Review of Economic Studies (2003) **70**, 87–113 0034-6527/03/00040087$02.00

Group Formation in Risk-Sharing Arrangements

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First version received December 2000; *final version accepted July* 2002 (*Eds.*)

We study informal insurance within communities, explicitly recognizing the possibility that subgroups of individuals may destabilize insurance arrangements among the larger group. We therefore consider self-enforcing risk-sharing agreements that are robust not only to single-person deviations but also to potential deviations by subgroups. However, such deviations must be credible, in the sense that the subgroup must pass exactly the same test that we apply to the entire group; it must itself employ some self-enforcing risk-sharing agreement. We observe that the stability of subgroups is inimical to the stability of the group as a whole. Two surprising consequences of this analysis are that stable groups have (uniformly) bounded size, a result in sharp contrast to the individual-deviation problem, and that the degree of risk-sharing in a community is generally non-monotonic in the level of uncertainty or need for insurance in the community.

1. INTRODUCTION

Risk is a pervasive fact of life in developing countries. As a response to the large fluctuations in their income, individuals—mainly in rural areas—often enter into informal insurance or quasi-credit agreements. To be sure, such arrangements are potentially limited by the presence of various incentive constraints. As a first cut, it appears that the most important constraint arises from the fact that such agreements are not written on legal paper, and must therefore be designed to elicit voluntary participation. To be self-enforcing, the expected net benefits from participating in the agreement must be at any point in time larger than the one time gain from defection.[1]

There is a growing body of literature, both theoretical and empirical, on self-enforcing risk-sharing agreements. Some important theoretical contributions are Posner (1980), Kimball (1988), Coate and Ravallion (1993), Kocherlakota (1996), Kletzer and Wright (2000), and Ligon, Thomas and Worrall (2002).[2] Our starting point is the following observation: all these studies (and to our knowledge all existing studies) define "self-enforcing" agreements as those that are proof from defection by *individual* members of the group. As a consequence, the common

1. Udry (1994), in his study of rural northern Nigeria, finds this constraint to be the most important in describing the structure of reciprocal agreements. While this does not prove that other informational asymmetries are of second-order importance (for instance, they may limit the choice of whom to transact with in the first place), we feel that the self-enforcement constraint represents a good first approximation.

2. The literature on risk-sharing without commitment in rural societies started with the suggestions of Posner (1980) and Kimball (1988) that schemes of mutual insurance with limited commitment were possible. In an important paper, Coate and Ravallion (1993) characterized mutual insurance arrangements with a restriction to stationary transfers for a symmetric two-household model. A recent strand of literature investigates efficient dynamic contracts in the absence of commitment (Kocherlakota (1996), Kletzer and Wright (2000), Ligon *et al.* (2002)).

practice in the literature is to define self-enforcing risk-sharing agreements as subgame perfect equilibria of a repeated game (in which self-insurance is always an option), and to characterize the Pareto frontier of such equilibria. But this raises the obvious question: if a "large" group—say the village community or a particular caste or kinship group within the community—can foresee the benefits of risk-sharing and reach an agreement, why might smaller groups not be able to do so? Why would subgroups not be able to agree to jointly defect and share risk among themselves? This concern implies that to be truly self-enforcing, an informal risk-sharing agreement needs to be immune to joint deviations by subgroups. At the same time, it seems only natural to require that deviating groups themselves satisfy the same criterion. To be of any value—or to pose a credible threat to the group at large—a deviating coalition should also employ self-enforcing arrangements. These embedded constraints characterize the concept of self-enforcing risk-sharing agreements and stable coalitions that we define in this paper.

Despite their importance, issues of participation and group formation have been little discussed in the literature on informal risk-sharing. As we shall see, this criticism is not just one of methodology, it has substantive implications. The most important of these is that the "individual deviations" framework places no bound on group size. For instance, in a homogeneous population, the larger the group the higher the *per capita* utility from risk-sharing. Barring *other* impediments to group size, the theory implies that any efficient agreement has to be at the level of the "community". That is why most empirical tests of insurance (Deaton (1992), Townsend (1994), Udry (1994), Grimard (1997), Jalan and Ravallion (1999), Gertler and Gruber (2002), Ligon *et al*. (2002)) take the unit of analysis as exogenous and study the extent of insurance at the level of the village or even larger groups.[3] Of course, this is not to say that group limits are not taken seriously. But other considerations—caste, kinship, or even the informational decay that must ultimately affect large groups—must be brought in to complete the picture. One could, of course, model this in several ways: for instance, by positing some cost of group formation which increases with the size of the group (see, *e.g.* Murgai, Winters, Sadoulet and de Janvry (2002)).

We abstract from all such factors. We endogenize not just the extent of insurance within given groups, but the process of group formation within a community. Without invoking any of the additional considerations described in the previous paragraph, we show that there are limits to group size (though at this time we have not obtained clear characterizations of these limits). Indeed, we show that for any parametric configuration of the environment, only a *finite* number of group sizes can be stable, even though—in principle—every potential group size is allowed for.

An important and paradoxical consequence of this observation is that an increase in the need for insurance—stemming either from a change in the environment or in some behavioural parameter such as the degree of risk aversion—can *decrease* the extent of risk-sharing among the population. Indeed, it can do so by reducing the maximal stable group size. It is important

3. These studies typically test for full insurance at the community (village) level in less developed countries, many of them inspired by Townsend's (1994) well-known study of risk and insurance in village India. In this study, Townsend finds evidence consistent with substantial insurance but rejects full insurance. Similar conclusions have been found by many authors such as Deaton (1992) in the context of Côte d'Ivoire, Ghana and Thailand, by Udry (1994) for northern Nigeria, and Jalan and Ravallion (1999) in villages of rural China. Ligon *et al*. (2002) test the constrained efficient or dynamic limited commitment model on the ICRISAT Indian households. Gertler and Gruber (2002) look at consumption insurance in case of major illnesses in Indonesia. Grimard (1997) tests the full insurance model among even larger groups defined along ethnic lines in Côte d'Ivoire. All the above reject complete risk-sharing at the level of the community or even larger ethnic group, but provide significant evidence of partial insurance. From a related perspective, Townsend (1994) and Chaudhuri and Ravallion (1997) find that the poor have only limited insurance against idiosyncratic shocks. Rosenzweig (1988) studies the transfers themselves in the same data set and estimates these to typically be less than 10% of the typical income shocks in bad periods. Morduch (1991) obtains similar results using the Indian ICRISAT data. He finds evidence of risk-sharing within groups, but not across groups. He finds that the poor have only limited insurance against idiosyncratic shocks. He also notes a caste effect. Cox and Jimenez (1991) find that just 40% of black South African households and fewer than 10% of white South African either give or receive transfers.

to note that such an observation cannot be obtained in a model with only individual deviations, even if considerations of kinship or information are brought in to close off group size. With an increase in output uncertainty, these considerations would be given somewhat less importance at the margin, so equilibrium group size can only expand.[4]

We do not attempt, in this paper, to provide empirical support for the possibilities raised here. But it is worth noting that the few papers actually addressing the issue of risk-sharing among subgroups find convincing evidence for it; see Fafchamps and Lund (2001) and Murgai *et al.* (2002).[5] Both these studies suggest that the explanation for the formation of these subgroups must lie in the existence of "group costs" that increase with the number of participants in the risk-sharing agreement. For instance, Murgai *et al.* (2002) observe that: "if establishing and maintaining partnerships is indeed costless, there is no reason for a mutual insurance group not to be community-wide or world-wide. Real world limits to group size must therefore be the result of costs relating to the formation and maintenance of partnerships".

We suggest that there may be more fundamental reasons for group splintering. Naturally, one can think of many reasons for which group size may be limited, and we have already mentioned some of these. Individuals belonging to a certain religion, caste or ethnicity may prefer not to transact with anyone outside their group. Asymmetries of information and the lack of enforcement may be less pronounced among people living next to each other, sharing the same relatives, having the same activities, etc. But although these motives may limit the number of people one may consider sharing risk with, that generally leaves a significant group of people still available.

Further theoretical and empirical investigation of risk-sharing agreements—with a sharp focus on strategic group formation—would be very useful, especially given the potential importance of the policy implications. From a theoretical perspective, a tighter characterization of stable group sizes is much needed. Once these limits are well established for the homogeneous case, heterogeneity among the agents can be better studied. For instance, the consequences of endogenous matching—*e.g.* along wealth levels (Hoff, 1997) or risk characteristics (Ghatak (1999), Sadoulet (1999))—may be investigated against this background.

2. THE FORMATION OF MUTUAL INSURANCE GROUPS

2.1. *Introduction*

A community of n identical agents is engaged in the production and consumption of a perishable good at each date. Each agent produces a random income which takes on two values: h with probability p and ℓ with probability $1 - p$. The terminology h and ℓ naturally suggests the ordering $h > \ell > 0$. Income realizations are independent and identical, over people as well as dates.

Each agent has the same utility function, increasing, smooth and strictly concave in consumption. We thus have an instance of a classical group insurance problem. The (symmetric) Pareto optimal allocation is reached by dividing equally—and among all members of the community—the aggregate available resources at each period. Naturally the larger the group the smaller the dispersion of *per capita* output, and the larger the potential value of insurance.

4. It is logically possible that an increase in output uncertainty can simultaneously increase informational asymmetries, so much so that equilibrium group size shrinks as a consequence. But one would need rather strong assumptions to derive such a prediction.

5. Fafchamps and Lund (2001) examine—in the context of the rural Philippines—whether gifts and loans circulate among networks of friends and relatives, or whether risk is efficiently shared at the village level. Murgai *et al.* (2002) investigate water transfers among households along a watercourse in Pakistan's Punjab. They find that reciprocal exchanges are localized in units smaller than the entire watercourse community. Unfortunately—these exceptions apart—issues of participation and group formation have been little discussed in the literature on informal risk-sharing.

As motivated and discussed in the Introduction, we focus on the theme that insurance arrangements must be self-enforcing, and that this requirement constrains the form of such arrangements. Briefly, the enforcement constraint refers to the possibility that at some date, an individual who is called upon to make transfers to others in the community refuses to make those transfers. The constraint is then modelled by supposing that the individual is excluded from the insurance pool, so that he must bear stochastic fluctuations on his own (there may be additional sanctions as well). If the power of such punishments is limited, then perfect insurance (which calls for extensive transfers) may not be possible. The literature cited above concentrates on the structure of "second-best" self-enforcing schemes.

