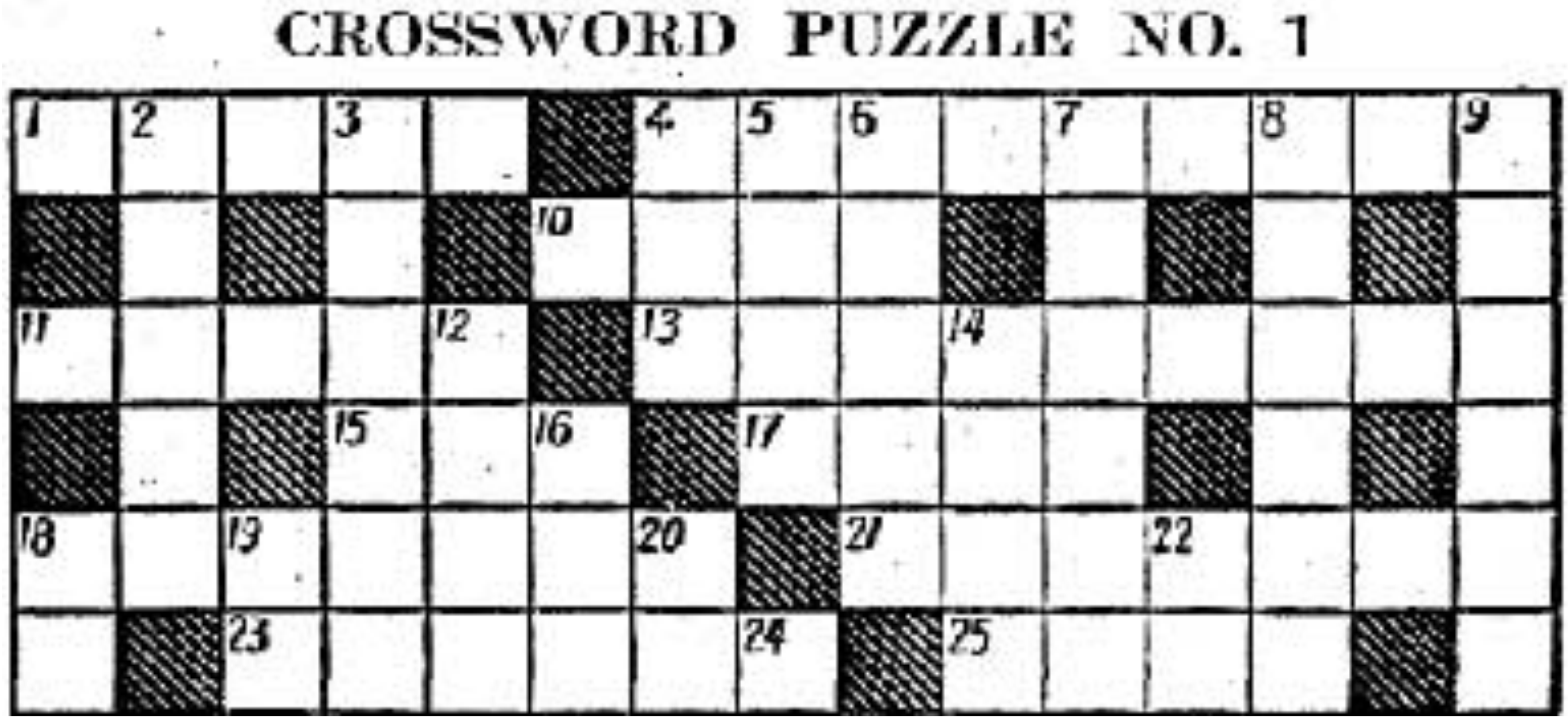
Choice and Order: or First Things First[1]

By D. BANERJEE

'There is certainly no obvious kind of market behaviour which can be called indifferent. How long must a person dither before he is pronounced indifferent?'—I. M. D. Little, *Oxford Economic Papers*, N.S., vol. 1, 1949.

There is some reason to think that the great simplifications in utility theory—achieved in the past dozen years or so—have passed a number of economists by. This is possibly due to the axiomatic presentation favoured by most workers in the field. The excellent survey of consumption theory by Professor H. S. Houthakker[2] has not altogether solved the problem of communication: breadth of scope is often the enemy of emphasis. Moreover, this remarkably lucid survey makes heavy demands on the reader, that he see through a newly-corrected pair of lenses and focus exactly and immediately. The sort of person who can do this easily is, in all probability, one who has been there before, and possesses a fair knowledge of the terrain.

2. The *London Times* crossword:



Crooked course of a Cockney courtship [7]

Status, Intertemporal Choice and Risk

Professor Dipak Banerjee Memorial Lecture

Debraj Ray

New York University

based on joint research with Arthur Robson, Simon Fraser University

Outline

- Study payoffs that depend on relative consumption or status.
- Embed these preferences into standard growth model.
- Describe the equilibrium of such a model.

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- Embed these preferences into standard growth model.

- Describe the equilibrium of such a model.

- Particular focus today: emergence of risk-taking

- lotteries, occupational choice, financial markets, property speculation . . .

- Friedman (1953) directly addressed the efficiency of risk-taking:

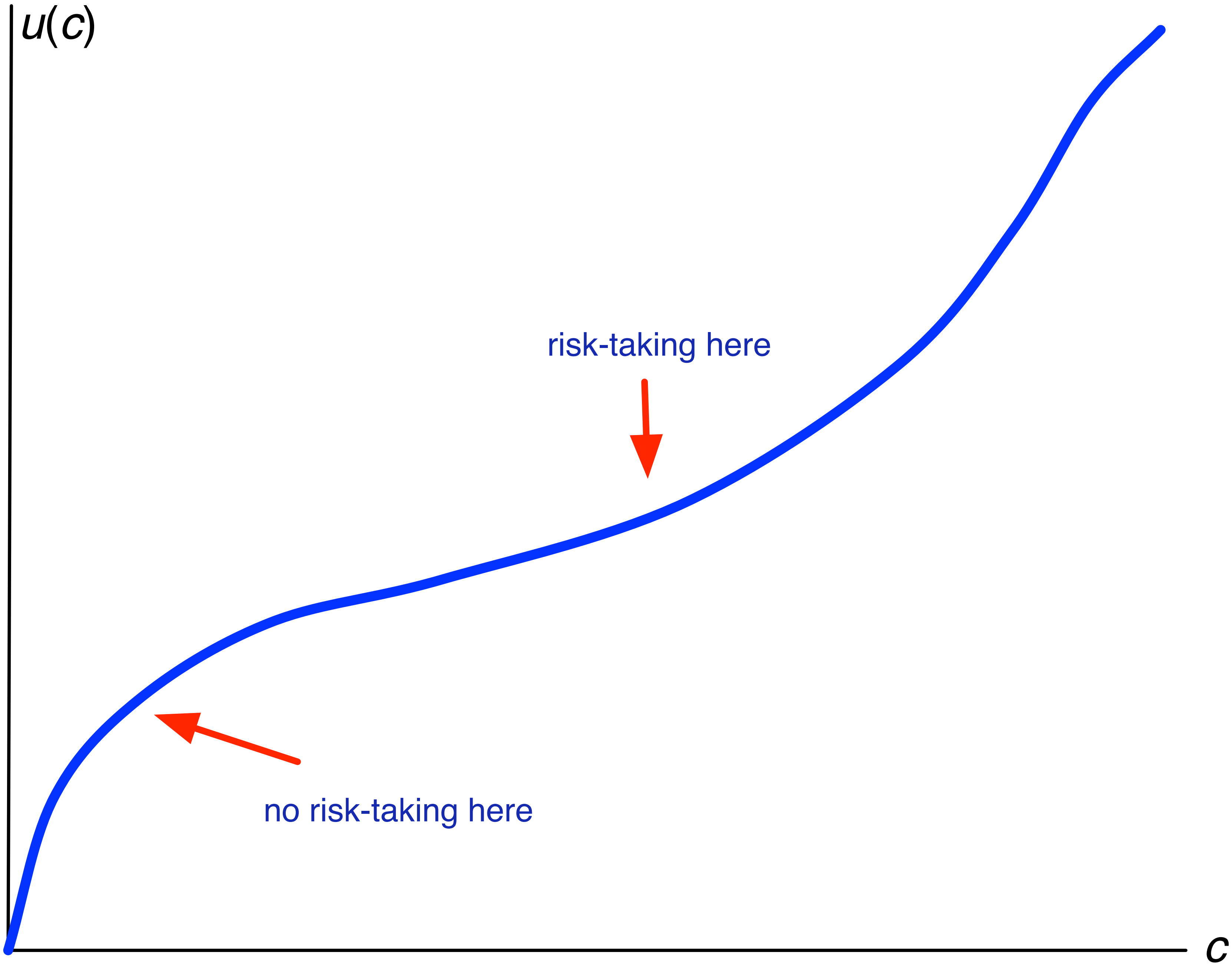
"Differences in tastes [for risk] will dictate different choices from the same alternatives.

"These will be reflected most clearly [in the] allocation of resources to activities devoted to manufacturing the kind of risk attractive to individuals . . .

"The inequality of income in a society may be regarded in much the same way as the kinds of goods that are produced, as at least in part — and perhaps in major part — a reflection of deliberate choice in accordance with the tastes and preferences of the members of the society . . ."

- For Friedman, all risk-taking was "deliberate" and therefore efficient
  - Including asymmetric treatment of downside/upside risk.

- The famous Friedman-Savage (1948) utility function:



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"Diminishing marginal utility plus maximization of expected utility would imply that individuals would always have to be paid to induce them to bear risk. But this implication is clearly contradicted by actual behavior. People not only engage in fair games of chance, they engage freely and often eagerly in such unfair games as lotteries."

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- In their paper, they launch an incisive attack on believers in diminishing marginal utility.

- The Friedman-Savage rationalization for increasing marginal utility:

"A possible interpretation of the utility function . . . is to regard the [concave] segments as corresponding to qualitatively different socioeconomic levels, and the [convex] segment to the transition between the two levels.

"On this interpretation, increases in income that . . . shift [the consumer] into a new class, that give [him] a new social and economic status, yield increasing marginal utility."

- There is also the "revealed preference" defence (a bit tricky here).

- Ideological agenda is clear:

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- Their justification of u motivates a related agenda:

  - derive Friedman-Savage-like outcomes from a more primitive model in which utility depends on status;

  - reexamine efficient risk-taking.

- This research has intrinsic value for us anyway:

- Robson's interests in relative income and risk-taking;

- A.J. Robson, "Status, the Distribution of Wealth, Private and Social Attitudes to Risk", *Econometrica* **60**, 837–857 (1992).

- My interest in endogenous evolution of inequality;

- D. Mookherjee and D. Ray, "Persistent Inequality," *Review of Economic Studies* **70**, 369–393 (2003), and "Is Equality Stable?," *American Economic Review* **92**, 253–259 (2002).

- My interest in socially conditioned aspirations.

- D. Ray "Aspirations, Poverty and Economic Change," in Banerjee et al (eds), What Have We Learnt About Poverty, Oxford University Press (2006); G. Genicot and D. Ray, "Aspirations and Growth," mimeo., Georgetown University (2009).

- Utility from relative status is an old idea (see Veblen (1899) and Duesenberry (1949)).

- Yet Robert Frank writes in the *New York Times* (2005):

"In the light of abundant evidence that context matters, it seems fair to say that Mr. Duesenberry's theory rests on a more realistic model of human nature than Mr. Friedman's. It has also been more successful in tracking actual spending. And yet, as noted, it is no longer even mentioned in leading textbooks."

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- A recent literature has begun to develop on the subject.

See, for example, Frank (1985), Robson (1992), Cooper, García-Peñalosa and Funk (1998), Corneo and Jeanne (1998), Kockesen, Ok and Sethi (2000), Brown, Oswald and Qian (2004), Becker, Murphy and Werning (2005), Hopkins and Kornienko (2004, 2006), Ray (2006), Dynan and Ravina (2007) . . .

Intertemporal Choice with Status Payoffs

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- Continuum of dynasties of unit measure.

- Typical dynasty has initial wealth w.

- Wealth divided between lifetime consumption and bequests:

$$w_t = c_t + k_t.$$

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- Wealth divided between lifetime consumption and bequests:

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- Bequests create new wealth via production function (human + physical investments):

$$w_{t+1} = f(k_t).$$

- Same production function for every agent.

- Assume f increasing and differentiable, with $f(0) = 0$.

