Aspirations, Inequality, Investment and Mobility

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1. Introduction

What individuals want for themselves, or what parents want for their children, is conditioned by society in fundamental ways. One such pathway is via the creation of individual aspirations (for their own future, or for their children's future). To some extent, such aspirations are drawn from past individual experience (as in the literature on habit formation or reference points), but at the same time they are profoundly affected by one's *social* environment. We look at others "around" us, and their experiences and achievements shape our desires and goals.

This is a view of individual preferences that isn't standard in economic theory. But it should be. Individual goals don't exist in social isolation as consumer preferences are so often assumed to do. Thus society-wide aggregates (say in intergenerational investment, which is our main interest here) may depend fundamentally on the ambient distribution of income and wealth. In short, aspirations affect income mobility and income distribution, but in turn, these latter ingredients shape aspirations. Thus aspirations, income (and its distribution), investment and economic mobility evolve jointly, and in many situations in a self-reinforcing way. A detailed examination of this proposition is the subject of our paper.

The relationship between macroeconomic outcomes (such as growth and mobility) and individual aspirations turns on three things. First, there is the question of how individuals react to the aspirations that they do have. Second, there is the issue of how aspirations are formed: how they vary with the current economic circumstances of the individual concerned as well as the world around her. Finally, we must aggregate back up from individual behavior to derive society-wide outcomes. The theory we propose and attempt to develop has these corresponding segments. We emphasize the first two, as they are relatively new.

First, individual aspirations determine one's to incentives to invest, accumulate, and bequeath. We argue below that the existence of realistic, attainable aspirations — targets that are currently beyond one but which are potentially "reachable" — are the most conducive to upward mobility. The "best" sort of aspirations are those that induce a "reasonable distance" between one's current living standards and where one wants to be, but not a gap that is so large so as to induce frustration. For instance, both Appadurai [2] and Ray ([14] and [15]) have argued that individuals with aspirations that are very far away from their current standards of living have little incentive to invest, because the gap would remain very large before and after. There are evidence from cognitive psychology (see Heath, Larrick and Wu [7]), sport and lab experiments (see Berger and Pope [8]) that goals that are ahead but achievable provide the most incentives. This argument can be used to create an aspirations-based theory of poverty traps. On the other hand, it can also be argued that individuals whose aspirations are closely aligned to their current standards of living have relatively little incentive to raise those standards. That may be the case for the very rich sections of society who derive their aspirations from their peers. It is often individuals in the middle — those with a good-sized aspirations gap as well as the resources to effectively close that gap — who might invest the most.

We develop an extremely simple framework that captures these ideas. We begin by defining utilities around a "reference point", following the lead of Kahneman and Tversky [9], Karandikar *et al.* [10], Kőszegi and Rabin [11], and others. We interpret our reference point as an *aspiration* and study its effect on accumulation and growth. Proposition 2 develops the idea that aspirations which are unattainable can serve to frustrate, while aspirations which are too low might breed complacency. The main goal of this proposition is to argue that aspirations which are challenging, yet attainable, are the most conducive to mobility. We show that as a result individual growth rates are inverted U-shaped in income.

The analysis then continues with the formation of aspirations from the underlying income distribution, and therefore allows us to study steady state distributions[1]. We argue that aspirations are likely to depend not only on one's living standards, as commonly assumed, but also on other people's experience though the expected income distribution. There are obviously many options here. We use the following two processes of aspirations formation as benchmarks. Under "common aspirations", individuals simply use the overall mean of the expected income distribution to form her aspirations. With "upward looking aspirations", an individual forms her aspirations based on the mean expected income of the people who are above her in the income distribution. In an equilibrium, the expected distribution matches that obtained by aggregating up from individual behavior, thereby closing the model. Note that the influence of the ambient income distribution on one's preferences is a common feature of the literature on status seeking and relative wealth (see Frank [5] and Corneo and Jeanne ([3], [4]) for instance) which is therefore related. However, in our paper the ambient distribution affects one's preference through a reference point – one's aspirations – introducing convexities and very different distributional effects.

We define "steady states" as sequences of equilibrium income distributions along which all incomes grow at the same rate. With common aspirations, steady state distributions tend to be bipolar: when aspiration effects are important, a steady state with perfect equality does not exist, and unequal steady states tend to have two mass points, one below and one above the mean. Moreover, more polarized steady states exhibit lower growth and mobility. When aspirations depend on one's income, continuous distributions may be possible in a steady state. We show that with upward looking aspirations, this is the case if and only incomes follow a Pareto distribution. Here again we see that the more unequal the distribution, the lower the growth and mobility.