In this paper, we extend the enforcement constraint in a natural way. There is no reason to believe that only an *individual* will deviate from an ongoing arrangement. An entire subset of individuals—a *coalition*—may instigate a joint deviation by refusing to contribute to the wider community and thereafter forming their own reciprocity "subgroup". To be sure, the subgroup itself may be vulnerable to further deviations and so may lack "credibility" in its threats (see Ray (1989)). Thus we only permit credible coalitions—those that are stable in their own right—to pose a threat to the community as a whole. We shall show that these threats have a dramatic effect on our predictions concerning the extent of insurance, and especially on the way the need for insurance varies with the amount of (exogenous) environmental risk.

Our insistence on immunity with respect to blocking coalitions is reminiscent of recent literature on endogenous coalition formation (see, *e.g.* Bloch (1996, 1997), Ray and Vohra (1997, 1999, 2001)). However, there is a difference. In the cited literature, coalitions respond to a proposed *ex ante* arrangement by blocking it, or by proposing alternatives, etc. In this case, a tighter constraint must be observed; namely, that created by the *realization* of income shocks at every date. Thus, in contrast to the "participation constraints" of the literature on endogenous coalition formation, these are truly "incentive constraints". In this sense, our approach also bears a close connection to coalition-proof Nash equilibrium (Bernheim, Peleg and Whinston, 1987)

In the main part of the paper, we develop the theory of group enforcement constraints under the simplifying assumption that each coalition or group, once formed, attempts to implement some *symmetric and stationary* risk-sharing arrangement. This assumption allows us to make the main points very cleanly. But it is not a critical assumption. In a later section, we point out how the theory can be extended to the case in which coalitions implement arbitrary (nonsymmetric, nonstationary, history-dependent) arrangements among their members.

This is not to say that our analysis is entirely devoid of restrictive assumptions. The most important of these is the assumption that only subsets of existing groups are permitted to deviate. We discuss this and other issues in Section 4.

2.2. *Stable groups*

Because the stability of a group is threatened by subgroups of individuals, it is possible to define group stability recursively. To this end, we begin with individuals (or singleton coalitions). The lifetime utility of an individual in isolation (normalized by the discount factor to a per-period equivalent) is simply

$$v^*(1) \equiv pu(h) + (1-p)u(\ell).$$

Because singleton groups have no proper subsets, this is the stable worth of an individual.

Recursively, having defined stable worths for all $m = 1, \ldots, n-1$, consider some coalition of size n. We first define a (symmetric and stationary) *transfer scheme*. This may be written as a vector $\mathbf{t} \equiv (t_1, \ldots, t_{n-1})$, where t_k is to be interpreted as the (non-negative) transfer or payment by a person in the event that his income is *h and k* individuals draw h. We only consider nontrivial schemes in which $t_k > 0$ for some k.

With a transfer scheme in mind we can easily back out what a person receives if his income draw is ℓ and k individuals produce h. The total transfer is then kt_k, to be divided equally among the $n-k$ individuals who produce l. Thus a transfer scheme $\mathbf{t}$ implies the following: if there are k high draws, then a person consumes $h - t_k$ if he produces h, and $\ell + \frac{kt_k}{n-k}$ if he produces ℓ. It follows that the expected utility from a transfer scheme $\mathbf{t}$ is given by

$$v(\mathbf{t}, n) \equiv p^n u(h) + (1-p)^n u(\ell) + \sum\nolimits_{k=1}^{n-1} p(k,n)\left[\frac{k}{n}u(h-t_k) + \frac{n-k}{n}u\left(\ell + \frac{kt_k}{n-k}\right)\right], \quad (1)$$

where $p(k, n)$ is just the probability of k highs out of n draws.[6] Define a (nontrivial) transfer scheme to be *stable* if for all $k = 1, \ldots, n-1$,

$$(1-\delta)u(h-t_k) + \delta v(\mathbf{t}, n) \geq (1-\delta)u(h) + \delta v^*(s) \quad (2)$$

for every stable $s \leq k$.

The interpretation of stability is quite simple. We require that for all possible income realizations, the stipulated transfers be actually carried out. If (2) fails for some k and $s \leq k$, this means that there is a stable coalition of size s who can credibly refuse to pay what they are required to pay (when k individuals draw high). Such a transfer scheme will, sooner or later, break down.[7]

Say that *a group of size n is stable* if a stable transfer scheme exists for a group of size n. Define $v^*(n)$ to be the maximum possible value of $v(\mathbf{t}, n)$, where the maximum is taken over the set of all stable transfer schemes for a group of size n. Otherwise, n is *unstable* and $v^*(n)$ is not defined.

3. GROUP STABILITY AND THE NEED FOR INSURANCE

Our main interest lies in examining the relationship between the "need for insurance" and the "stability of insurance groups". We begin by making these phrases more precise in our particular context.

The need for insurance is, of course, a composite object: it will vary with the extent of environmental uncertainty (proxied by the gap between h and ℓ), and given the environment, it will vary with the degree of risk aversion. For our purposes, it will turn out that a useful measure of the need for insurance is the ratio $\frac{u'(\ell)-u'(h)}{u'(h)}$, which we henceforth denote by θ. Keeping everything else constant, notice that a mean preserving spread between h and ℓ increases θ. Moreover, for the same income distribution, a utility function that exhibits a higher risk aversion throughout its domain will translate into a higher need for insurance. Hence, our measure incorporates both environmental uncertainty and attitudes towards risk, albeit in summary form.

The "stability of insurance groups" is a more problematic concept. One might be interested, for instance, in the entire *set* of stable groups, which can behave in a complicated fashion. We simply assume that there is some given population size in the community, and ask how close we can get to this without forsaking stability. In other words, we are interested in the largest stable group.

6. That is, $p(k, n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$.

7. It might be objected that this definition of stability is too stringent, in the sense that it requires the group of size n to be immune to *every* possible realization of the random shocks, and not simply those which occur with "high enough" probability. However, provided expected payoffs are calculated properly, it is easy enough to see that the two definitions are equivalent. For suppose that a transfer scheme is stable under the weaker definition, noting that in the "unstable" low probability states no transfers will be implemented. Then a new transfer scheme which is the same as the old except for suitably chosen small transfers in the previously "unstable" states will be stable in our sense.

3.1. *The individual enforcement problem*

It will be useful to begin by considering the well-known problem in which only the individual enforcement constraint must be respected. Fix a population of n individuals, and let $\hat{v}(n)$ denote the maximum value of (1) when (2) is only invoked for $s = 1$. In other words, $\hat{v}(n)$ is the solution to the following problem:

$$\max_{\mathbf{t}} v(\mathbf{t}, n) \tag{3}$$

subject to

$$(1-\delta)u(h-t_k) + \delta v(\mathbf{t}, n) \geq (1-\delta)u(h) + \delta v^*(1) \tag{4}$$

for all $k = 1, \ldots, n-1$.

If there is some nontrivial transfer scheme that solves this problem, say that a group of size n is *individually stable*, or i-stable for short.

The following proposition fully characterizes i-stability for symmetric stationary equilibria.

Proposition 1. *A group size n is* i-stable *if and only if*

$$\theta > \frac{1-\delta}{\delta p(1-p^{n-1})}. \tag{5}$$

Proposition 1 states that some minimal need for insurance is required for a group to be i-stable; indeed, when the need for insurance is very low no group is i-stable. To see this, note that the R.H.S. of (5) is bounded below (uniformly in n) by the expression $\frac{1-\delta}{\delta p}$, so that if the need for insurance is lower than this value *no* group size, however large, can be stable.

As the need for insurance increases above this absolute minimum, i-stable groups begin to appear. Indeed, they appear in a particular order. An easy consequence of Proposition 1 is

Corollary 1. *If n is* i-stable, *so is n' for all $n' > n$.*

This tells us that larger groups do better in terms of i-stability than smaller groups do.

3.2. *Low need for insurance*

The preceding discussion has an immediate implication. Fix a community of size n, and carry out the thought experiment of raising the need for insurance (for instance by creating a mean-preserving spread of the income distribution) from a value close to zero. By Proposition 1, a group of two or more individuals must initially not be i-stable, and *a fortiori*, it must be unstable. At some point θ crosses the critical threshold given by the R.H.S. of (5). *Just* at this point, the only i-stable group size is n itself (and all larger sizes, but these are irrelevant). It follows that n must be stable as well, because all smaller groups are unstable.

We may summarize this discussion in the following proposition, which is an obvious corollary of Proposition 1.

Proposition 2. *As the need for insurance increases, the first group to attain* (*full*) *stability is the entire community.*

3.3. *High need for insurance*

As the need for insurance continues to rise, smaller groups become i-stable. The question is: does the newly acquired stability of these groups threaten to disrupt the stability of larger groups?

TABLE 1

Stability for various values of θ

θ	2 stable?	3 stable?
$\leq 0{\cdot}6096$	$\times$	$\times$
$0{\cdot}7195$	$\times$	$\surd$
$0{\cdot}9136$	$\surd$	$\surd$
$1{\cdot}6375$	$\surd$	$\times$
$2{\cdot}6601$	$\surd$	$\surd$

General arguments that apply to the comparison of any two i-stable groups are hard to obtain, as the following example shows.

Example 1. Set community size n equal to 3, and assume that individuals have the CRRA utility indicator

$$u(c) = \frac{1}{1-\rho} c^{1-\rho}$$

where ρ is the Arrow–Pratt coefficient of relative risk aversion.

The following parameters are set through the example: $\delta = 0{\cdot}83$, $\rho = 1{\cdot}6$ and $p = 0{\cdot}4$. We consider several options for ℓ and h, keeping mean income constant throughout,[8] but progressively raising the value of θ. The results are reported in Table 1.