- F_t = cdf of consumption in society at generation t

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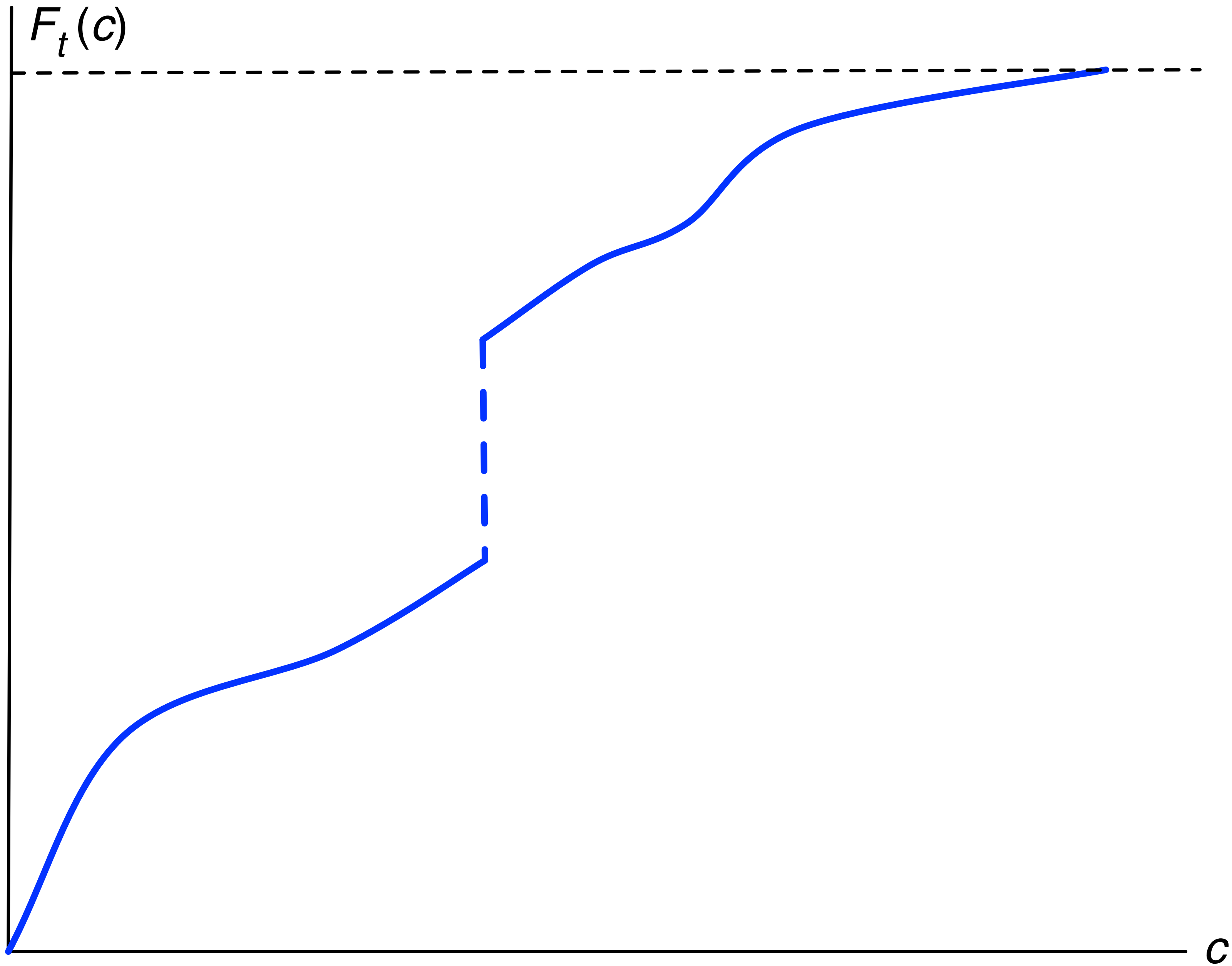

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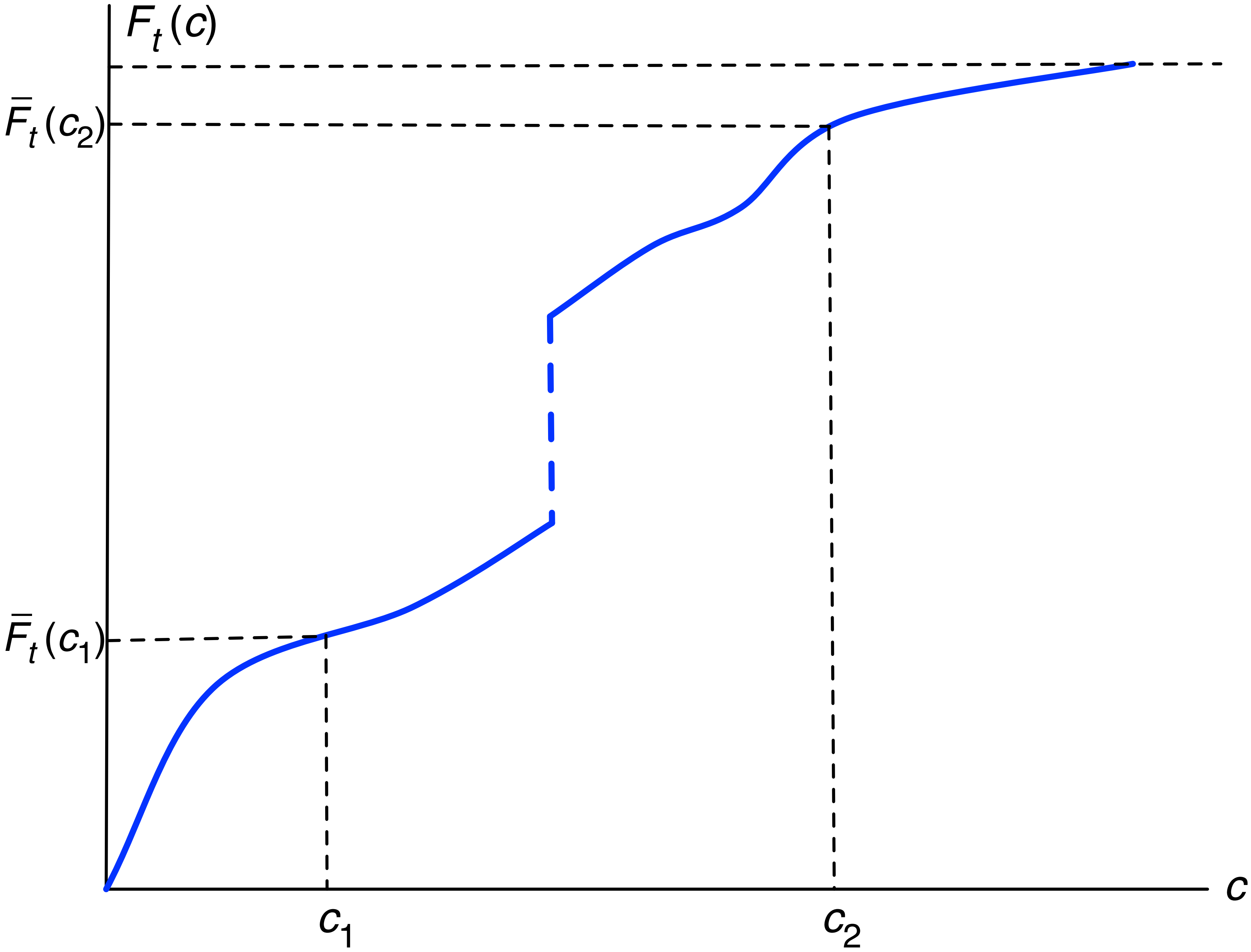

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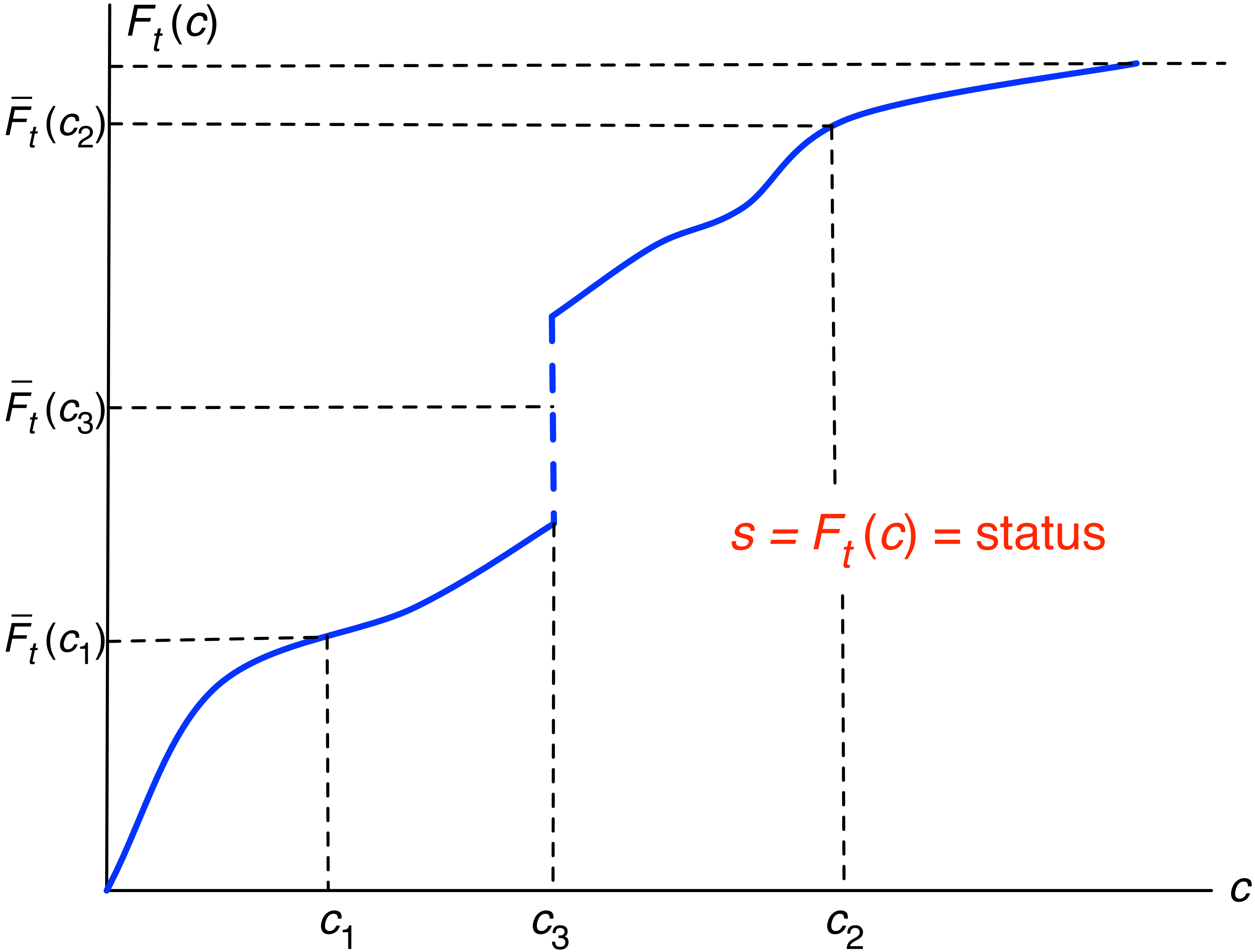

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  - $c =$ consumption and
  - $s = \bar{F}_t(c) =$ status.
- Agent's objective: choose a policy to max

$$\sum_{t=0}^{\infty} \delta^t \mathbb{E} u\left(c_t, \bar{F}_t(c_t)\right)$$

where $\delta \in (0, 1)$ is the discount factor (or cross-generational tie).

  - (what's the $\mathbb{E}$ for?)

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- Zero profit "lottery creators" can create a fair gamble from any fixed upfront payment.

- Setting up a business with risky outcomes, playing the stock market, etc.

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  - Zero profit "lottery creators" can create a fair gamble from any fixed upfront payment.

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- No exogenous uncertainty: risk-taking decisions made endogenously.

  - Agent policy: a (possibly time-dependent) map from starting wealth to a randomization over wealth, then division of realizations into consumption and bequests.

Equilibrium

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- An equilibrium is a sequence of cdfs $\mathbf{F} \equiv \{F_t\}$ for consumption and $\mathbf{G} \equiv \{G_t\}$ for wealth, and a policy for each individual such that

- (i) Each individual policy maximizes expected utility given $\mathbf{F}$.

- (ii) Given G_t, individual policies together generate F_t and G_{t+1}.

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- Production function is convex.

    - Nonconvergent. Deterministic equilibrium.

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    - Convergent. Endogenous randomization.

- Paper studies both cases. Here we emphasize the second.

- Assumption on production function (Solow, Ramsey, Loury):

[fconc] f smooth, increasing, strictly concave, with $\delta f'(0) > 1$, and $f(k) < k$ for all k large.

Convergence and Randomization

- Assumption on production function (Solow, Ramsey, Loury):

[fconc] f smooth, increasing, strictly concave, with $\delta f'(0) > 1$, and $f(k) < k$ for all k large.

- Assumption on utility function:

[ugen] u smooth, nondecreasing and concave in c, increasing in s with $u_s(c, s) > 0$.

If $u_c(c, s) > 0$, then $u_c(c, s) \downarrow$ in c, with $u(c, 1)/c \to 0$ as $c \to \infty$.

- Note 1: no assumption on curvature of u in status.

- Note 2: allow for u to depend on status alone (pure status).

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where b_t is lifetime consumption "budget".

- This budget can be used by the individual to buy any fair bet.

  - The outcomes c_t are "realized consumption".

  - Distribution of c_t over everyone is F_t at date t.

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■ (C) This steady state is unique in the class of all steady states in which almost all individuals have positive wealth.