[1] See Macours and Renos [12] for evidence of the importance of social interactions in the formation of aspirations

Our theory can also be used to make predictions regarding individual growth rates. We illustrate this with a numerical example. Using a lognormal approximation of the income distribution in some Latin American countries over the 1990s, we find the return to capital that matches (in the model) the observed growth rate over a fifteen-year period. Our model then predicts individual growth rates for different economic percentiles within the society. These are predictions that can — in principle — be tested empirically with longitudinal data. We hope that our paper will encourage this sort of empirical research.

2. Preferences and Aspirations

Our economy is assumed to be populated by infinitely many families or dynasties. Each family has one individual per generation. Time is discrete and will be measured in generations. For each individual in generation t, lifetime income y_t is divided between consumption c_t and investment in her child, k_t :

$$y_t = c_t + k_t,$$

Investment gives rise to fresh output for the next period:

$$y_{t+1} = f(k_t).$$

So far, this is a standard growth model. Our interest lies in the explicit incorporation of aspirations into preferences. Specifically, we denote the utility of a time-t individual by

$$u(c_t) + \Omega(y_{t+1}, a_t),$$

where a_t is the *aspiration* of the individual at time t. One could also think of a as a "reference point" as in the work of Kahneman and Tversky [9] and Kőszegi and Rabin [11], but whereas in those models (see especially Kőszegi-Rabin [11]) a is determined by the own experience of the individuals, we are going to view these aspirations as coming from what individuals see around them.

There are many possible specifications of the utility function Ω; we adopt one that can capture complacency and frustration (as per our informal discussion earlier) in a reasonably tractable way.

First, we write Ω as follows:

$$\Omega(z, a) = v(z) + w(z/a),$$

where v might be thought of as an intrinsic utility of "target income" tomorrow (denoted by z) and w is a term that depends — in an increasing way — on the *ratio* of that target income z to aspirations a.

We presume that v satisfies all the standard assumptions, and study the term w.

Let σ denote the ratio of z to a. We assume that $w(\sigma)$ is continuous, smooth and strictly increasing: the greater the extent to which one meets (or exceeds) aspirations, the better off she is. The important restriction, however, is the one we capture in Figure 1: we presume that the utility excess (or shortfall) as one moves away from the aspirations target is *increasing at a decreasing rate in either direction*. If I am far ahead of my aspirations, an extra gain is not going to create much additional satisfaction, and likewise if I am way below my aspirations, an

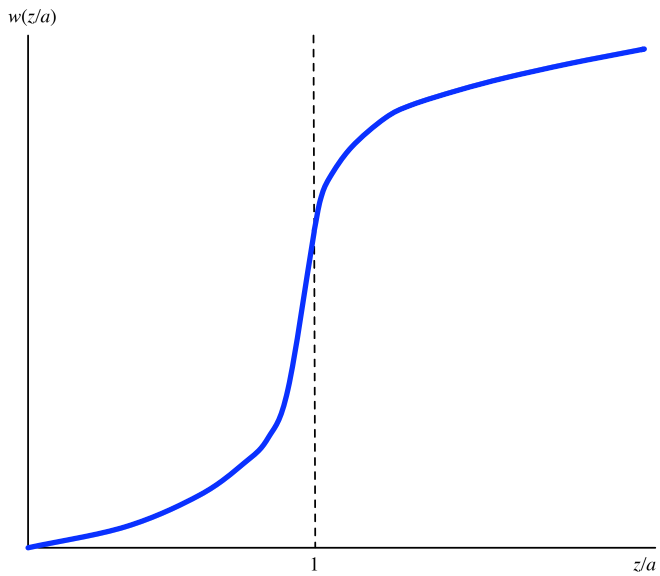

FIGURE 1. THE FUNCTION w

increase or decrease is not going to make much of a difference. It is in the region of the aspiration itself that utility gains are most sensitive to an increase in income. Formally we suppose that w is strictly convex (with $w''(\sigma) > 0$) for $\sigma < 1$, and that it is strictly concave (with $w''(\sigma) < 0$) for $\sigma > 1$.

3. THE ASPIRATIONS RATIO

Let γ denote the *aspirations ratio*; that is, the ratio of an individual's starting income y to her aspirations a. As far as the individual is concerned, this is exogenously given at any point of time, but there is a second comparison between income and aspirations that we have already carried out. This is σ, the ratio of "target" income to aspirations; call this the *target ratio*. Unlike the aspirations ratio, the target ratio is influenced by an individual's current decisions. Below, we will be referring to an aspiration as "unattained" when the target ratio remains below 1, and as "exceeded" if the target ratio is larger than 1.