The table shows that stability is a complex object to check for. When the need for insurance is low, the three-person community is stable, in line with Proposition 2. Thereafter, two-person groups also acquire stability, but fail to generate enough insurance to threaten the community as a whole. The situation changes, however, when the need for insurance is still larger. While both two- and three-person groups gain in i-stability, the gain enjoyed by the former is large enough to render the three-person community unstable. In contrast to the notion that more uncertainty generates larger insurance groups, the maximal stable group size *falls*. Yet, as the final row of Table 1 shows, the fall is not inevitable: for *still* higher degrees of uncertainty, the three-person community regains its stability.[9]

It should be clear from this example that general results regarding stability will be hard to come by, though the possibility that stable group size responds perversely to greater uncertainty is well illustrated even in this special case. In the remainder of this section, we outline some general findings.

In what follows, it will initially be useful to fix the need for insurance and ask a preliminary question: in a *given* environment, are there an infinity of stable groups? To be sure, the answer must be in the negative when the need for insurance is "small"; to be precise, when $\theta < \frac{1-\delta}{\delta p}$. For we know from Proposition 1 that in this case there is no i-stable group (other than the singletons), and consequently no stable group either. Once this critical bound is passed, however, Corollary 1 tells us that an infinity of i-stable groups appear. The lowest i-stable size is the smallest value of n for which (5) holds, and every group size exceeding this bound is also i-stable. The question of whether there is an infinite number of *stable* groups now becomes nontrivial, and is answered in

8. We set mean income equal to $2{\cdot}4$ through the example.
9. The detailed numbers are available on request from the authors.

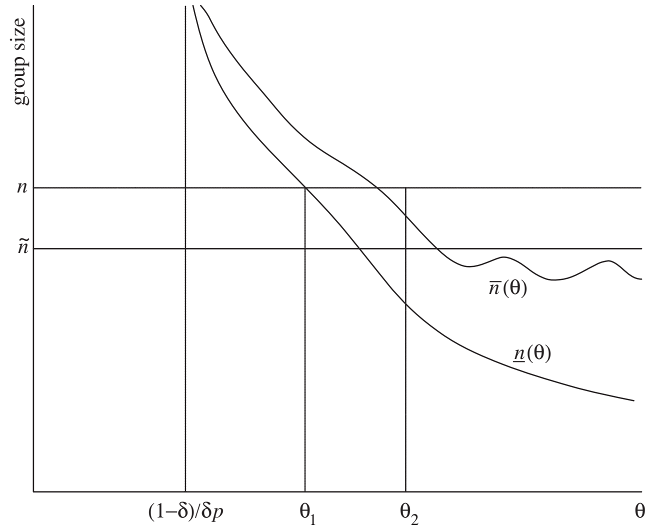

FIGURE 1
An illustration of Proposition 3

Proposition 3. *For each level of need for insurance* $\theta > \frac{1-\delta}{\delta p}$, *there are thresholds* $\bar{n}(\theta)$ *and* $\underline{n}(\theta)$ *such that*

$$2 \leq \underline{n}(\theta) \leq n \leq \bar{n}(\theta) < \infty, \tag{6}$$

for every stable group size n. *Moreover,*

$$\underline{n}(\theta) \uparrow \infty \qquad \text{as} \qquad \theta \downarrow \frac{1-\delta}{\delta p}. \tag{7}$$

Figure 1 provides a graphical description of the proposition. The lower bound $\underline{n}(\theta)$ is easy enough to obtain: it is simply the size of the smallest i-stable group and is therefore the smallest value of n such that (5) holds. This immediately explains why $\underline{n}(\theta) \geq 2$ (once θ lies above the critical threshold $\frac{1-\delta}{\delta p}$), and why this threshold becomes infinitely high as θ descends to the critical threshold.

The nontrivial part of the proposition asserts the existence of an *upper* bound on stable groups. This observation, by the way, contrasts with the existence of infinitely many stable sizes in the coalition formation literature (see, *e.g.* Bloch (1996), Ray and Vohra (1997) for results on stable cartels in oligopoly, and Ray and Vohra (2001) for results on the efficient provision of public goods). It is peculiar to the insurance problem.

To see why, consider this intuitive account of the proof. If the assertion were false, there would be (for some need for insurance) an infinity of stable sizes. But we do know that the marginal "diversification gain" from an increase in size ultimately tends to zero. Therefore, we may pick a stable size n such that a coalition of size n is able to reap most of the benefits of sharing risk: a larger stable group improves the *per capita* utility of its members by only a small amount. It follows from the enforcement constraint that in any larger stable coalition, the transfers made whenever at least n people have a good shock have to be close to 0. Because the set of stable sizes is infinite, we can choose this stable coalition sufficiently large such that the probability that at least n people have a good shock is close to 1. Therefore the worth of such a coalition can be brought arbitrarily close to autarkic utility, but this contradicts the presumed stability of that coalition.[10]

In addition Proposition 3 allows us to derive a general "nonmonotonicity" result akin to the sort indicated in Example 1.

Proposition 4. *There is an integer $\tilde{n}$ with the following property. For every community size $n \geq \tilde{n}$, there exist degrees of uncertainty θ_1 and θ_2, with $\theta_2 > \theta_1$, such that n is stable under θ_1 but is unstable under θ_2.*

The proof follows immediately from an examination of Figure 1. The lower bound $\tilde{n}$ can be taken to be any value that lies above the "stable correspondence" at some point. Then for any $n \geq \tilde{n}$, it is obvious that stability obtains at the value of θ that corresponds to the inverse image of $\underline{n}$ (evaluated at n)—θ_1 in the figure—while stability fails once $\bar{n}(\theta)$ falls below n — *e.g.* at θ_2 in the figure.

These propositions are silent on just how restrictive coalition considerations can be. Unfortunately, we have no general results to this effect. But an example may be illustrative.

Example 2. Consider a community of 10 individuals with the same functional form for utility as in Example 1;

$$u(c) = \frac{1}{1-\rho}c^{1-\rho},$$

where ρ is the Arrow–Pratt coefficient of relative risk aversion. We also use the same specific parameters as in Example 1: $\delta = 0{\cdot}83$, $\rho = 1{\cdot}6$, $p = 0{\cdot}4$, $\ell = 2$ and $h = 3$.

We evaluate—for each group size ranging from 1 to 10—the return to informal insurance. One natural way to do this is to look at the gain over and above autarky, compared to the corresponding *per capita* gain that the first-best provides *in the community of all* 10. If $\tilde{v}$ denotes this latter value and $\hat{v}(n)$ is the i-stable value for a group of size n, then the i-stable gain may be reported as

$$\frac{\hat{v}(n) - v(1)}{\tilde{v} - v(1)} \times 100$$

in percentage terms. Similarly, if $v^*(n)$ is the stable value for a group of size n, then the stable gain is described as

$$\frac{v^*(n) - v(1)}{\tilde{v} - v(1)} \times 100,$$

again in percentage terms. The results for this example are reported in Table 2.

It turns out that within this population of 10 and for the parameter values described, only individuals and groups of size 2 and 3 are stable. The question arises then: which groups do we expect to see and if there are groups of different sizes which payoffs do we look at? Since we are looking at constrained efficient schemes among identical agents, a good contender is the partition of the population into stable groups that maximizes the expected utility of an agent, under the assumption that his probability to be in any given group is proportional to the size of the group. In this example, this rule predicts that the population would break into three stable groups of three and one individuals (which means a 90% chance to get $v^*(3)$ and a 10% probability to get

10. The argument is much more subtle when history-dependent and asymmetric strategies are involved. The subtlety arises from the fact that to prevent a group deviation by n individuals, only a subset of them (one, at best!) needs be deterred. With general strategy spaces, the additional flexibility arising from unequal treatment becomes available. We return to these issues below.

TABLE 2

Stable gains are limited

n	Stable?	i-stable gain (%)	Stable gain (%)
1	$\surd$	0	0
2	$\surd$	10	10
3	$\surd$	50	38
4	$\times$	61	Ø
5	$\times$	69	Ø
6	$\times$	75	Ø
7	$\times$	78	Ø
8	$\times$	81	Ø
9	$\times$	84	Ø
10	$\times$	85	Ø

$v(1)$). That is, an individual's stable payoff gain is 38% (see Table 2) with probability $9/10$ and 0 otherwise. This implies a stable gain of only 34% which is less than *half* the return (85%) were we not to account for coalition formation.

Much has been written on "social capital" in the past few years. In the insurance context one could measure the return to such capital very much as we have done here. Clearly, recognizing the possibility of coalition deviations dramatically reduces the estimated return on social capital.

One might object that Example 2 is only described for a very special set of parameter values. Computations for several parameter values[11] reveal both robustness and sensitivity, in the following sense.

Equilibrium group sizes and the need for insurance are very sensitive to the parameters. As Coate and Ravallion (1993) observed in their computations, "[o]ne striking feature of the results . . . is how sharply the performance varies. Even quite a successful risk-sharing arrangement may vanish with certain seemingly modest perturbations to parameter values, such a small decline in the participants' aversion to risk". These observations are compounded by an order of magnitude in our model. Even an *increase* in risk (or in the aversion to it) may destroy previously successful insurance arrangements as previously non-viable subgroups now become viable, destroying the viability of the larger community. In this example, for instance, increasing the need for insurance θ from 0·91 to 1 causes a group of size 3 to become unstable. Several perturbations of θ and p cause the stable gain to fluctuate from 20% (a fifth of the corresponding i-stable gain) to 37% (45% of the corresponding i-stable value). This suggests a great deal of sensitivity in the quantitative magnitudes. However, the results are surprisingly robust in the sense that potential coalition deviations inevitably cause a large fraction of the potential benefits from insurance not to be reaped.

4. LIMITATIONS AND POSSIBLE EXTENSIONS

We have made several assumptions in the analysis so far, some of them implicit. In this section, we discuss the more important restrictions (several of them will be taken care of as we move on to the nonstationary case in Section 5).[12]

11. The computations, conducted on MATLAB, are available on request from the authors, and are also available at `http://www.restud.com/supplements.htm`.

12. Some of the obvious suspects are not crucial at all. For example, it is easy to generalize the ideas of this paper to the case in which there are several income realizations, not just two, as well as to the case in which income shocks are correlated across agents, as in Kocherlakota (1996).