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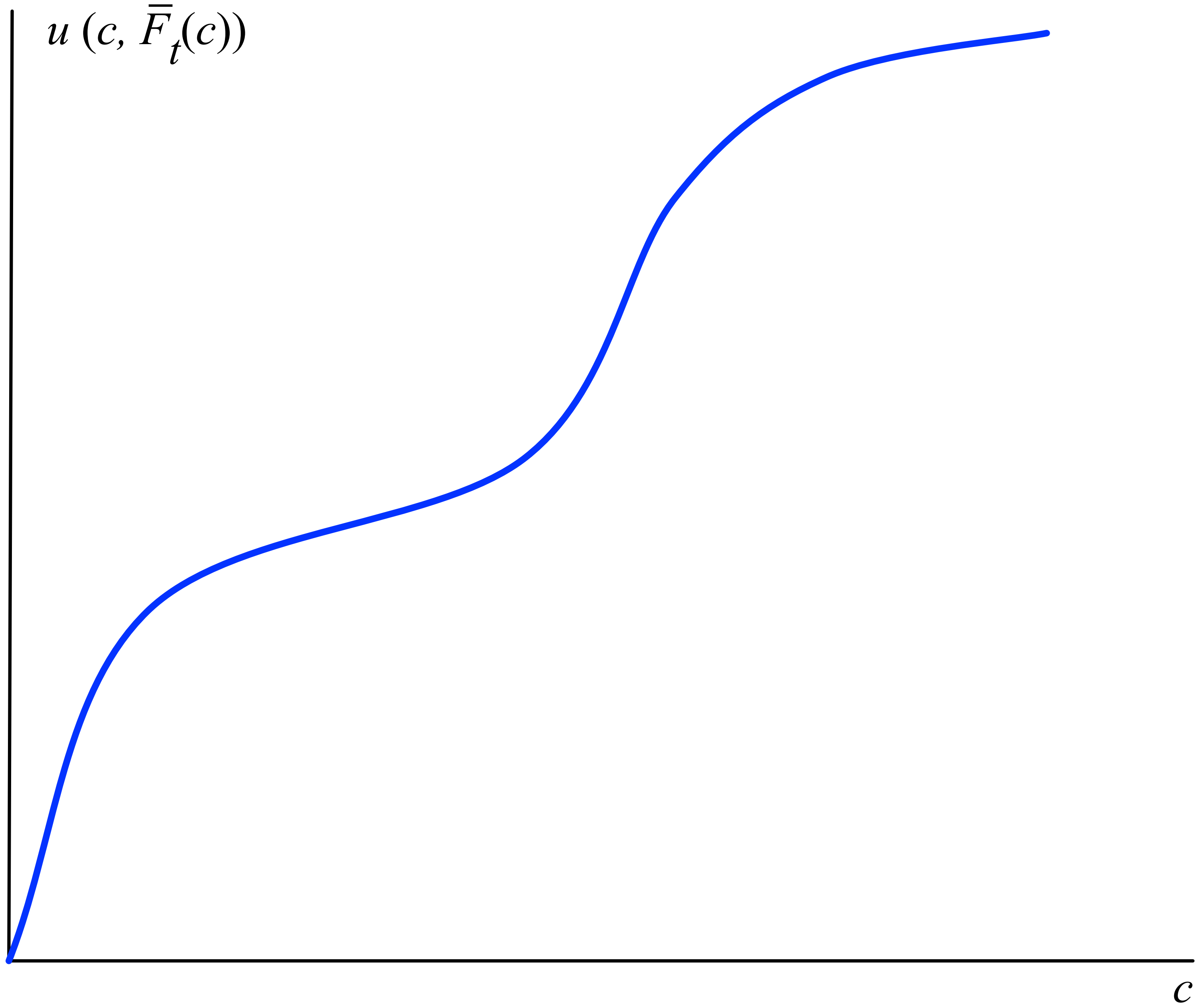

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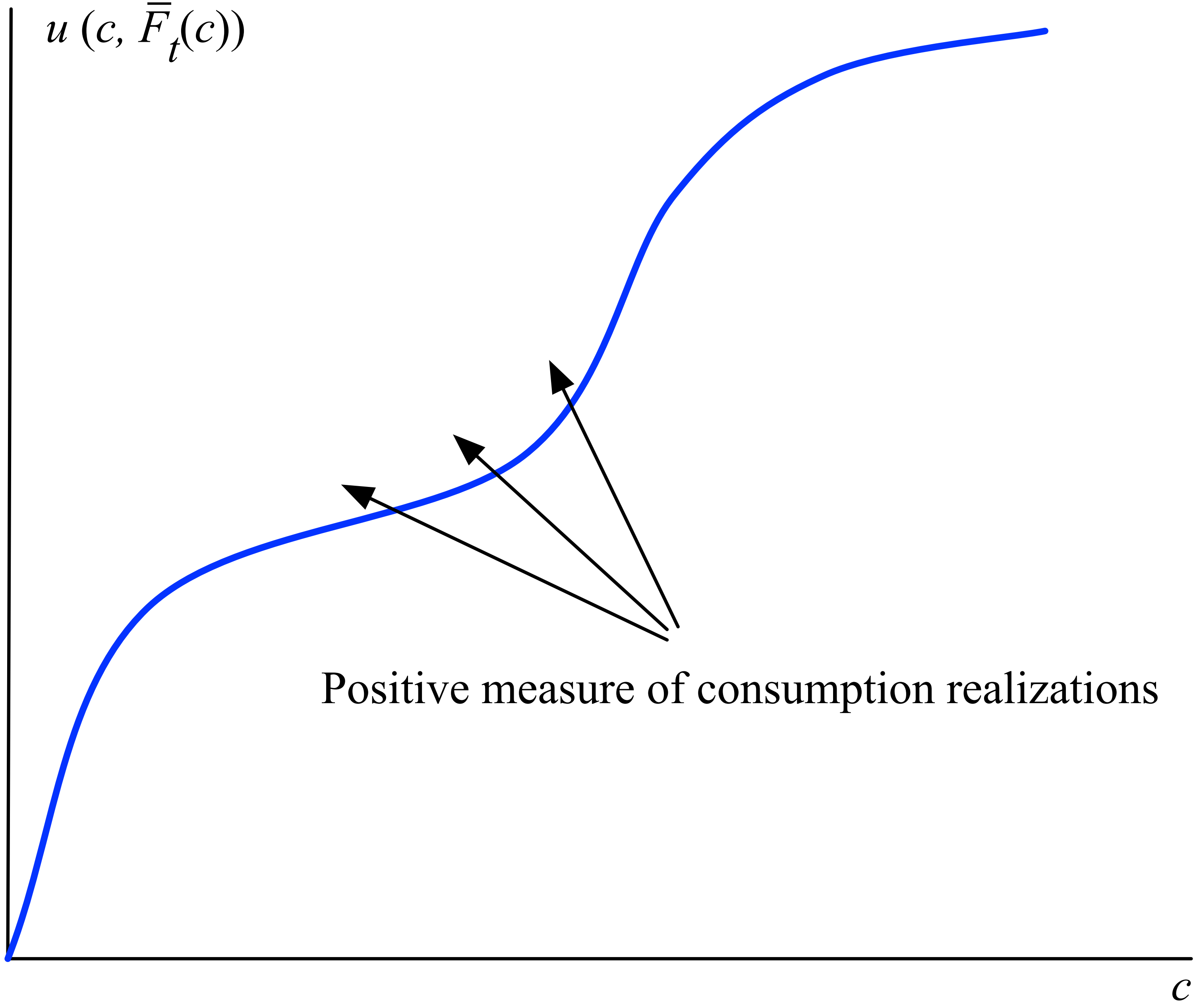

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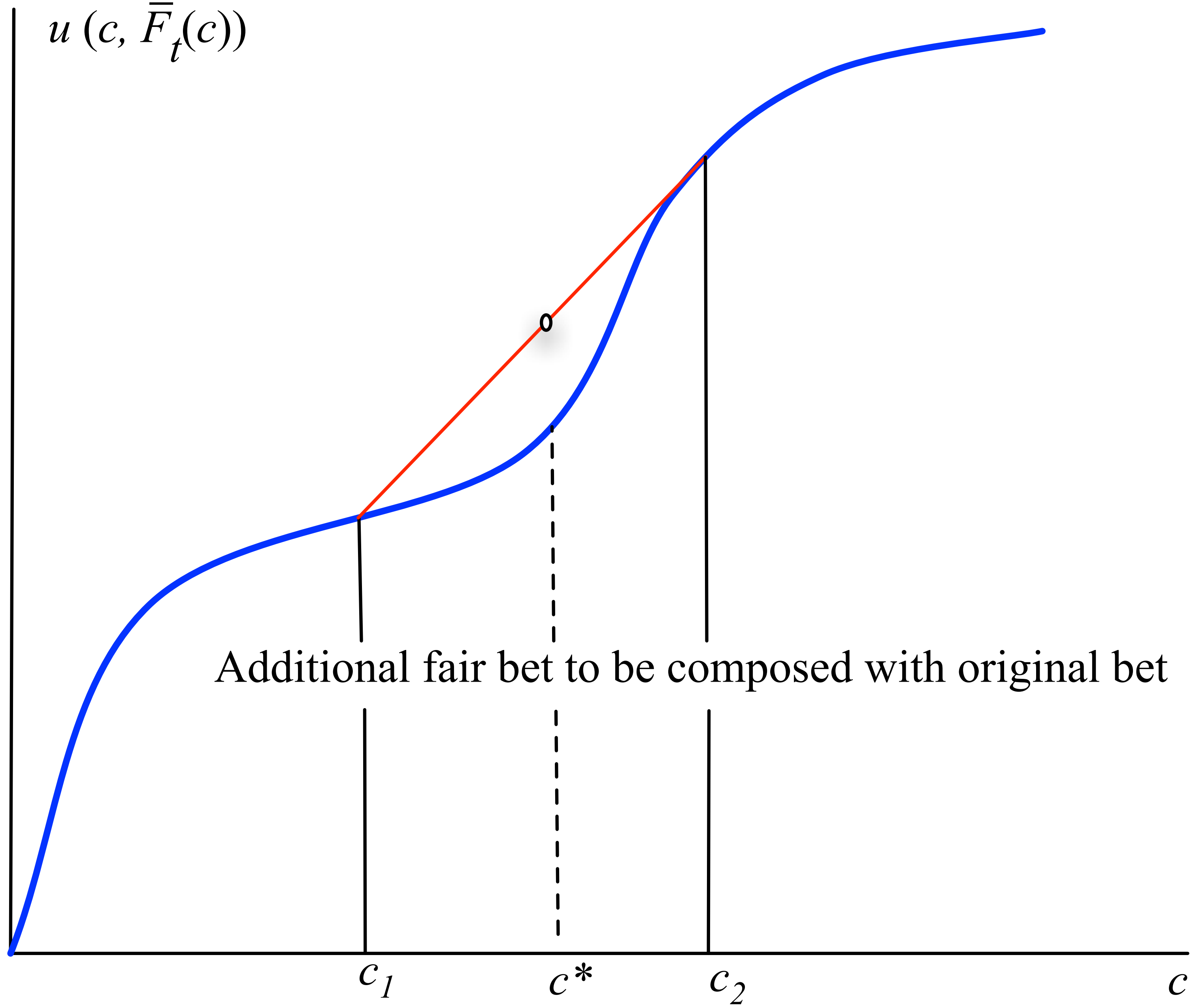

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■ Define $\mu(b) = u(b, F(b))$. By Observation 1, $\mu(b)$ is lifetime utility if consumption budget is b.

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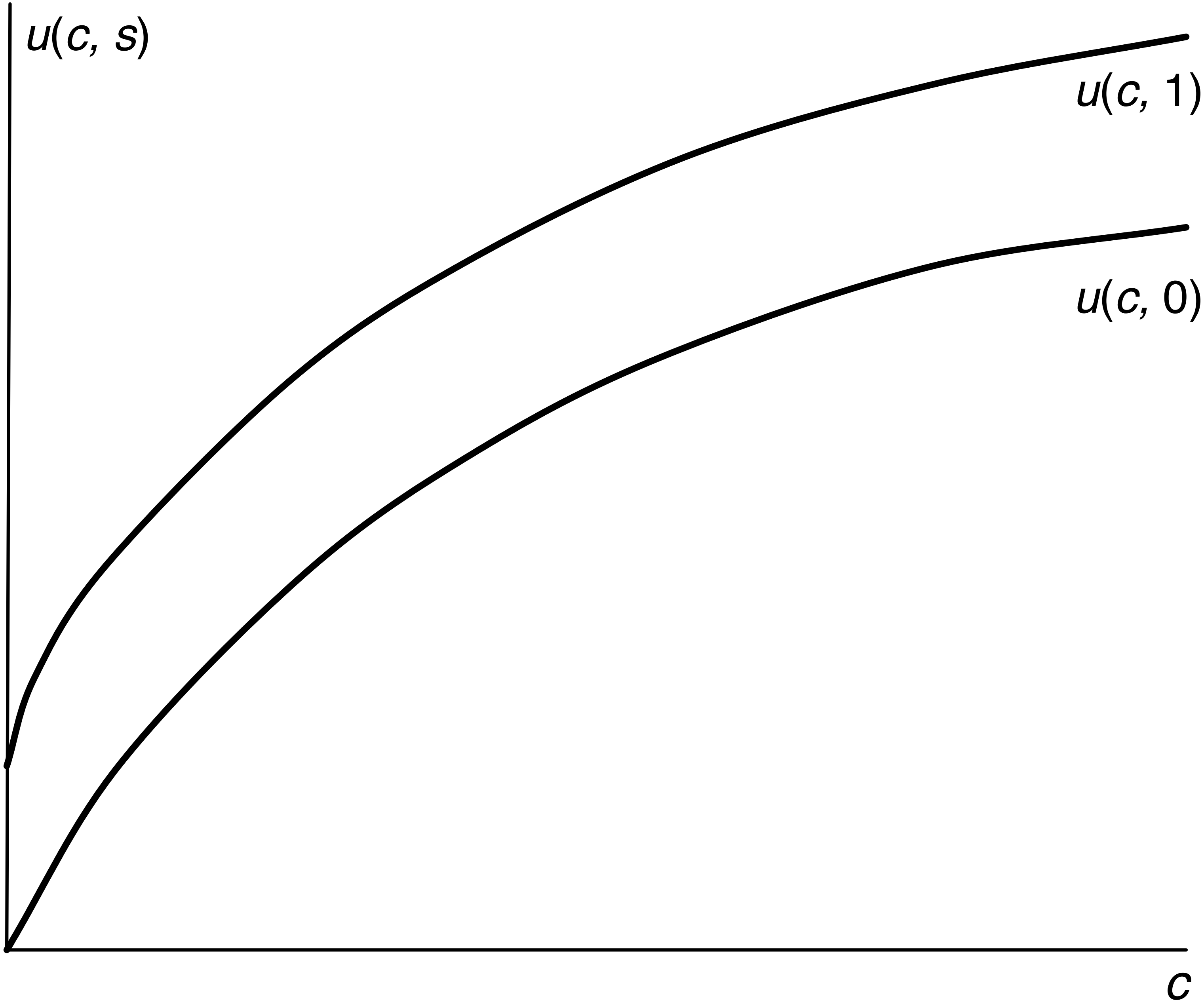