Of course, these concepts are all wholly contingent on the time period; depending on the interpretation, we may think of the distance between t and $t + 1$ as small (e.g., in the case of investments for self-improvement) or large (e.g., in the case of intergenerational altruism).

Notice that we haven't said anything yet about just how a is formed and how it varies with one's income. That will come later.

4. Aspirations and Incentives

Let's look at this first segment of the theory — how aspirations affect incentives — more closely. To do so, it will be useful to provide some more structure on the utility functions u and v, as well as the production function f. Let us suppose that u and v are logarithmic: $u(c) = \ln c$, while $v(y) = \rho \ln y$ for some $\rho > 0$. Suppose, moreover, that the production function is linear: $f(k) = (1 + r)k$, where r is the rate of return on investment.

Our implicit presumption is that the maximization of the function *without aspirations*

$$u(c) + v(y) = \ln c + \rho \ln y$$

yields socially suboptimal investment and growth levels, especially when starting income is relatively low. For instance, there might be credit market imperfections, or individuals may not adequately internalize the welfare of their children (unmodeled here). Thus we view aspirations as having the instrumental effect of increasing the rate of investment towards the socially optimal level. We are therefore interested in the effects of maximizing the function

$$u(c) + v(z) + w(z/a) = \ln c + \rho \ln z + w(z/a), \tag{1}$$

subject to the constraint

$$z = (1 + r)(y - c). \tag{2}$$

for a given starting income level y and aspiration level a.

The following proposition is intuitive:

Proposition 1. *The target ratio is increasing in the aspirations ratio.*

In particular, if aspirations are exceeded at the "aspirations ratio" γ, an increase in γ raises σ and aspirations remain exceeded.

We can say more. The solution to problem (1) is obviously interior in the choice of z and c and is therefore described[2] by the first-order condition

$$\frac{1}{c} = \frac{\rho}{y - c} + w'(\sigma)\frac{1 + r}{a},$$

where σ, as before, is the endogenous "target ratio" z/a. Rewriting this equation in terms of the growth rate $g = (z/y) - 1$, we see that

$$\frac{1}{r - g} - \frac{\rho}{1 + g} = w'\left([1 + g]\gamma\right)\gamma. \tag{3}$$

The optimally chosen growth rate will lie between a minimum of -1 and a maximum of r.

The condition (3) permits us to study the effect of the aspirations ratio on the (chosen) rate of growth g. To gain intuition, Figure 2 describes how the rate of growth g is determined by this first-order condition. The upward-sloping bold line

[2]The first-order condition does not fully characterize the solution because the problem is non-convex and may exhibit more than one solution. Our formal analysis takes this fully into account.

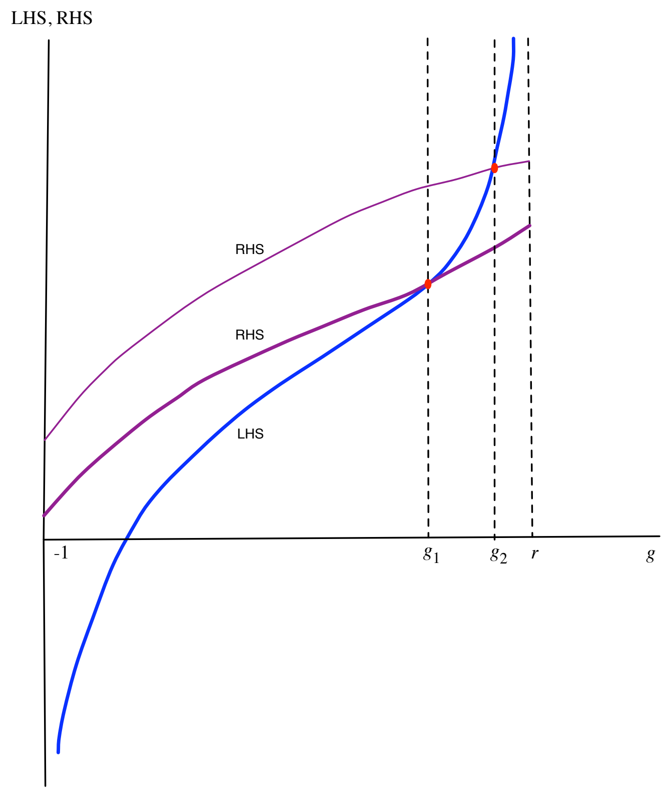

FIGURE 2. THE OPTIMUM WHEN ASPIRATIONS GAPS ARE NEGATIVE

is the left hand side of the first-order condition, it is obviously increasing in g.[3] The right-hand side (which is the other bold line) is *also* increasing in g, at least as long as aspirations are unattained. Once aspirations are exceeded, the right-hand side will decline in g.