4.1. *On group formation*

An important restriction runs through the entire exercise, and it is unclear how crucial this is for the results we obtain. This is the assumption that only *subgroups* of existing groups can deviate. In other words, *when we consider further deviations from subgroups*, we do not permit new groups to form by putting together fragments from several existing groups, nor do we permit wholesale mergers of existing groups. But the emphasized phrase in the previous sentence is important: we certainly allow groups of any size to form at the outset; no restriction is imposed at that stage.

The problem arises because it is unclear how to formalize arbitrary deviations, and not because there is some *a priori* suspicion that this will invalidate our results. One issue is the potential threat of "cyclical blocking chains", which often frustrates the basic task of formulating a satisfactory solution concept.[13] Suppose that there are three agents. Consider all possible groupings of two individuals each. At first blush, each such group is unstable: for instance, the group {12} could disintegrate when person 1 gets a good shock: 1 could refuse to make any transfers, team up with person 3, and replicate whatever it is that {12} planned to do in the future. The same argument holds with each pair of individuals. But this approach is inconsistent: if all pairs are known to lack viability, then a particular pair should be able to exploit the consequent instability to attain viability on its own (under the credible threat that any alliance with an outside agent will itself disintegrate).

There are two ways around this conceptual problem. First, symmetry may need to be broken: coalition {12} may be deemed stable while all other pairwise groups are deemed unstable.[14] (The idea generalizes to communities with more agents.) This is consistent: groups {13} and {23} are both unstable because of the perceived threat from {12}, while this last pair is stable because there is no credible threat from the other pairs. Second, one might entertain *symmetric* probabilistic solutions, in which deviations occur stochastically so that each pair has an uncertain lifetime. This will require, of course, that a potential deviator be indifferent between remaining in the group and joining hands with an outsider, and this indifference condition will pin down group value (see Konishi and Ray (2002) for an approach to coalition formation along these lines).

Note that neither approach compromises our results in any qualitative sense, though it is possible that the list of stable group sizes will be altered.

4.2. *Who deviates?*

Notice that in our definition, we consider potential deviations only by agents whose income realizations are high. Might people with a bad shock never want to deviate? There are two responses to this question. First, once we consider asymmetric or history-dependent equilibria there is no reason to exclude low draws from the list of potential deviants: they may well be called upon to make transfers. We take this fully into account in Section 5.

Second, it is possible that low draws may participate in group deviations even when we consider symmetric, stationary equilibria. Our analysis makes an implicit—but, in our view, reasonable—assumption that rules out this possibility. To clarify this, suppose that a subgroup of two agents in a community of four receive a good shock. Suppose, moreover, that the two-person enforcement constraint is satisfied, either because a two-person coalition is unstable and does not pose a threat, or it is stable but the gains from deviation are not worth the cost of moving from four to two persons. However, assume that a *three*-person group is indeed stable, and that our

13. The restriction to "internal deviations" is common ever since the possibility of cyclical chains of blocking was raised by Shenoy (1979).

14. Bernheim and Ray (1989, p. 307) discuss this issue of asymmetric labelling in a related context.

two high drawers would prefer to deviate and enjoy the benefits of a three-person arrangement thereafter. Now, there is no apparent reason why a person with a low draw should join this group: she is about to *receive* money and she will thereafter stay with a four-person group, rather than with three. But it still might happen: if each of the two low drawers were to believe that the other would join if she didn't, it might pay to agree. But this sort of behaviour depends crucially on a coordination failure among the low drawers. If no agent with a low draw were to join the deviants, then no such agent would *want* to join.

The same reasoning applies for any number of agents. Given this equilibrium selection, checking the enforcement constraints only for stable subsets of highs is necessary and sufficient when considering symmetric stationary strategies.

4.3. *Stability relative to some given scheme*

The analysis so far goes some way towards explaining the role of group deviations in fully bounding stable group size. It also explains why the extent of insurance might change in paradoxical ways as the need for insurance changes. But the arguments we employ are limited, in that we do not provide a tighter characterization of stable groups. We believe this is a very difficult question, though certainly not an impossible one.[15]

One possible approach is to specify some insurance scheme, and then to attempt to describe the set of stable groups relative to that scheme. An obvious candidate is full insurance, in which individuals within a group must attempt to equalize their consumption in every period. Such a scheme may be socially determined, for instance: it may be incumbent on all members, *conditional on being in the same community*, to share their resources to the maximum extent. To be sure, an insistence on such norms may ultimately cause the community to split up. Using a notion of stability for equal sharing—*first-best stability*—it is possible to make some progress.

Let $\tilde{v}(n)$ denote the expected utility from a first-best transfer scheme.[16] By definition, individuals are first best stable and the worth of a singleton group is $\tilde{v}(1)$. Recursively, having assessed first-best stability for all $m = 1, \dots, n-1$, a coalition of size n is said to be first-best stable if, for all $k = 1, \dots, n-1$,

$$(1-\delta)\left(u(h) - u\left(\frac{k}{n}h + \frac{n-k}{n}\ell\right)\right) \le \delta(\tilde{v}(n) - \tilde{v}(s)) \tag{8}$$

for every first-best stable $s \le k$. If n is first-best stable then its worth is simply $\tilde{v}(n)$. Note that for a given first-best stable size s it actually suffices to check the constraint for $k = s$ since the L.H.S. is decreasing in k.

With quadratic preferences, it is possible to show that the set of first-best stable sizes is a "connected" set of integers:

Observation 1. *Let u have the special form $u(x) = -(B-x)^2$ for some $B > h$. Then n is stable if and only if for every $1 \le k \le n-1$,*

$$\frac{C}{k} + \frac{k}{n} \ge \frac{2}{\theta} + 1, \tag{9}$$

where $C = \frac{\delta}{1-\delta}p(1-p)$ and $\theta = \frac{h-\ell}{B-h}$ is the need for insurance, as defined earlier.

15. Yi (1996), Bloch (1997) and Ray and Vohra (2001) all contain applications in which characterizations of stable groups are provided. These applications include Cournot oligopoly, public goods coalitions, and customs unions. However, apart from the general methodology, the analysis is typically application-specific.

16. That is $\tilde{v}(n) \equiv \sum_{k=0}^{n} p(k,n)u(\frac{k}{n}h + \frac{n-k}{n}\ell)$.

In particular, if a group of size n is not first-best stable then a group of size $n' > n$ is not first-best stable either.

Given (9), the reason for the set of first-best stable sizes being connected is simple: if condition (9) fails for some group size n and some subgroup size k, then it must fail for the very same subgroup size at all values $n' > n$.

A corollary of this observation is that a necessary and sufficient condition for the existence of *some* nontrivial first-best stable size is the same as the condition for a *two*-person group to be nonempty.[17] A specialization of (9) to the case of $n = 2$ and $k = 1$ reveals that the required necessary and sufficient condition is

$$\frac{\delta\theta p(1-p)}{1-\delta} \geq 3/2. \tag{10}$$

The condition (9) may also be used to obtain a tighter description of the maximal first-best stable group. We illustrate this by neglecting integer constraints (which are easily accounted for). Observe that the L.H.S. of (9) is minimized (in k) when $k = \sqrt{nC}$, this condition being applicable when $n > C$. Solving for the minimum value, we see that the maximal group size M is bounded above by the inequality

$$M \leq \max\left\{C, \frac{4C}{\left(\frac{2}{\theta}+1\right)^2}\right\}. \tag{11}$$

Note that M is bounded *uniformly* in θ. Whether or not this is a general observation is an open question (our proposition for stability establishes "pointwise" bounds).

4.4. *Strong stability*

One might wish to drop the requirement that blocking groups be credible. This is a "strong equilibrium" notion. In our view, this is very restrictive. It places severe constraints on the original group, but does nothing to subcoalitions. (The concept is not consistent in a very basic sense.) It is true that if something survives strong blocking, it passes a big test, but we may be throwing a lot out.

Briefly, one could define the concept as follows. Say that a (nontrivial) stationary transfer scheme $\mathbf{t}$ is *strongly stable* if for all $k = 1, \ldots, n$,

$$(1-\delta)u(h-t_k) + \delta v(\mathbf{t}, n) \geq (1-\delta)u(h) + \delta\tilde{v}(s) \tag{12}$$

for every $s \leq k$.[18]

The set of nontrivial strongly stable sizes may or may not be empty. For instance, the members of a two-person group can divide their income equally each period with no incentive to deviate alone, given sufficient patience. Under those circumstances a two-person coalition would be strongly stable. Less trivially, one can write down examples of three-person insurance schemes which are immune to two-person deviations *that employ the first-best insurance scheme thereafter*.

More generally, observe that if n is strongly stable it has to exhibit perfect risk-sharing, otherwise the peculiar logic of strong stability renders it vulnerable to a deviation by n itself! So strong stability is very close to examining the stability of first-best schemes, as discussed in

17. With first-best stability, a two-person group reaches stability at a lower threshold of θ than any group of larger size (at least with quadratic preferences). While this superficially runs against the grain of Corollary 1, there is no contradiction here. First-best stability is different from i-stability.

18. Recall that $\tilde{v}(n)$ is the expected utility from a first-best transfer scheme.

Section 4.3. In fact, when the set of stable sizes is connected (as in the quadratic case discussed in Section 4.3), the two concepts can be shown to be equivalent.

The set of strongly stable sizes has a finite upper bound. Since first-best risk-sharing is stationary and symmetric, the proof of this statement is very similar to the proof of Proposition 3 and is therefore not included.

4.5. *Group deviations and asymmetric treatment*

Our use of symmetric equilibrium may be criticized along the following lines. When group deviations are important, symmetry necessitates that we must compensate *all* potential deviants in a subgroup in order to prevent a deviation. If the symmetry is broken (perhaps stochastically), then not all deviants need to be so treated. A subset will suffice, containing the minimal number that must be compensated in order to avoid the deviation. It appears, then, that asymmetric treatment is a conceivable reaction to group deviations, possibly widening the scope of stability for a community.

As an illustration, consider an environment similar to Example 1. We solve this numerically so specific parameters are involved: a population of three individuals with constant relative risk aversion of 1·6 and a discount factor of 0·83 face an income distribution with mean 2·4 and a probability $p = 0{\cdot}4$ of a low income. It is easy enough to pick a level of insurance need $\theta = 1{\cdot}13$ ($h = 3{\cdot}1$, $\ell = 1{\cdot}9$) such that, using symmetric and stationary agreements, the only (nontrivial) stable group size is 2. The stability of groups of size 2 prevents the stability of the full community.