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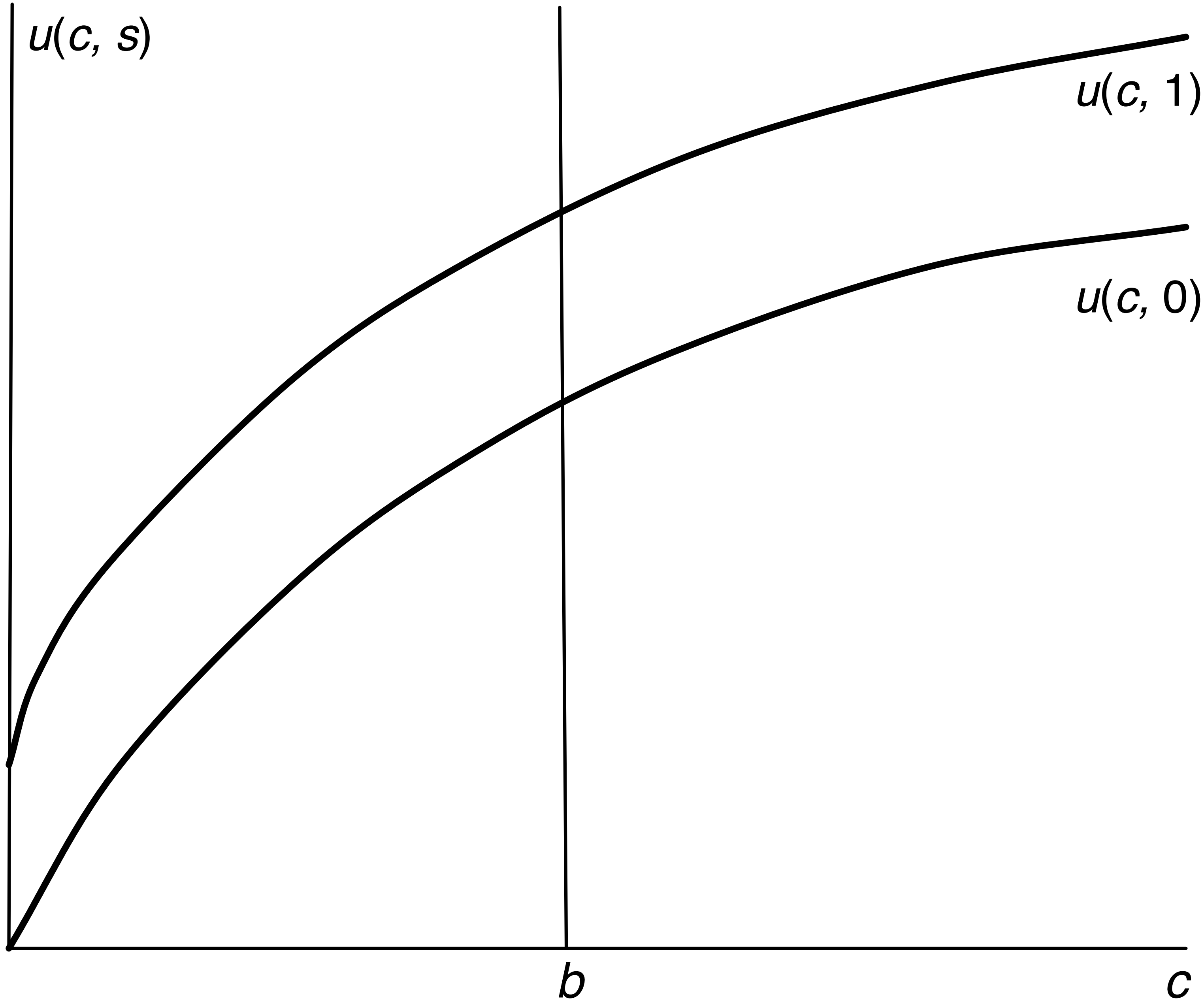

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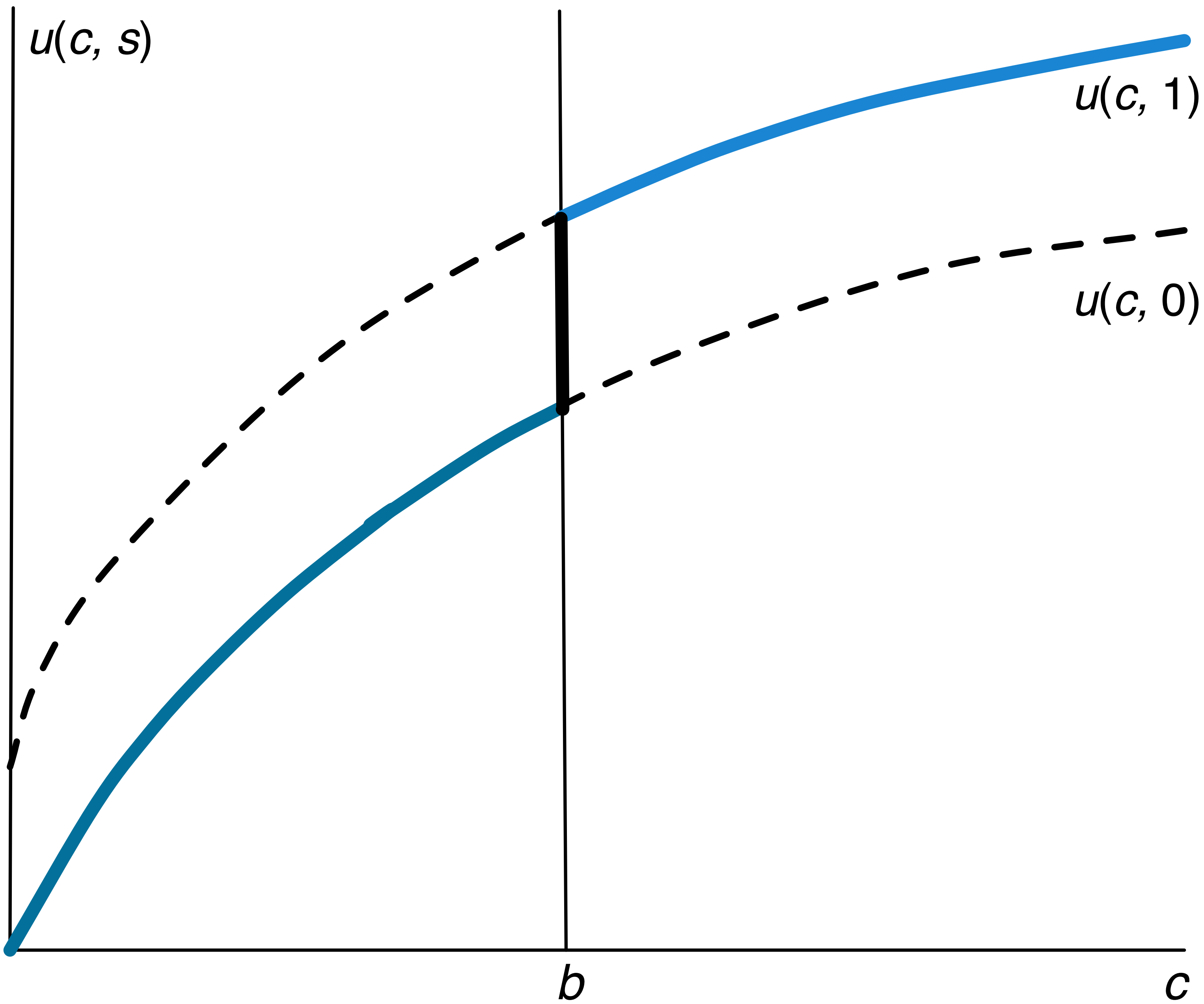

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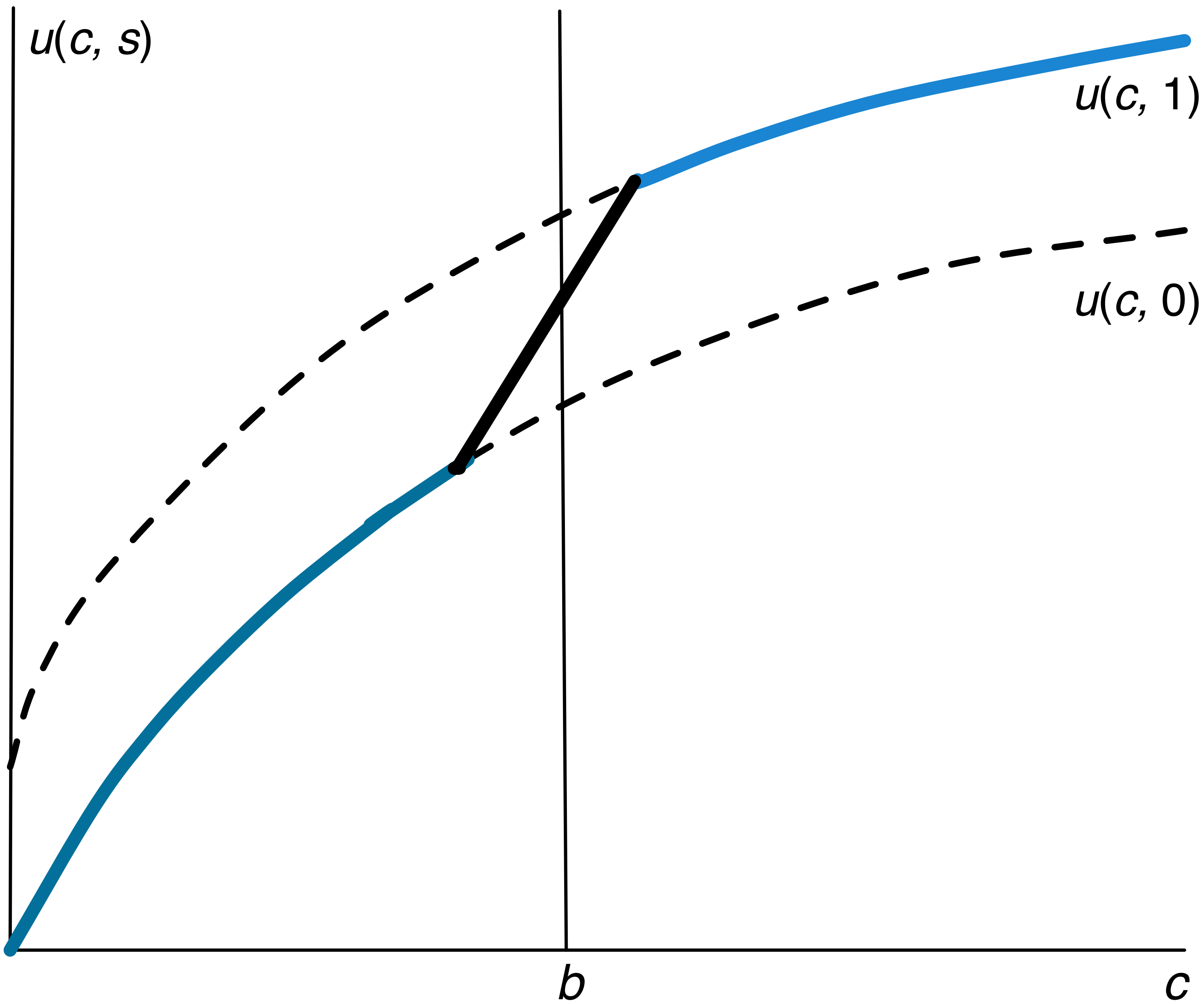