There could, in principle, be several intersections between the two lines. The second-order condition, however, assures us that we only need to consider those intersections in which the right-hand side cuts the left-hand side "from above". (Even that isn't enough to fully pin the solution down, but it is certainly necessary.) For ease of exposition, the diagram only has the two lines intersecting once, at g_1.

Consider an aspirations ratio low enough such that aspirations remain unattained. Figure 2 carries out the exercise of raising γ but keeping aspirations unattained; ($\sigma = (1+g)\gamma \leq 1$). The right-hand side of (3) is unambiguously shifted upwards, and we see that the new growth rate is higher, at g_2.

[3]Indeed, the left-hand side tends to minus (or plus) infinity when g tends to -1 (or r), with curvature switching from concavity to convexity in between.

Now look at aspirations ratios at which aspirations are exceeded. By exactly the same logic described above, everything hangs on what happens to the right-hand side of the first-order condition (3) as γ changes. Consider the following restriction:

[W] $w'(\sigma)\sigma$ is declining in σ when $\sigma > 1$.

How reasonable is [W]? If w has unbounded steepness at $\sigma = 1$, the condition certainly holds in some region above $\sigma = 1$. Whether it holds more globally will depend on the specific form of w, and in particular on the degree of concavity exhibited by it when $\sigma > 1$. It is easy to see that [W] is equivalent to the requirement that relative risk aversion $-w''(\sigma)\sigma/w'(\sigma)$ exceed unity when $\sigma > 1$.

In what follows, we will occasionally use [W] as a provisional assumption but keep track of the consequences when it does not hold.

Proposition 2. *If aspirations are unattained, growth rates are locally increasing in* γ.

In contrast, if aspirations are exceeded, an increase in γ *lowers growth under* [W].

The second part of the proposition does depend on [W]. Without it, the effect of a change in γ is ambiguous, as the discussion above suggests and the formal proof in the Appendix makes clear.

Proposition 2 represents a formal statement of our informal assertion that an "attainable" aspirations gap, which can be closed by a round of sustained growth, is relatively conducive to investment. A larger gap breeds frustration, while a negative gap can breed complacency.

We can map the (presumably unobserved[4]) relationship between γ and growth rates into an observable relationship between starting *income* and growth rates, provided we can make a plausible connection between income y and aspirations ratio γ. Recall that γ is just y/a, and that as y increases a increases as well. This creates a potentially ambiguous movement of γ along the income distribution.

First consider the case in which γ is increasing in income over some interval I. That is, while aspirations rise with income, they do not grow faster than income. Then, invoking Proposition 1, if aspirations are unattained for some income level y, then they must also be unattained at all smaller income levels in I. We conclude, then, that we can find some y_I in I such that for all $y < y_I$, aspirations are unattained, while for all $y > y_I$, they are exceeded. (To be sure, y_I might lie on the edge of the support.) Now, as long as aspirations are unattained, we know from Proposition 2 that an increase in income within I (and therefore in γ) must increase the growth rate. To the right of y_I aspirations are exceeded, so that under Condition W, an increase in income must lower the growth rate.

Next, suppose that γ decreases in income over some interval J. That is, aspirations rise even faster than income as new horizons come rapidly into view. Then Proposition 1 implies that if aspirations are unattained for some income level y, they must also be unattained at all *higher* income levels in J. Once again, there is a threshold

[4]We write "presumably unobserved", because it may be hard to measure aspirations directly, which enter γ.

y_J in J which partitions J into incomes with attained and unattained aspirations, but notice the ordering of the two sets is reversed relative to the previous case.

Nevertheless, the same "inverted-U" relationship holds between growth rates and incomes in the interval J. To the left of the threshold y_J, aspirations are exceeded. Because an increase in incomes lowers γ, it must therefore increase the growth rate (provided that [W] holds). In contrast, to the right of y_J aspirations are unattained, so that an increase in income — which lowers γ in J — must depress the growth rate.