Now we can show that a stable stationary insurance scheme exists for the entire community: one that makes use of *asymmetric* transfers among the three agents. Define $\tau_1 = 0{\cdot}33$ and $\tau_2 = 0{\cdot}13$ and consider the following scheme. When only one individual draws high, he pays τ_1 which is divided equally among the two lows as before. The difference appears when two members have high incomes. The scheme selects randomly one of the highs and requires him to pay τ_1 to the agent with a low income. The other agent with a high draw is asked only to transfer τ_2. Under this scheme, it is easy to see that the second agent will not want to deviate in a group of 2 while the first (who makes the larger transfer) would not want to deviate alone. Hence, the asymmetric scheme is stable. Clearly, different agents with the same income realization are treated differently although the randomization ensures that symmetry is respected *ex ante*.

The points made here and in the previous subsections strongly suggest that an examination of asymmetric and fully history-dependent schemes is called for as a robustness check on the results. Fortunately, we are able to report some progress on this important question.

5. ON NONSTATIONARY INSURANCE ARRANGEMENTS

The reader familiar with the insurance literature might ask whether our results extend to non-stationary schemes. As we have already noted, Section 4 provides additional motivation for such a query. To this it must be added that history-dependent schemes are of interest even in the case of individual deviations. Indeed, as Fafchamps (1996), Kocherlakota (1996), Kletzer and Wright (2000), Ligon *et al*. (2002) and others have observed, second-best i-stable schemes are generally history dependent once the first-best fails to be self-enforcing.

In addition, as several authors have argued, history-dependent schemes appear to receive considerable empirical support. Numerous studies in the economic and anthropological literature provide evidence that informal risk-sharing agreements and informal credit arrangements are not clearly separated (see, for instance, Evans-Pritchard (1940), Platteau and Abraham (1987), Udry (1994)). These studies report a large reliance on what is observationally equivalent to informal loans with an implicit repayment scheme contingent on the lender's needs and the borrower's ability to repay. This is actually well reflected in the structure of history-dependent schemes. Ligon *et al*. (2002), in their analysis of ICRISAT data, observe that the history-dependent "dynamic commitment model does better than any of several alternatives in explaining actual consumption allocations. It provides a better explanation than the benchmark of perfect risk-pooling; it also performs better than the (stationary) limited commitment model". Foster and Rosenzweig (2001) extend this framework to include altruism:[19] this permits them to uncover variations in history dependence as a function of, say, within-family vs. cross-family arrangements. Such variation is borne out in their analysis, lending further credence to the view that history dependence is an outcome of lowered commitment ability (proxied here by altruism). Fafchamps and Lund (2001) also obtain evidence that the "quasi-credit" nature of insurance appear to fit the data best.[20] They conclude that "(the) bulk of the evidence appears in agreement with the theoretical predictions of (limited commitment) models: risk sharing takes place through repeated informal transactions based on reciprocity; mutual insurance takes place through a mix of gifts and no interest loans; and informal indebtedness reduces borrowing".

5.1. *Insurance arrangements*

We first define general insurance arrangements. For a group of size n, let $\mathbf{y}$ be a vector of *realized incomes*; that is, y_i is either h or ℓ for each $i = 1, \ldots, n$. Let $\mathbf{c}$ be a non-negative vector of consumptions. Say that $\mathbf{c}$ is *feasible* (under $\mathbf{y}$) if $\sum_i c_i = \sum_i y_i$. For any date s, an s-history—call it H_s—is a list of all past income realizations and (feasible) consumption vectors. (At $s = 0$, simply use any singleton to denote the 0-history.)

Define $\mathcal{M}(\mathbf{y})$ to be the set of all probability measures over consumption vectors $\mathbf{c}$ such that $\mathbf{c}$ is feasible for $\mathbf{y}$. An *insurance arrangement* is a list of functions $\sigma = \{\sigma_s\}_{s=0}^{\infty}$ such that for all $s \geq 0$, σ_s maps the product of s-histories and current income realizations $\mathbf{y}$ to lotteries in $\mathcal{M}(\mathbf{y})$. We will say that an insurance arrangement is *nontrivial* if it places positive probability on schemes that involve nonzero transfers for some states.

Observe that an insurance arrangement generates a vector of expected payoffs following every s-history H_s: call this vector $\mathbf{v}(H_s, \sigma, n)$. (These are discounted normalized expected payoffs for each individual in the group, *before* the realization of current incomes and, of course, the consumption lottery.) Also observe that by standard dynamic programming arguments, an insurance scheme may be viewed as a lottery over *current* consumption schemes, followed by a vector of continuation payoffs, all contingent on the realization of the income state.

Notice that this general definition allows for history-dependence, asymmetries and randomization. Note that nothing is to be gained from randomization when individual deviations alone are considered (the set of stable payoffs from deterministic schemes is convex in any case). However, as discussed in Section 4.5, randomization makes an appearance when group deviations pose a threat.

5.2. *Stability*

Just as in the stationary case, we proceed recursively. Individuals (or singleton coalitions) are automatically branded stable. Indeed, there is only one stable payoff for an "individual coalition",

19. They use three different data sets, all from South Asia: the ICRISAT village level studies (VLS) survey, the Additional Rural Income Survey of the National Council of Applied Economic Research (India), and the IFPRI's Pakistan Food Security Survey.

20. They study transfers in a panel of Philippine rice farmers.

which is just the no-insurance payoff at every date. That is, if we define

$$v^*(1) \equiv pu(h) + (1-p)u(\ell), \tag{13}$$

then the set of stable payoffs is just $V^*(1) \equiv \{v^*(1)\}$.

Now suppose that we have defined stable payoff sets $V^*(m)$ for all $m = 1, \ldots, n-1$ (some of these may be empty). Pick a group of size n and a nontrivial insurance arrangement σ for this group. Say that σ is *stable* if the following two conditions are satisfied:

[PARTICIPATION]. For no s-history H_s is there a subgroup of individuals (of size $m < n$) and a stable payoff vector $\mathbf{v} \in V^*(m)$ such that $v_i(H_s, \sigma, n) < v_i$ for all $i = 1, \ldots, m$.

[ENFORCEMENT]. The following is a zero-probability event under σ: there is an s-history H_s, an income realization $\mathbf{y}$, and a prescribed consumption allocation $\mathbf{c}$ such that for some subgroup of individuals (of size $m < n$) and some stable payoff vector $\mathbf{v} \in V^*(m)$,

$$(1-\delta)u(y_i) + \delta v_i > (1-\delta)u(c_i) + \delta v_i(H_{s+1}, \sigma, n), \tag{14}$$

where H_{s+1} is the $(s+1)$-history obtained by concatenating H_s with $\mathbf{y}$ and $\mathbf{c}$.

If σ is stable, then say that $\mathbf{v}(h_0, \sigma, n)$ is a *stable payoff vector* for n. If no such vector exists, we say that n is *unstable* and set $V^*(n)$ to the empty set.

Observe that our recursion yields stable payoff sets that only depend on group size, and that such stable payoff sets must be symmetric (if a payoff vector $\mathbf{v}$ is in the set, then so are all permutations of $\mathbf{v}$).

Next, with these definitions in hand it is easy enough to put the usual enforcement constraints in perspective. Say that an insurance scheme (or group) is *individually stable*, or i-stable for short, if the participation and enforcement constraints are satisfied for singleton subgroups. In fact, in this case, we can ignore the participation constraint because it will be implied by the enforcement constraint. Moreover, as already discussed, no randomization will be necessary.

5.3. *General results*

The following proposition extends our main result to the general case.

Proposition 5. *For every value of θ such that some stable group exists, the maximal stable group size is finite.*

This proposition is the key result that establishes the finiteness of stable groups without taking recourse to parameters such as the cost of group formation. It is a considerably more complex result than its counterpart for the stationary case. There are two reasons for the increased complexity: equilibria make possible use of history and of asymmetric strategies in subgames. The two, acting in concert, significantly reduce the bite of the no-deviation constraint for groups. Recall that in a symmetric and stationary equilibrium, *every* member of a potentially deviant subgroup must be *simultaneously* compensated for staying with the ambient group. Such compensations become impossible because the marginal gains to group size vanish, while deviation gains are bounded away from zero, precipitating the boundedness result in the stationary case. In the general case, it is possible to carefully switch to asymmetric strategies following appropriate histories of good and bad draws. With these asymmetries, it is no longer necessary to compensate *every* member of every potential subgroup; it is only necessary to compensate *some* member of every potential subgroup. The question is: does this effective relaxation of the constraints permit very large groups to form? The answer is still no (which is the substance of the proposition).

The main idea of the proof (see Section 6 for details) is the construction of a particular set of average payoffs. For each stable size n and $v \in V^*(n)$, let a denote the *average* payoff under v, and let $a^*(n)$ stand for the maximum value of a over $V^*(n)$. Now for each integer k (stable or not), consider the maximum stable size no larger than k—call it $m(k)$—and consider the value $a(m(k))$. The point is this: if $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ is a payoff vector in a group of size n, arranged in increasing order of payoffs, the k-th entry v_k must exceed $a^*(m(k))$. Otherwise such a payoff would surely be "blocked" (see the proof of Lemma 6). In other words, this construction limits the extent of asymmetric treatment that can be generated in equilibrium.

Once these limits are set, there are bounds on the average payoffs that can be generated for any subgroup of a larger group of size n. The average value of these sub-payoffs cannot exceed $a^*(n)$ by "too much" because then the remainder would have to suffer enough asymmetric treatment so that they necessarily block the outcome. To be sure, small groups can be still treated asymmetrically but the degree of asymmetric treatment shrinks with relative subgroup size. (This is the subject of Lemma 9.)

Once this hurdle is cleared, the rest of the proof is relatively straightforward, and follows a modified version of the stationary case. This is the remainder of the formal argument starting with Lemma 10, and continuing through the end of the proof for the proposition.