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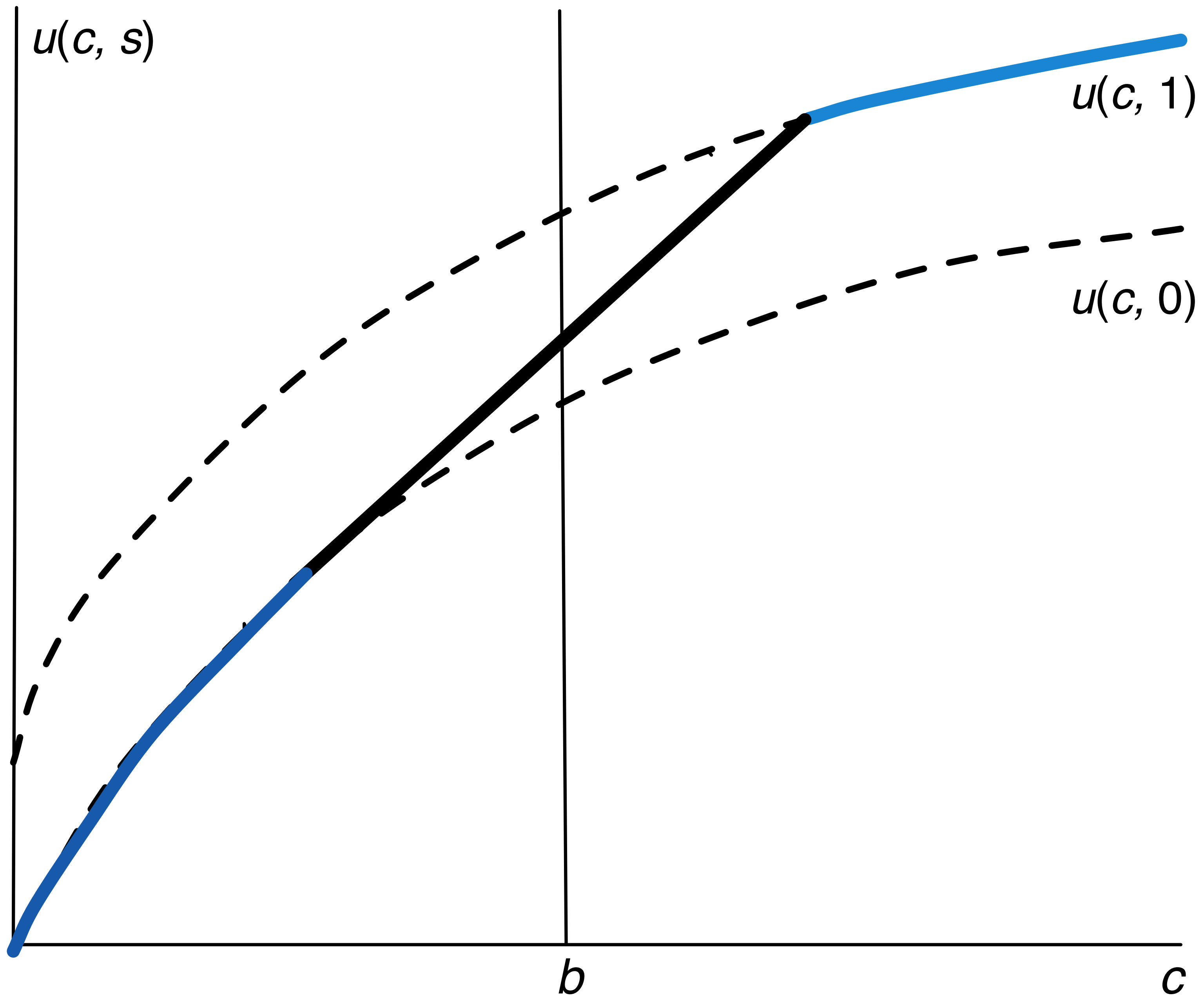

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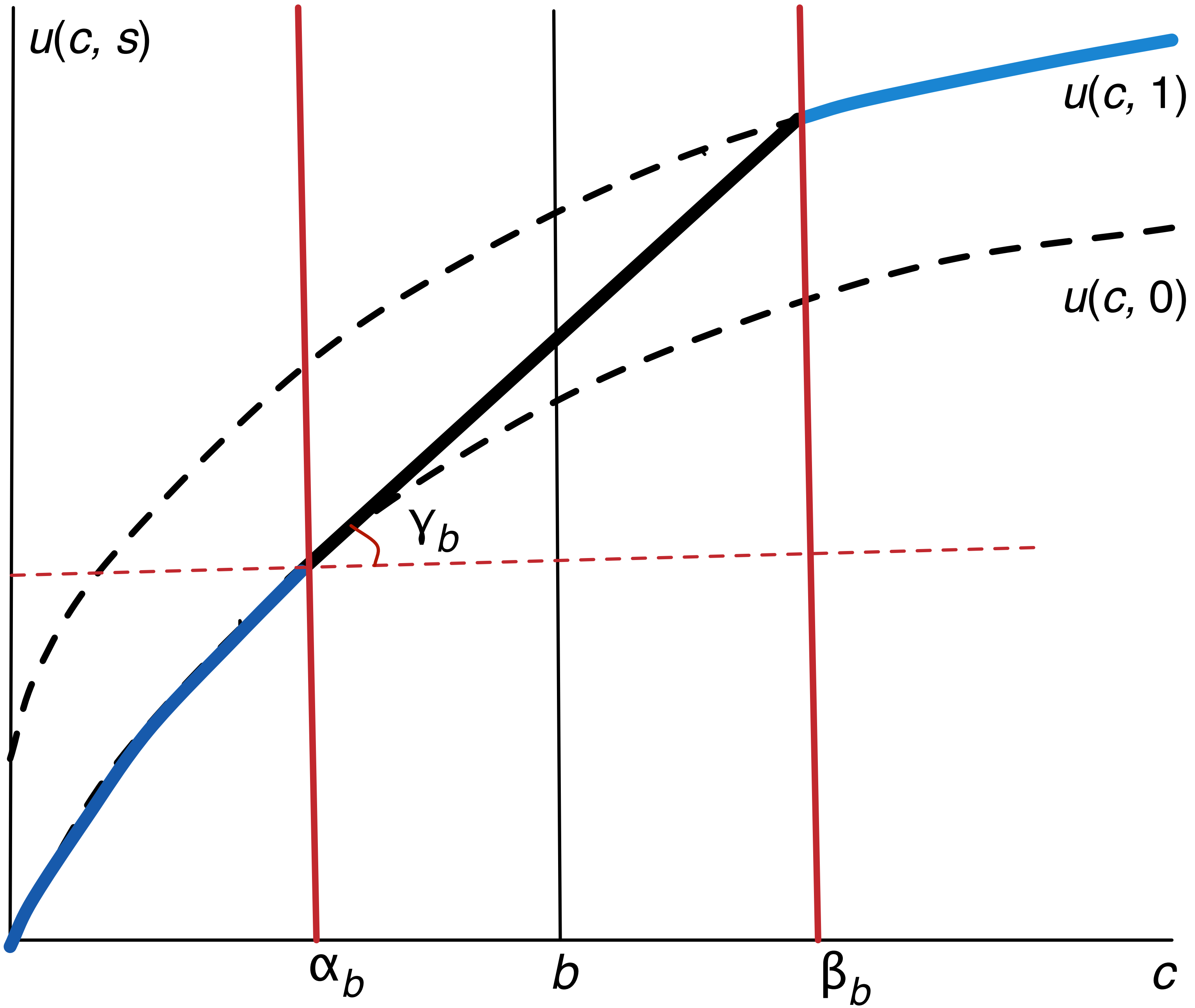

Convergence

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- Proposition. Make Assumptions [fconc] and [ugen].

- Fix any initial wealth distribution, bounded, infimum wealth positive.

- Then any equilibrium sequence of consumption distributions must converge over time to our unique steady state distribution.

Proof

Step 1. $\sup_{i,j} |k_t(i) - k_t(j)| \longrightarrow 0$ and $\sup_{i,j} |b_t(i) - b_t(j)| \longrightarrow 0$ as $t \longrightarrow \infty$.

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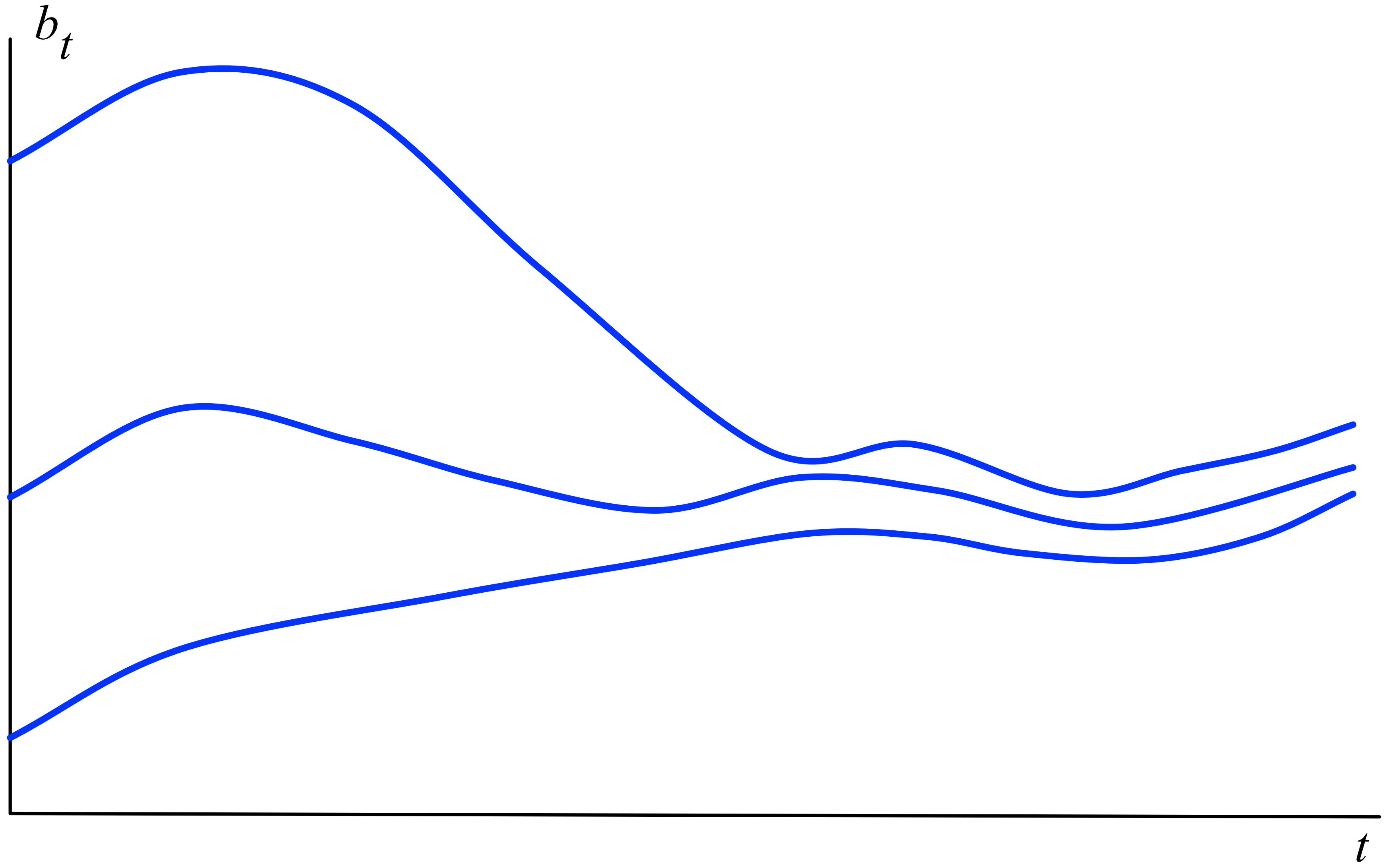

- Mitra-Zilcha turnpike theorem (1981); extension by Mitra (2009).