Can this proposition be employed—as it was in the stationary case—to establish the nonmonotonicity of maximal stable groups in the need for insurance? To a large extent, it can, but unfortunately it is not enough to settle the question. It would be, if we could show that the minimum i-stable group size becomes unboundedly large as the need for insurance θ descends to the minimum level necessary for viability (for *some* group size). In turn, all this means is that (on the grounds of single-person deviations alone) larger groups should do better than smaller ones, in the sense that they should exhibit lower thresholds (in θ) for i-stability. Then larger groups would be i-stable strictly "before" any of their smaller counterparts, and would consequently be (fully) stable in our sense.

We believe that this observation is true. (It *is* true for stationary equilibria; see Proposition 1.) But we have not been able to prove it,[21] and so leave matters open in the form of the following conjecture.

Conjecture. The minimum i-stable group size goes to infinity as the need for insurance descends to its critical lower bound for which *some* group is stable.

6. PROOFS

6.1. *The stationary case*

Lemma 1. *Suppose that a transfer scheme* $\mathbf{t}$ *satisfies* (4). *Then so does the transfer scheme* $\lambda\mathbf{t}$ *for every* $\lambda \in (0, 1)$.

21. It is *not* enough to prove that larger groups are viable whenever smaller groups are. This leaves open the possibility that the threshold viability conditions are exactly the same. Once this is the case, however, there is no guarantee that the minimum *stable* group size becomes unboundedly large as θ descends to the minimum threshold. Indeed, all this discussion motivates the search for a *necessary and sufficient* condition that guarantees the viability of a given group of size n. This is a hard question. Some new results are reported in the supplementary material: see `http://www.restud.com/supplements.htm`.

Proof. It is easy to see, using (1), that $v(\mathbf{t}, n)$ is concave in $\mathbf{t}$. Because $u(h - t_k)$ is concave in t_k (and therefore in the vector $\mathbf{t}$), the expression

$$(1 - \delta)u(h - t_k) + \delta v(\mathbf{t}, n)$$

is concave in $\mathbf{t}$ for each $k = 1, \ldots, n - 1$. Finally, note that $v^*(1)$ is simply $v(0 \cdot \mathbf{t}, n)$. Using all this information, we see that (4) must hold for $\lambda\mathbf{t}$ if it holds for $\mathbf{t}$. ‖

Let $\mathbf{t}(t)$ denote the (symmetric stationary) transfer scheme in which t_k is set equal to a constant $t \geq 0$, for all $k = 1, \ldots, n - 1$. Define, for $t \geq 0$ and small,

$$d(t, n) \equiv (1 - \delta)u(h - t) + \delta v(\mathbf{t}(t), n). \tag{15}$$

We denote by $d'(t, n)$ the partial derivative of t with respect to t.

Lemma 2. *A nontrivial transfer scheme satisfying the individual enforcement constraint* (4) *exists if and only if* $d'(0, n) > 0$.

Proof. [Necessity] Suppose that there is some nontrivial transfer scheme $\hat{\mathbf{t}}$ satisfying (4). Choose $\lambda > 0$ such that $\lambda\hat{t}_k \leq [h - \ell]/n$ for all k. By Lemma 1, $\lambda\hat{\mathbf{t}}$ satisfies (4). Now observe that $v(\mathbf{t}, n)$ is increasing in any component t_k as long as $t_k < [h - \ell]/n$; it follows that the constant scheme $\mathbf{t}(t)$ in which t is set equal to the maximum value of $\lambda\hat{t}_k$ (over k) also satisfies (4). Noting that $v^*(1)$ is just $v(\mathbf{t}(0), n)$, we have therefore shown that

$$d(t, n) \geq d(0, n).$$

To complete the proof of necessity, observe that d is *strictly* concave in t.

[Sufficiency] If $d'(0, n) > 0$, then for small positive t we have $d(t, n) > d(0, n)$. But this means that we have found a nontrivial transfer scheme (with $t_k = t$ for all $k = 1, \ldots, n - 1$) such that

$$(1 - \delta)u(h - t) + \delta v(\mathbf{t}(t), n) \geq (1 - \delta)u(h) + \delta v^*(1),$$

so that (4) is satisfied. ‖

Proof of Proposition 1. By Lemma 2, a necessary and sufficient condition for the i-stability of n is $d'(0, n) > 0$. Using the definition of d and (1), simply unpack this condition. It is equivalent to the requirement that

$$-(1 - \delta)u'(h) + \delta[u'(\ell) - u'(h)] \sum_{k=1}^{n-1} p(k, n)\frac{k}{n} > 0,$$

which, on rearrangement, yields (5). ‖

Proof of Corollary 1. Simply note that if (5) is satisfied for some n, then it is satisfied for all $n' > n$. ‖

Proof of Proposition 2. Let m be the size of the community. Pick θ such that (5) is satisfied at $n = m$ but fails at $n = m - 1$. Then every (nonsingleton) group below m fails to be i-stable and is therefore unstable. Thus the stability of m is only to be assessed using individual deviations, and its i-stability implies stability. ‖

We now prepare for the proof of Proposition 3.

Lemma 3. *If n and n' are both stable and $n' < n$, then* $v^*(n) \geq v^*(n')$.

Proof. Simply use the constraint (2) for $s = k = n'$ when the group size is n, and the fact that $t_{n'} \geq 0$. ‖

Lemma 4. *For any $\epsilon > 0$, define $t(\epsilon)$ by*

$$u(h) - u(h - t(\epsilon)) \equiv \frac{\delta}{1-\delta}\epsilon, \tag{16}$$

and let

$$t(\epsilon, x) \equiv \min\{t(\epsilon), (1-x)(h-\ell)\} \tag{17}$$

for $\epsilon > 0$ and $x \in [0, 1]$. For $\epsilon > 0$ and each positive integer n define

$$v(\epsilon, n) \equiv \sum\nolimits_{k=0}^{n} p(k,n)\left[\frac{k}{n}u(h - t(\epsilon, k/n)) + \frac{n-k}{n}u\left(\ell + \frac{kt(\epsilon, k/n)}{n-k}\right)\right]. \tag{18}$$

Then there exists $\psi(\epsilon)$ with $\psi(\epsilon) \to 0$ as $\epsilon \to 0$, such that

$$\limsup\nolimits_{n\in\mathcal{N}} |v(\epsilon, n) - v^*(1)| \leq \psi(\epsilon). \tag{19}$$

Proof. Consider any sequence of integers in $\mathcal{N}$. By a simple large-numbers argument for Bernoulli trials (see, *e.g.* Feller (1968, p. 152, equation (4.1))), it must be the case that for fixed ϵ,

$$\lim\nolimits_n v(\epsilon, n) = pu(h - t(\epsilon, p)) + (1-p)u\left(\ell + \frac{pt(\epsilon, p)}{1-p}\right).$$

It is easy to check from (16) and (17) that $t(\epsilon, p) \to 0$ as $\epsilon \to 0$. It follows that

$$\lim\nolimits_n v(\epsilon, n) \to v^*(1) \text{ as } \epsilon \to 0,$$

which proves the lemma. ‖

Proof of Proposition 3. The only nontrivial part of this proposition (given the previous propositions) is the assertion that $\bar{n}(\theta)$ is finite. Suppose that Proposition 3 is false. Then there exists an infinite set $\mathcal{N}$ such that for all $n \in \mathcal{N}$, n is stable. By Lemma 3, if n and n' are both in $\mathcal{N}$ and $n < n'$, then $v^*(n) \leq v^*(n')$. Moreover, $\{v^*(n)\}_{n\in\mathcal{N}}$ is bounded. It follows that for any $\epsilon > 0$, there exists $n(\epsilon) \in \mathcal{N}$ such that for all $n \in \mathcal{N}$ with $n > n(\epsilon)$,

$$v^*(n) - v^*(n(\epsilon)) < \epsilon. \tag{20}$$

Moreover, it is easy enough to choose $n(\epsilon)$ satisfying both (20) and the requirement that

$$v^*(n(\epsilon)) \geq v^*(1) + A \tag{21}$$

for some $A > 0$ and independent of ϵ.

Now consider some stable $n > n(\epsilon)$, and let t_k be the optimal transfer in this coalition when there are k successes. Applying the constraint (2) when $k \geq n(\epsilon)$, we see that

$$(1-\delta)u(h - t_k) + \delta v^*(n) \geq (1-\delta)u(h) + \delta v^*(n(\epsilon)),$$

so that—rearranging terms—

$$(1-\delta)[u(h) - u(h - t_k)] \leq \delta[v^*(n) - v^*(n(\epsilon))] \leq \delta\epsilon,$$

using (20). It follows (using (16)) that $t_k \leq t(\epsilon)$. Consequently, applying the definition of $t(\epsilon, x)$, we can conclude that

$$\sum\nolimits_{k=n(\epsilon)}^{n} p(k,n)\left[\frac{k}{n}u(h - t_k) + \frac{n-k}{n}u\left(\ell + \frac{kt_k}{n-k}\right)\right]$$

$$\leq \sum_{k=n(\epsilon)}^{n} p(k,n)\left[\frac{k}{n}u(h - t(\epsilon, k/n)) + \frac{n-k}{n}u\left(\ell + \frac{kt(\epsilon, k/n)}{n-k}\right)\right].$$

It follows that for some finite constant B,

$$v^*(n) \leq v(\epsilon, n) + \Pr\{k < n(\epsilon)\}B. \tag{22}$$

Now it is obvious that as $n \to \infty$, $\Pr\{k < n(\epsilon)\} \to 0$. That is, we may use (22) to conclude that there exists a function $h(n)$ such that $h(n) \to 0$ as $n \to \infty$ (in $\mathcal{N}$) and such that

$$v^*(n) \leq v(n, \epsilon) + h(n). \tag{23}$$

Combining (23) with the conclusion—see (19)—of Lemma 4, we see that

$$\limsup_{n \in \mathcal{N}}[v^*(n) - v^*(1)] \leq \psi(\epsilon) + h(n).$$

This shows, in particular, that for large enough n and small enough ϵ,

$$v^*(n) - v^*(1) < A. \tag{24}$$

Combining (21) and (24), we see that

$$v^*(n) < v^*(n(\epsilon)),$$

which contradicts Lemma (3). ||

6.2. *Observations in Section* 4.3

Proof of Observation 1. With $u(x) = -(B - x)^2$, it is easy to see that $\tilde{v}(n) = -(B - \mu)^2 - \frac{p(1-p)}{n}(h - \ell)^2$. It follows that

$$\delta(\tilde{v}(n) - \tilde{v}(k)) = \delta \frac{p(1-p)(n-k)}{nk}(h - \ell)^2,$$

while routine computation reveals the L.H.S. of (8) to be

$$(1-\delta)\frac{n-k}{n}\left[2B(h-\ell) - \frac{k+n}{n}h^2 + \frac{n-k}{n}\ell^2 + 2\frac{k}{n}h\ell\right].$$

Recall that to check stability, it is necessary and sufficient to put $k = s$ in (8). Combining the expressions above with this observation, we see that the required condition is

$$(1-\delta)\left[2B(h-\ell) - \frac{k+n}{n}h^2 + \frac{n-k}{n}\ell^2 + 2\frac{k}{n}h\ell\right] \leq \delta \frac{p(1-p)}{k}(h-\ell)^2$$

for all $1 \leq k \leq n-1$. Some tedious simplification shows this to be equivalent to (9).

Clearly, for any given k the L.H.S. of (9) is decreasing in n. It follows that if (9) is violated for some n and k, will be violated for a group of size $n' > n$ and the same k. This proves the observation. ||

6.3. *The nonstationary case*

Before we proceed to a proof of Proposition 5, the following preliminary lemma is useful.

Lemma 5. *For any $\zeta > 0$, define $t(\zeta)$ by*

$$u(h) - u(h - t(\zeta)) \equiv \frac{\delta}{1-\delta}\zeta. \tag{25}$$

For $\zeta > 0$ and each positive integer n, define $a(n, \zeta)$ to be the maximum value of average expected utility (in one period, and neglecting any incentive constraints) over a group of size n, assuming that in no event can a total transfer exceeding $t(\zeta)n$ be made. Then there exists $\psi(\zeta)$ with $\psi(\zeta) \to 0$ as $\zeta \to 0$, such that

$$\limsup_{n\to\infty}[a(n, \zeta) - v^*(1)] \leq \psi(\zeta). \tag{26}$$

Proof. By symmetry and strict concavity of u, and because no incentive constraints are involved, it must be the case that for each state, consumption should be equalized over successes, consumption should be equalized over failures, and the transfer between success and failure individuals should aim at equality of consumption *across* both types of persons, subject to the constraint that the total transfer not exceed $t(\zeta)n$. That is, an optimal *per capita* transfer from any successful individual, defined for each n and each number of successes k, is given by

$$t(k, n) = \min\left\{\frac{n}{k}t(\zeta), \frac{n-k}{n}\Delta\right\}, \tag{27}$$

where $\Delta \equiv h - \ell$. Moreover,

$$a(n, \zeta) = \sum_{k=0}^{n} p(k, n)\left[\frac{k}{n}u(h - t(k, n)) + \frac{n-k}{n}u\left(\ell + \frac{k}{n-k}t(k, n)\right)\right]. \tag{28}$$

By the same large-numbers argument as in the proof of Lemma 4, it must be the case that if $\zeta > 0$ and small enough,

$$\lim_n a(\zeta, n) = pu\left(h - \frac{t(\zeta)}{p}\right) + (1-p)u\left(\ell + \frac{t(\zeta)}{1-p}\right).$$

Because $t(\zeta) \to 0$ as $\zeta \to 0$,

$$\lim_n a(\zeta, n) \to v^*(1) \text{ as } \zeta \to 0,$$

which proves the lemma. ||

Proof of Proposition 5. Suppose that the proposition is false. Then there exists an infinite subset of indices M such that for all $n \in M$, n is stable. We will employ the following notation. For any integer n, we let $m(n)$ stand for the maximum stable size not exceeding n. For each stable size n and $v \in V^*(n)$, let a denote the *average* payoff under v, and let $a^*(n)$ stand for the maximum value of a over $V^*(n)$. (This is well defined because $V^*(n)$ is compact by standard arguments.) Also, define for each n (stable or not),

$$\tilde{a}(n) \equiv \frac{1}{n}\sum_{k=1}^{n} a^*(m(k)). \tag{29}$$

Lemma 6. *For every stable $n > 1$,*

$$a^*(n) \geq \tilde{a}(n), \tag{30}$$

and in particular, the sequence $\{\tilde{a}(n)\}$ is nondecreasing in n.

Proof. Let $n > 1$ be stable. Now, there exists a *stable* payoff vector $\mathbf{v}$ in $V^*(n)$ with average value precisely equal to $a^*(n)$. Without loss of generality $v_k \leq v_{k+1}$ for all $k = 1, \ldots, n-1$. We claim that

$$v_k \geq a^*(m(k)) \tag{31}$$

for all $k = 1, \dots, n$. Suppose not. Then there is some first index k such that (31) fails. That is,

$$(v_1, \dots, v_k) \ll \{a^*(m(k)), \dots, a^*(m(k))\} \tag{32}$$

where there are k entries on the R.H.S. (Note that if $k = n$, $m(k)$ *must* be less than n.) If the equal payoff $a^*(m(k))$ is stable for $m(k)$, (32) immediately contradicts the stability of $\mathbf{v}$. Therefore the equal payoff must be unstable.

Notice that—by the symmetry of $V^*(m(k))$—the equal payoff $a^*(m(k))$ *can* be generated as a convex combination of vectors in $V^*(m(k))$. Therefore, if the equal payoff is unstable, this can *only* be because of the participation constraint. Randomization cannot affect the *ex post* enforcement constraint, which applies *ex post* and would be satisfied by every vector making up the required convex combination. Consequently, there is some $n' < m(k)$ and a stable payoff $\mathbf{v}' \in V^*(n')$ such that

$$\{a^*(m(k)), \dots, a^*(m(k))\} \ll \mathbf{v}' \tag{33}$$

(where there are n' entries on the L.H.S.). Combining (32) and (33), we contradict the stability of $\mathbf{v}$. Therefore our claim is true, and (31) holds. Averaging this inequality over $k = 1, \dots, n$, we conclude that

$$a^*(n) = \frac{1}{n}\sum_{k=1}^{n} v_k \geq \frac{1}{n}\sum_{k=1}^{n} a^*(m(k)) = \tilde{a}(n-1),$$

which is (30).

Now notice from (29) that for any n,

$$\tilde{a}(n+1) = \frac{1}{n+1}a^*(m(n+1)) + \frac{n}{n+1}\tilde{a}(n). \tag{34}$$

If $m(n+1) = n+1$, then—using (30)—it follows immediately from (34) that $\tilde{a}(n)$ is nondecreasing. Otherwise $m(n+1) = K$ for some stable $K \leq n$. Then it is easy to see that $\tilde{a}(n) = \lambda a^*(K) + (1-\lambda)\tilde{a}(K)$ for some convex combination λ. Applying (30) again, we may conclude that

$$a^*(m(n+1)) = a^*(K) \geq \lambda a^*(K) + (1-\lambda)\tilde{a}(K) = \tilde{a}(n),$$

and using this information in (34), we conclude that $\tilde{a}(n)$ must be nondecreasing. ‖

Lemma 7. *There exists $A > 0$ such that for all $n \in M$ with $n > 1$,*

$$a^*(n) \geq v^*(1) + A. \tag{35}$$

Proof. For any stable $n > 1$, a stable payoff vector must be supported by transfer schemes that are nonzero for some states: consequently, by the enforcement and participation constraints applied to singleton deviations, the average payoff under a stable vector in any stable (nonsingleton) group must *strictly* exceed $v^*(1)$.

To establish that this excess is *uniform* over the set of stable groups, observe that $\tilde{a}(n)$ (being a convex combination of the a^*'s) is also strictly in excess of $v^*(1)$, as long as n exceeds some stable group size that exceeds unity. Because $\tilde{a}(n)$ is nondecreasing (Lemma 6), the excess is uniform. Now (30) of the same lemma yields the desired result. ‖

Lemma 8. *For each $\epsilon > 0$, there exists a stable $n(\epsilon)$ and an infinite subsequence of stable sizes exceeding $n(\epsilon)$—call the set $M(\epsilon)$—such that for all $n \in M(\epsilon)$,*

$$a^*(n) - a^*(n(\epsilon)) < \epsilon. \tag{36}$$

Proof. By Lemma 6, $\tilde{a}(n)$ is nondecreasing in n. Obviously it is bounded. So it converges. Moreover, it is easy to verify (see, *e.g.* Knopp (1956, Exercise 2.4.1.2)) that

$$\lim_n \tilde{a}(n) \geq \lim\inf_{n \in M} a^*(n).$$

So for each $\epsilon > 0$, there is a stable size $n(\epsilon)$ and a subsequence $M(\epsilon)$ such that

$$a^*(n) - \tilde{a}(n(\epsilon)) < \epsilon$$

for all $n \in M(\epsilon)$. By Lemma 6, the desired property (36) follows immediately. ‖

Lemma 9. *Let* $\epsilon > 0$ *and* $x \in (0, 1)$ *be given. Then there exists an integer* $m(x, \epsilon) > n(\epsilon)$ (*the latter given by Lemma* 8) *such that for any stable group* $n \geq m(x, \epsilon)$, *any subgroup of players of size* $k \geq xn$ *and any vector of continuation payoffs* $\mathbf{v}(n) \in V^*(n)$, *the average payoff to the subgroup cannot exceed* $a^*(n) + \epsilon/x$.

Proof. First we describe $m(x, \epsilon)$. To this end, let a^* denote the supremum of $a^*(n)$ over stable n, and let $m(x, \epsilon)$ be any positive integer no smaller than $n(\epsilon)[a^* + \epsilon]/x\epsilon$.

Consider any $n \geq m(x, \epsilon)$, and let $\mathbf{v}(n) \in V^*(n)$. Let the average payoff under this vector be denoted by $a(n)$. Of course, $a(n) \leq a^*(n)$.

Consider any subgroup of k people, where $k \geq nx$. Let $a(k)$ denote the average payoff they receive under $\mathbf{v}(n)$. Now consider two cases. First, suppose that $k \geq n - n(\epsilon)$. Note that

$$a(k)k \leq na(n) \leq na^*(n).$$

Rearranging and using the fact that $k \geq n - n(\epsilon)$,

$$a(k) \leq a^*(n) + \frac{n(\epsilon)}{n - n(\epsilon)} a^* \leq a^*(n) + \epsilon,$$

where the last inequality uses the fact that $n \geq m(x, \epsilon)$. Because $x \in (0, 1)$, we are done in this case.