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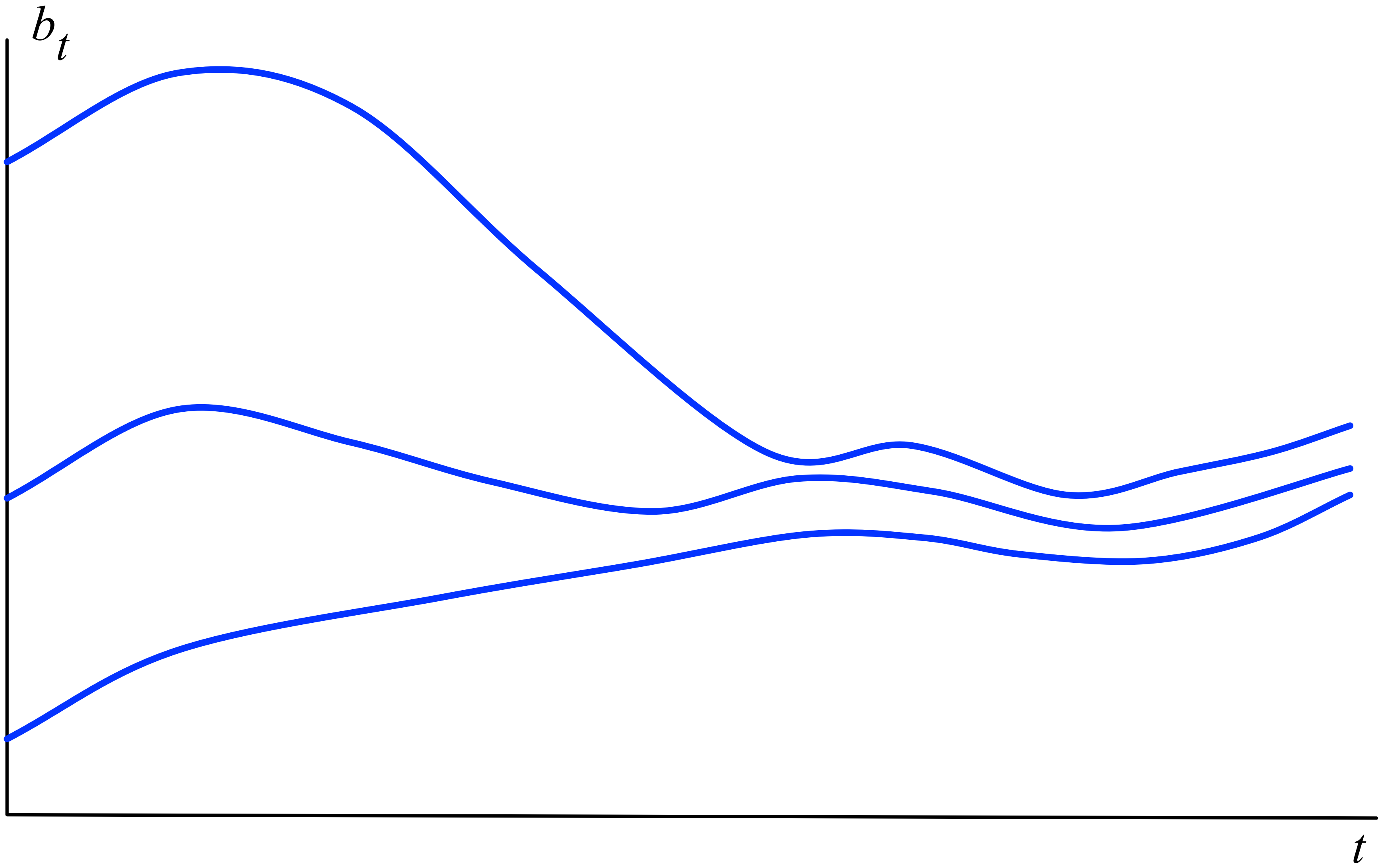

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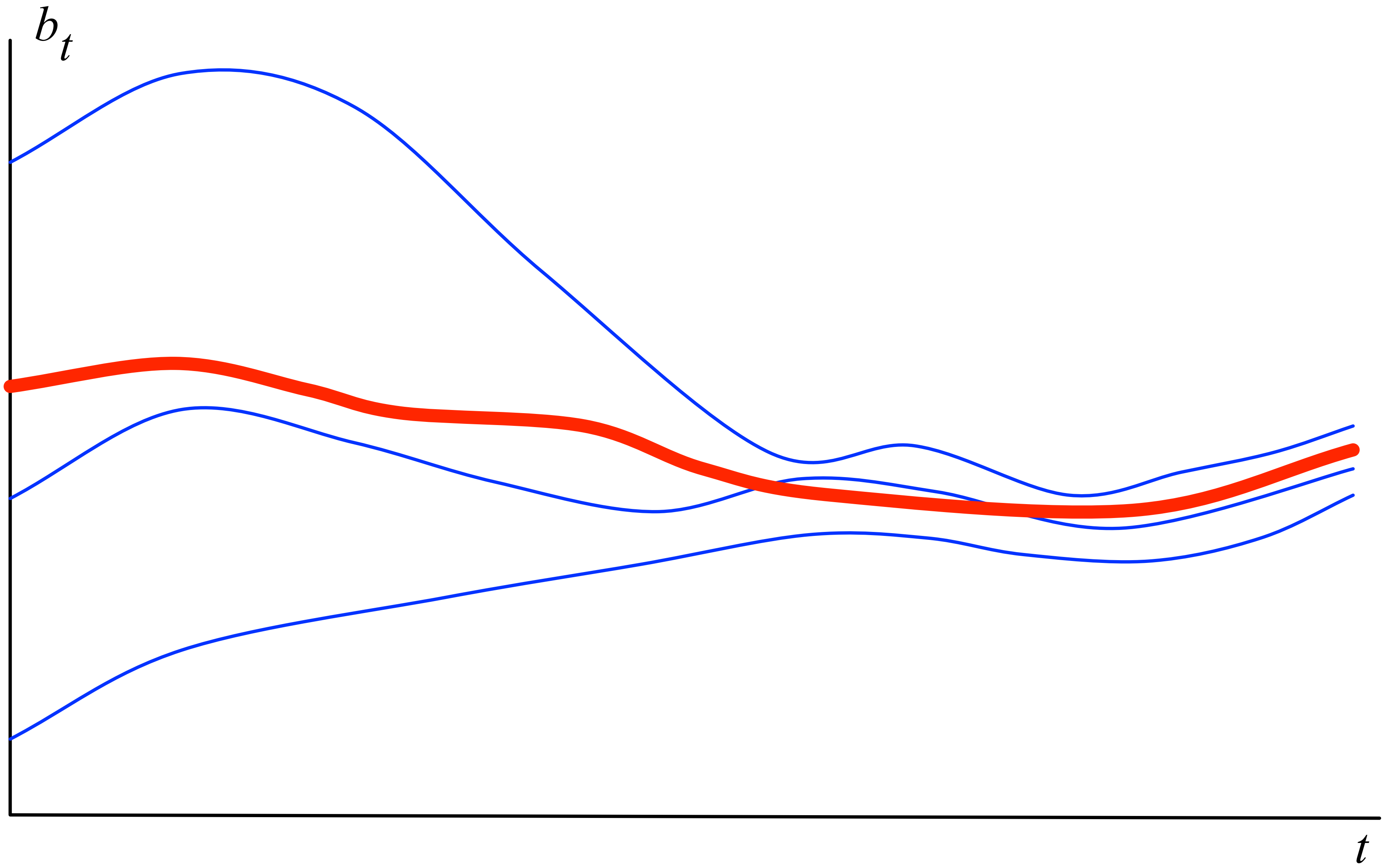

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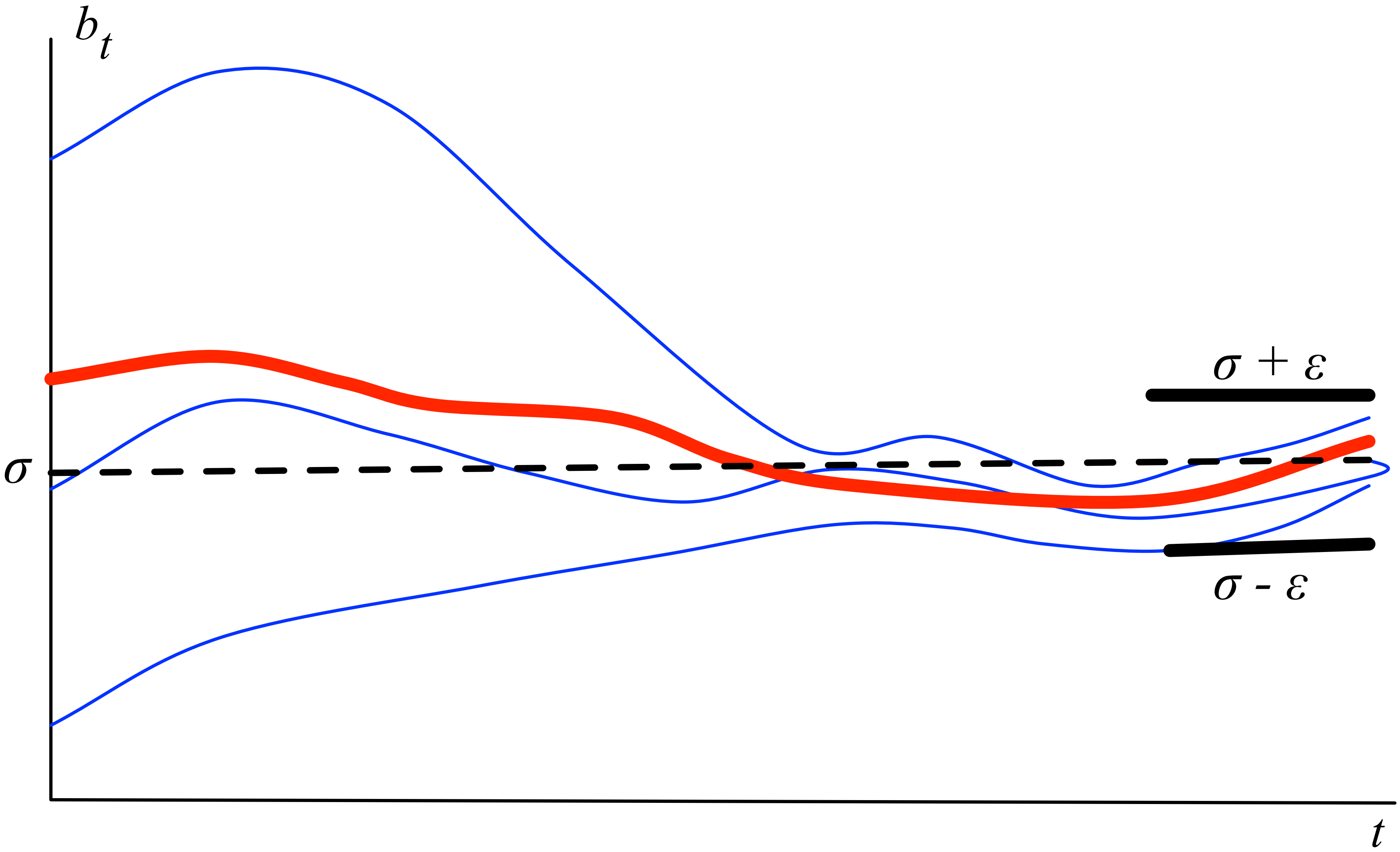

■ Step 3. For every $\epsilon > 0$, there is T such that $b_t \in [\sigma - \epsilon, \sigma + \epsilon]$ for all $t \geq T$ and all individuals.

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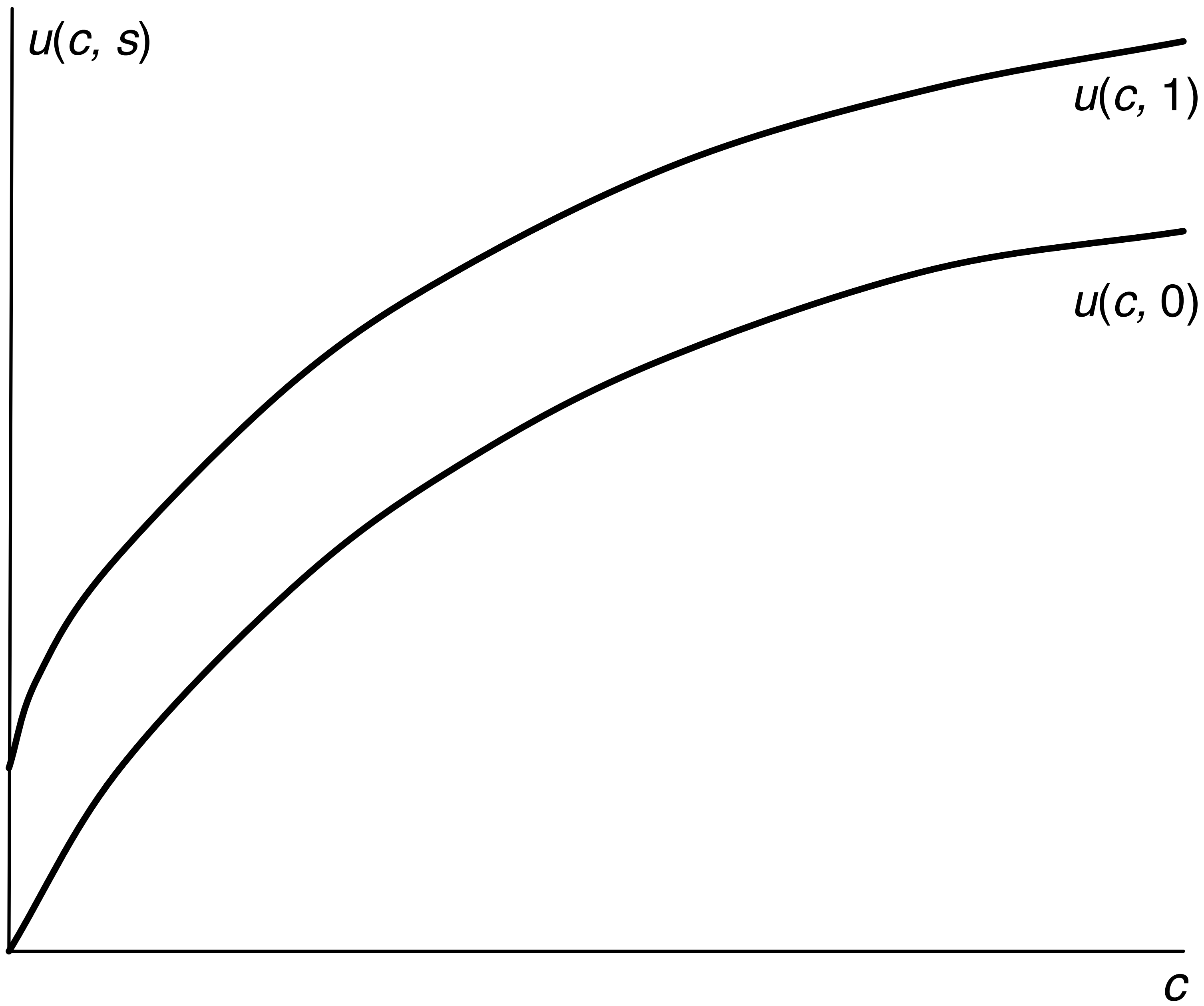

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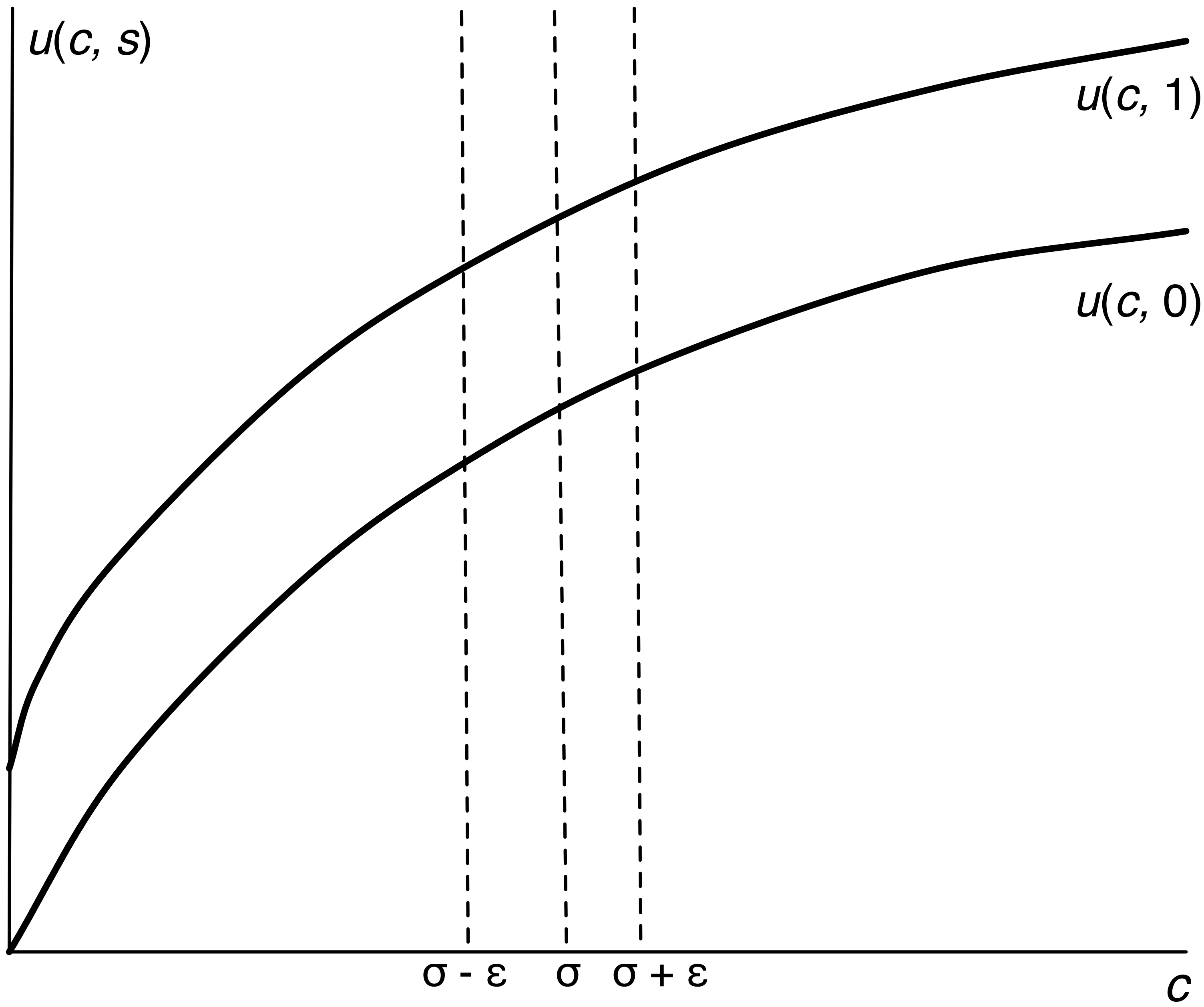

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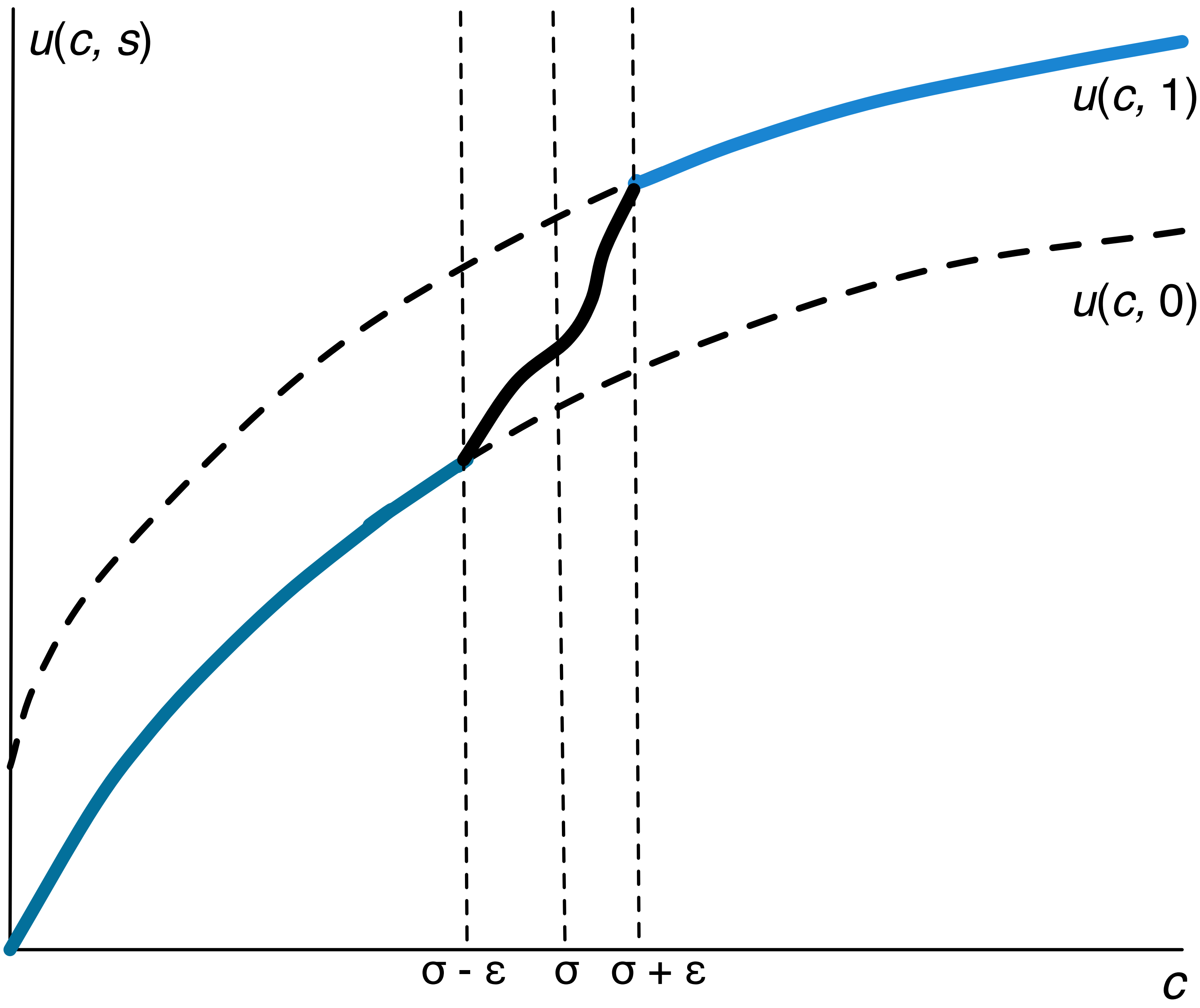

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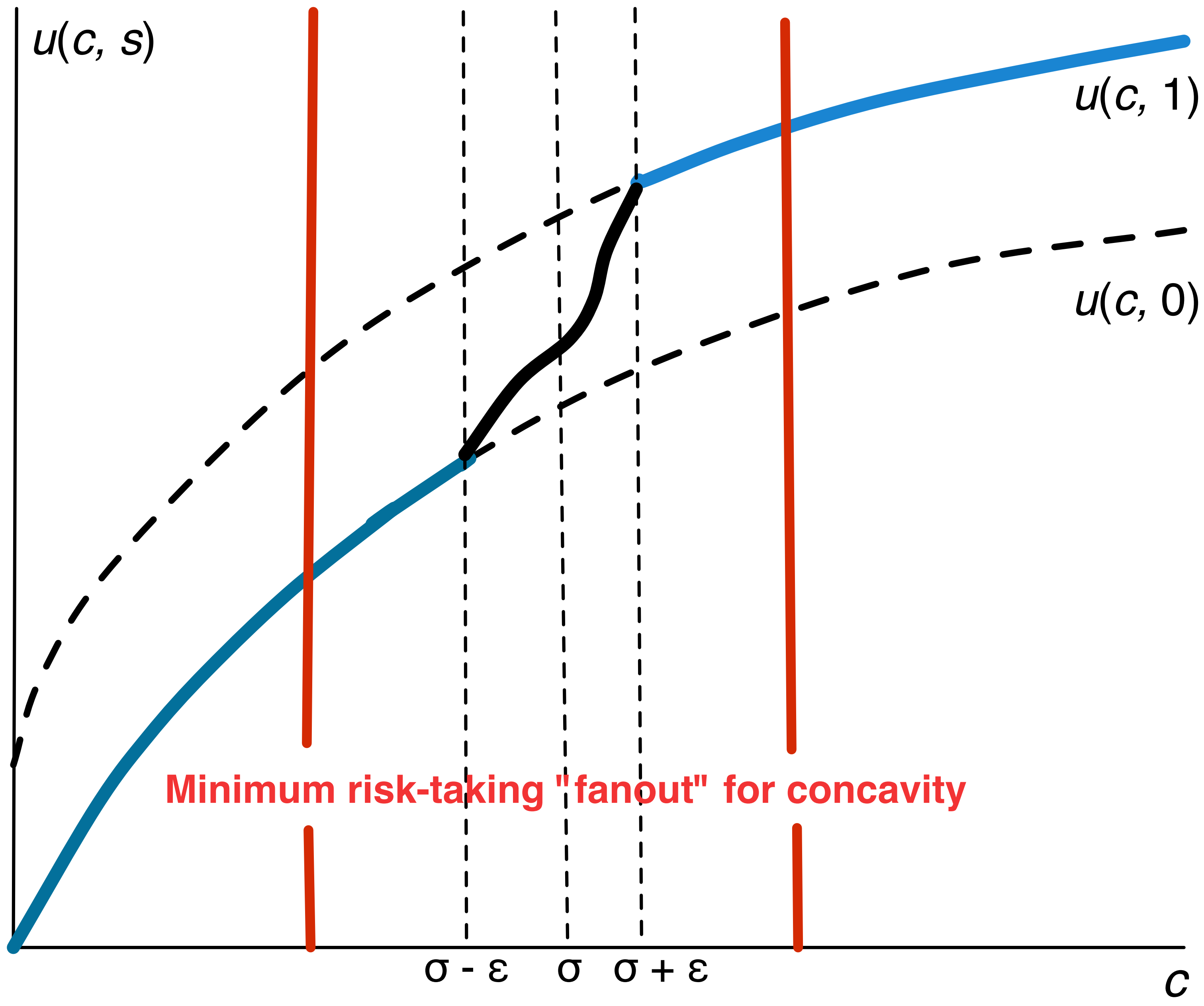