Otherwise, $k < n - n(\epsilon)$. Now, by the stability of the payoff vector $\mathbf{v}(n)$, no more than $n(\epsilon)$ individuals can receive strictly less than $a^*(n(\epsilon))$.[22] Since we are proving an upper bound on average payoffs, we can suppose without loss of generality that our subgroup of k is drawn from the remaining individuals (who receive at least $a^*(n(\epsilon))$ each). In this case,

$$a(k)k + [n - n(\epsilon) - k]a^*(n(\epsilon)) \leq na(n) \leq na^*(n).$$

Combining this inequality with the fact that $a^*(n(\epsilon)) \geq a^*(n) - \epsilon$ (see Lemma 8), we can conclude that

$$\begin{aligned} a(k) &\leq \frac{\epsilon n}{k} + \frac{n(\epsilon)[a^*(n) - \epsilon]}{k} + a^*(n) - \epsilon \\ &\leq \frac{\epsilon}{x} + \frac{n(\epsilon)[a^*(n) - \epsilon]}{nx} + a^*(n) - \epsilon \\ &\leq a^*(n) + \frac{\epsilon}{x}, \end{aligned}$$

where the second inequality uses the fact that $k \geq nx$, and the third inequality uses the fact that $n \geq m(x, \epsilon)$. Thus in both cases, the proof is complete. ‖

22. The proof is very similar to the argument used to establish (31). If $a^*(n(\epsilon))$ is a stable (constant) payoff for $n(\epsilon)$, the assertion is obviously true. If $a^*(n(\epsilon))$ is not stable, this can only happen because of the participation constraint. But then there is a still smaller group—say n'—and a stable payoff vector $\mathbf{v}'$ for n' such that $\mathbf{v}'$ dominates $a^*(n(\epsilon))$ in every component. The existence of this vector now establishes the assertion.

Lemma 10. *For any $\epsilon > 0$ and $x \in (0, p)$, suppose that $n \geq m(x, \epsilon)$ (where $m(x, \epsilon)$ is given by Lemma* 9) *and $n \in M(\epsilon)$, where $M(\epsilon)$ is given by Lemma* 8. *Consider the supporting insurance scheme for any $\mathbf{v}(n) \in V^*(n)$. Then the total transfer T made across agents in any realization must be bounded as follows:*

$$T \leq \max\{t(\epsilon(1+x)/x)n, xhn\} \tag{37}$$

where the function $t(\cdot)$ is defined by (25).

Proof. Let K be a group of people called upon to make transfers. Suppose first that $|K| \equiv k < nx$. Then the total transfer in this case can obviously not exceed xhn, so the lemma is trivially true in this case.

Now suppose that $k \geq nx$. Recalling how $m(x, \epsilon)$ was defined in the proof of Lemma 9 and noting that $n \geq m(x, \epsilon)$, we see that $k \geq n(\epsilon)$, where $n(\epsilon)$ is given by Lemma 8. Therefore all stable subcoalitions of K of size $n(\epsilon)$ or less are *potential* deviants.

Arrange the members of K in decreasing order of the transfer t_i that they make, so that $t_i \geq t_{i+1}$. Recall that by familiar dynamic programming arguments, the associated transfer scheme (once the state is realized and all lotteries have been resolved) can be described by a set of current consumptions and a set of continuation payoff vectors. In this particular state, let v_i' stand for the continuation utilities of members of K.

We claim that for every $i \in K$,

$$u(h) - u(h - t_i) \leq \frac{\delta}{1-\delta}[v_i' - a^*(m(i))], \tag{38}$$

where $m(i)$, it will be recalled, is defined as the largest stable group size *not* exceeding the integer i. The proof of this claim is easy (and is similar to the proof of (31)): if it were to fail for some integer i, then by the ordering of the group members by transfer size, we have for *every* $j \in K$, $j \leq i$, that

$$u(\ell) - u(\ell - t_j) \geq u(h) - u(h - t_i) > \frac{\delta}{1-\delta}[v_i' - a^*(m(i))].$$

Thus, if $a^*(m(i))$ is stable for group $m(i)$, this is an immediate violation of the enforcement constraint: a group of size $n(i)$ can now profitably form and deviate. Otherwise, if it is not stable, this can *only* be due to the participation constraint (recall our discussion of randomization). But then a subgroup smaller than $m(i)$ will deviate.

So (38) must be true.

Let us add (38) over all individuals in K, and divide by the group size k. We then have

$$u(h) - \frac{1}{k}\sum_{j \in K} u(h - t_j) \leq \frac{\delta}{1-\delta}[a(n, K) - \tilde{a}(k)], \tag{39}$$

where $a(n, K)$ simply stands for the average continuation payoff accorded to members of K, and $\tilde{a}(k)$ is the moving average defined in (29). Now by the concavity of u,

$$\frac{1}{k}\sum_{j \in K} u(h - t_j) \leq u(h - t_K),$$

where t_K is the average transfer made by K. Moreover, $a(n, K) \leq a^*(n) + \epsilon/x$, by Lemma 9. Finally, because $k \geq n(\epsilon)$ and $n \in M(\epsilon)$, we know from (36) that $\tilde{v}(k) \geq a^*(n) - \epsilon$. Using all these three inequalities in (39), we may conclude that

$$u(h) - u(h - t_K) \leq \frac{\delta}{1-\delta}\frac{\epsilon(1+x)}{x}.$$

Invoking (25), we see that

$$t_K \leq t\left(\frac{\epsilon(1+x)}{x}\right),$$

and recalling that the total transfer T equals $kt_K \leq nt_K$, the lemma is established. ‖

Lemma 11. *For some stable* $\mathbf{v}(n) \in V^*(n)$ *and some supporting insurance scheme, let* $w(n)$ *denote the expected current utility averaged across all agents associated with that scheme. Then, if* $a(n)$ *is the average payoff under* $\mathbf{v}(n)$*, we have*

$$a(n) \leq (1-\delta)w(n) + \delta a^*(n). \tag{40}$$

Proof. Recall that the supporting insurance arrangement is expressible as a randomization over various current consumption schemes and continuation payoffs, both contingent on realizations of the income state. Fix any such scheme in the support of the arrangement. For each individual i, let $c_i(n, S)$ describe his current consumption and $v_i(n, S)$ his continuation payoff, where S is the set of currently successful people. Then

$$v_i(n) = (1-\delta)\sum_S \Pr(S)u(c_i(n, S)) + \delta \sum_S \Pr(S)v_i(n, S).$$

If we average this over all individuals, we see that, by definition,

$$a'(n) = (1-\delta)w'(n) + \delta \sum_S \Pr(S)\left[\frac{1}{n}\sum_{i=1}^n v_i(n, S)\right],$$

where $a'(n)$ is average payoff after the lottery is realized (but not the state) and $w'(n)$ is average *current* payoff evaluated under exactly the same conditions. However, $(1/n)\sum_{i=1}^n v_i(n, S) \leq a^*(n)$ for every S. Using this in the equation above, we see that

$$a'(n) \leq (1-\delta)w'(n) + \delta a^*(n).$$

Finally, take expectations over all schemes in the support of the insurance arrangement to establish the lemma. ‖

We now complete the proof of the proposition. To this end, fix a small positive number ϵ, and then choose $x \in (0, 1)$ small enough so that $xh < t(\epsilon)$, where $t(\cdot)$ is defined in (25). Now recall $M(\epsilon)$ from Lemma 8, $n(\epsilon)$ from Lemma 8 and the consequent definition of $m(x, \epsilon)$ in the proof of Lemma 9. Pick any n in $M(\epsilon)$ with $n \geq m(x, \epsilon)$.

Consider any stable $\mathbf{v}(n) \in V^*(n)$, and some insurance arrangement associated with it. By Lemma 10, the total transfer T under any realization cannot exceed

$$\max\{t(\epsilon(1+x)/x)n, xhn\}.$$

But our choice of x guarantees that the first of the two terms above always binds, so that

$$T \leq t(\epsilon(1+x)/x)n$$

for any total transfer T under any realization. It follows that any scheme in the support of the insurance arrangement is a *feasible* scheme over which maximization occurs in Lemma 5. Consequently, (current) expected utility averaged across all agents and over all schemes in the support of the insurance arrangement—call it $w(n)$—must satisfy the inequality

$$w(n) \leq a(n, \epsilon(1+x)/x). \tag{41}$$

Now consider any sequence of stable n such that $n \geq m(x, \epsilon)$ and $n \in M(\epsilon)$, and any sequence $\mathbf{v}(n) \in V^*(n)$ such that the associated average value

$$a(n) = a^*(n). \tag{42}$$

Lemma 5 and (41) together tell us that for any associated sequence of current consumption schemes,

$$\limsup_{n \in M(\epsilon); n \to \infty} w(n) \leq \limsup_{n \in M(\epsilon); n \to \infty} a(n, \epsilon(1+x)/x) \leq v^*(1) + \psi(\epsilon(1+x)/x). \tag{43}$$

Now choose ϵ and then x such that the R.H.S. of (43) is smaller than $v^*(1) + A$, where A is given by (35). We may then conclude that for large enough $n \in M(\epsilon)$,

$$w(n) < a^*(1) + A.$$

Combining this inequality with (40) of Lemma 11, we see that

$$a(n) \leq (1 - \delta)[v^*(1) + A] + \delta a^*(n),$$

where $a(n)$ is the (lifetime) expected utility averaged over all agents under the scheme. But we have chosen this so that $a(n) = a^*(n)$ (see (42)). Consequently, we may conclude that for large enough $n \in M(\epsilon)$,

$$a^*(n) < v^*(1) + A,$$

but this inequality contradicts (35) of Lemma 7 and completes the proof of the proposition.

Acknowledgements. The authors are grateful for comments by seminar participants at the University of Chicago, Princeton University, the INRA/Delta in Paris, the University of California at Irvine, the Summer Meeting of the Econometrics Society at the University of Maryland and the NEUDC at Cornell University. We also thank two anonymous referees for useful comments on an earlier draft.

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