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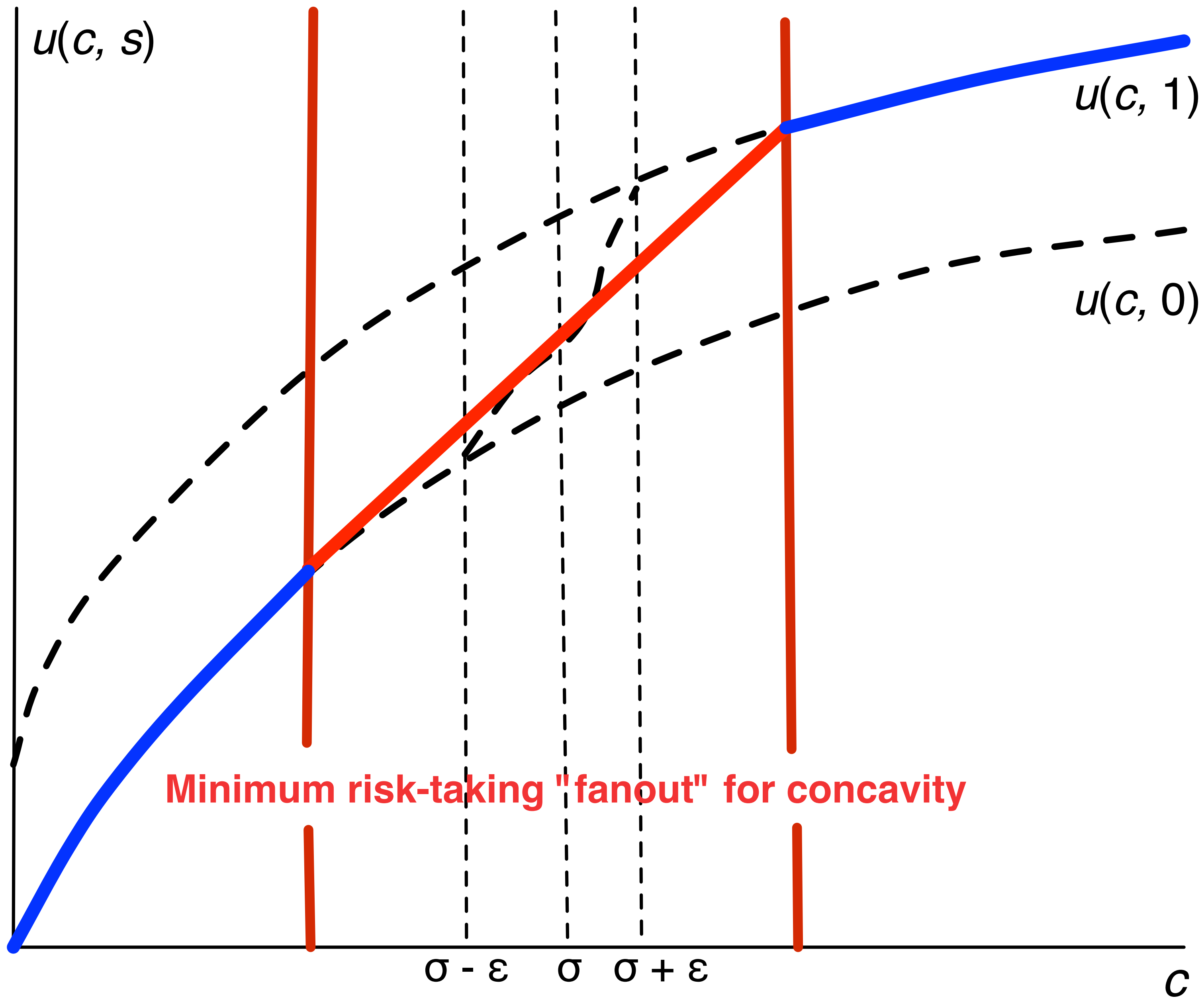

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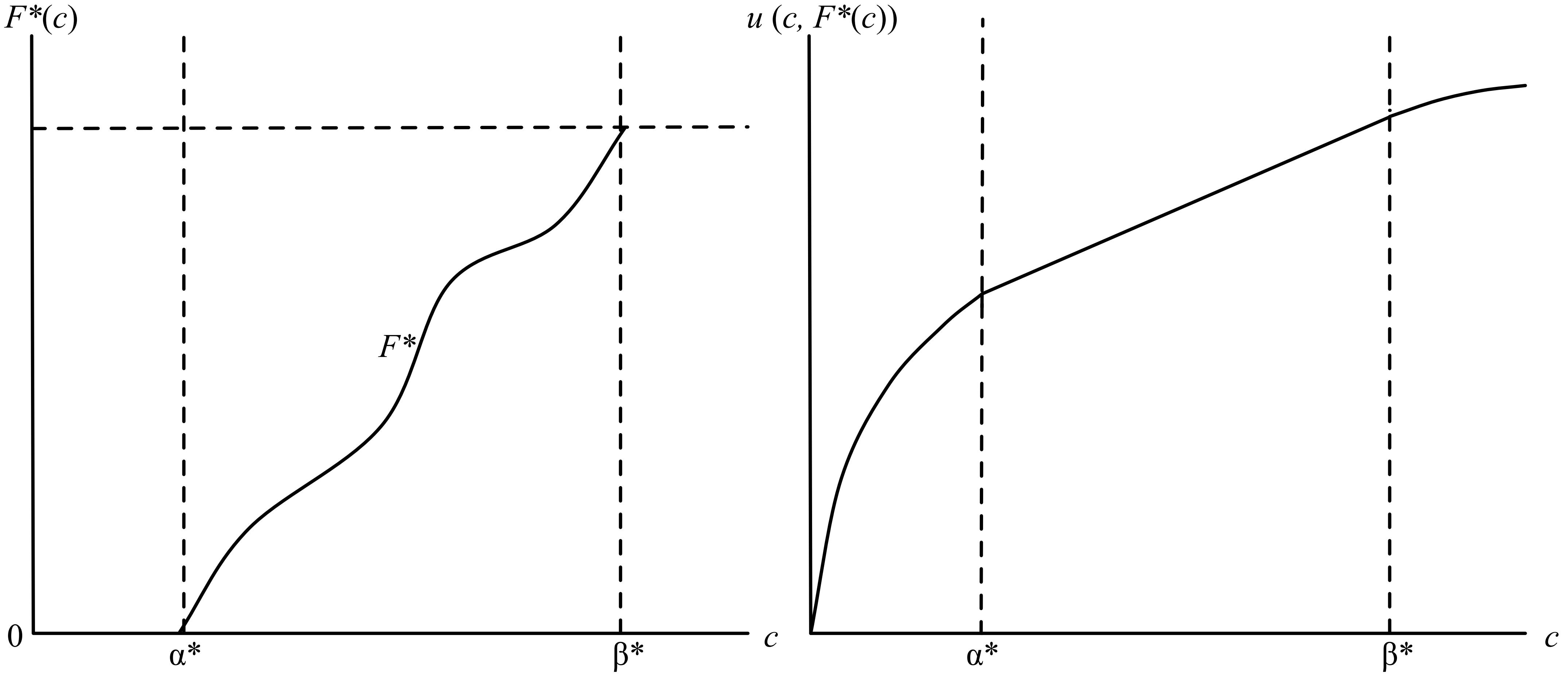
- Step 7. The limit value is k^*.

- Friedman-Savage (1948) revisited.



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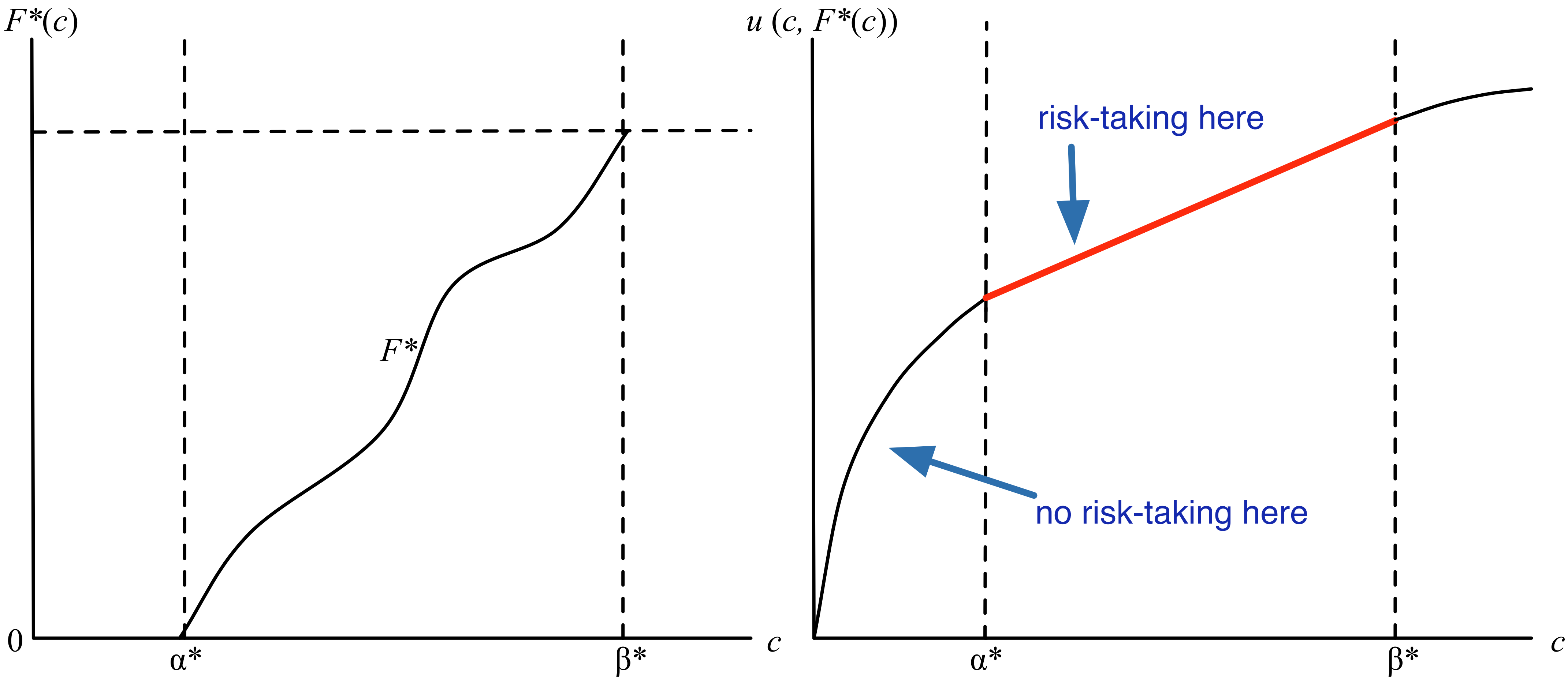

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Three Differences from Friedman-Savage

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- 1. Phenomenon arises "naturally" when utility depends on status.

- No dependence on *ad hoc* description of preferences with varying curvature.

- 2. The argument is scale-neutral.

- Two insulated societies two different production technologies will generally settle into two different steady states.

- *Both* the steady states will generally exhibit the Friedman-Savage property, even though overall wealths are different.

Three Differences from Friedman-Savage

- 3. Risk-taking is inefficient

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- Then our unique steady state is Pareto-inefficient.

- Proof. In steady state, lifetime expected utility is given by

$$\int u(c, F^*(c))dF^*(c) < u\left(\int cdF^*(c), \int F^*(c)dF^*(c)\right) = u(c^*, 1/2).$$

Final Remarks

- Concern for status in a conventional model of economic growth.

- Equilibrium involves persistent randomization and ex-post lifetime inequality.

- Generates Friedman-Savage with no reliance on changing utility curvature, and is scale-neutral.

- Friedman-Savage informally justified their utility function using relative status.

- That reformulation leads to *inefficient* risk-taking (in contrast to the main agenda of Friedman).

- Situation different if the production function is *convex*

- (as in Romer, Lucas, new growth theory).

- Inequality does not have to be recreated. Equilibrium is deterministic in the pure-status model

- (independent of curvature in u or f!)

- A theme in common with the endogenous inequality literature:

"If inequality didn't exist, it would have to be invented."

The End

P.S. Crooked course of a Cockney courtship [7]

Me/and/'er

These Notes Not Used in the Talk

Non-Convergence and Deterministic Equilibrium

- Proposition.

- Make the following assumptions:

- (i) u is pure-status (only depends on s).

- (ii) f is convex.

- (iii) G has full support and u(G(w)) is strictly concave.

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- Make the following assumptions:

- (i) u is pure-status (only depends on s).

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- Then there exists an equilibrium with deterministic policy

$$c = (1 - \delta)w \text{ and } k = \delta w,$$

and equilibrium status for every individual constant over time.

Idea of Proof

- Suppose everyone uses suggested policy.
- Say one individual deviates *only* at date t (Blackwell).
- Status for ever after is $G_{t+1}(w')$, where w' = wealth at $t+1$.
- Then, for every initial w at date t, chooses $k \in [0, w]$ to max

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$$u\left(F_t(w-k)\right) + \delta V_{t+1}\left(f(k)\right) \quad = \quad u\left(F_t(w-k)\right) + \frac{\delta}{1-\delta} u\left(G_{t+1}\left(f(k)\right)\right)$$

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Idea of Proof, contd.

- $$u\left(F_t(w-k)\right)+\delta V_{t+1}\left(f\left(k\right)\right)=u\left(G_t\left(\frac{w-k}{1-\delta}\right)\right)+\frac{\delta}{1-\delta}u\left(G_t\left(k/\delta\right)\right)$$

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- *Proof.* If everyone uses suggested policy, then for all w,

$$G_{t+1}(w)=G_t\left(f^{-1}(w)/\delta\right),$$

- Use recursion and convex f.

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- Use recursion and convex f.

- Now maximize ■ to get $k=\delta w$. ■

Remarks

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  - despite convexity in f and no assumption on curvature of u.
  - Intuition: wealth distribution stays dispersed, no bunching.
- 2. Equilibrium policy independent of wealth distribution or u/f.
  - Same as planner using logarithmic utility and linear production.

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  - *regular* if at each date, some person uses a strict best response at all but possibly a countable number of wealths;

  - *smooth* if every i uses a sequence of differentiable policies $\{c_t^i\}$, with $\epsilon^i \leq {c_t^i}'(w) < 1$ at all w and t, for some $\epsilon^i > 0$.

Converse, contd.

- Proposition.
- Make the following assumptions:
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- Proposition.

- Make the following assumptions:

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- Then any regular, smooth equilibrium must display the policy

$$c = (1 - \delta)w \text{ and } k = \delta w,$$

(common to all individuals and time stationary).