In summary, we have the same shape as in the previous case but for entirely different reasons. In the first case, the relatively poor are frustrated, while the relatively rich are complacent. In the second case, it is the relatively rich who have excessive aspirations and are frustrated, while the poor suffer from a failure of aspirations. Both cases, however, lead to the same observable relationship:

Proposition 3. *Assume* [W] *and suppose that* γ *is monotonic in* y *over some interval* I *of incomes. Then there exists a threshold* $y_I \in I$ *such that for all* $y \in I$ *with* $y < y_I$*, growth rates must increase in incomes, while for all* $y \in I$ *with* $y > y_I$*, growth rates must decrease in incomes.*

Notice that the behavior of γ along the cross-section of incomes depends on both the aspirations formation process as well as the shape of the current distribution of income. We discuss this connection in more detail in the next section.

5. The Determination of Aspirations

We now turn to a discussion of how aspirations might be determined. We specifically have in mind the possible influence of an ambient income distribution. In turn, the aggregate of individual behavior will determine that income distribution so that the full complex of aspirations and distribution jointly constitutes an equilibrium for the society at large.

We suppose that an individual in the support of the current income distribution derives her aspirations from the incomes she perceives "around" her. We write aspirations as a weighted average of some transform of these incomes, where the weights may depend on the relative incomes, the individual's own position, as well as the ambient income distribution. That is,

$$a(y, F) \equiv \int_0^\infty \frac{\alpha(y, y', F)}{\int_0^\infty \alpha(y, x, F) dF(x)} z(y') dF(y'). \tag{4}$$

where F is the current distribution of income and $\alpha(y, y', F) \geq 0$ (with $\alpha(y, y, F) > 0$) depends only on the relative incomes ($\alpha(\lambda y, \lambda y', F_\lambda) = \alpha(\lambda y, \lambda y', F_\lambda)$ if F_λ is the income distribution resulting from multiplying all income in F by λ). Notice that if F is a degenerate function with all mass on y then $a_t(y) = z(y)$.

Several interpretations of the transform z are possible; let us mention two of them. First, $z(y')$ may be merely the identity function y', in which case an individual's aspirations are simply some weighted combination of *current* ambient incomes. Second, $z(y')$ may be an individual's prediction of the income earned by the next generation, when the parent earns y'. This interpretation is particularly salient when individual's aspirations are viewed as her hopes for her children: she then attempts to "predict" what other next-generation incomes will look like and takes the weighted average of *those* incomes.

With either of these interpretations in mind, consider some particular processes of aspirations formation:

Common Aspirations: While not entirely realistic, it will serve as useful exposition to begin with the simplest case in which all individuals have exactly the same aspirations, which are given by the unconditional average of the expected income distribution in the economy. That is $\alpha(y, y', F) = 1$ for all y and y' so that

$$a(y, F) = \int z(y) dF(y).$$

Upward-Looking Aspirations: Aspirations are likely to depend on one's position in the income distribution. In particular, a natural assumption may be that individuals only look "upwards" at all families who are richer than them. Then $\alpha(y, y', F) = 1$ for all $y' \geq y$ and 0 otherwise, so that

$$a(y, F) = \frac{\int_y^\infty z(y') dF(y')}{1 - F(y)}.$$

Local Aspirations With Population Neighborhoods: Next, suppose that aspirational weights are placed only on the next d percentiles of the richer population. Then $\alpha(y, y', F) = 1$ if and only if $0 \leq F(y') - F(y) \leq d$, so that

$$a(y, F) = \frac{1}{d} \int_y^{\bar{y}} z(y') dF(y'),$$

where $\bar{y}$ is the largest value of y' (including ∞) such that $F(y') - F(y) \leq d$.

Local Aspirations With Income Neighborhoods: Now suppose that aspirational weights are placed instead only on incomes within a relative neighborhood $m > 1$ of the individual's income. That is, $\alpha(y, y', F) = 1$ if and only if $1 \leq y'/y \leq r$, so that

$$a(y, F) = \frac{1}{F(y[1+m]) - F(y)} \int_y^{[1+m]y} z(y') dF(y').$$

There is an interesting distinction to be drawn between these last two examples. With population neighborhoods, an individual's cognitive window includes a certain percentage of the *population*, which may contain individuals far richer than her. In particular, if she occupies a sparsely populated income segment, then this sort of cognitive window may lead to unduly high aspirations and consequently frustration, as discussed earlier. In contrast, with income neighborhoods, the possibly sparse population around an individual's income is of no consequence to her: she anchors her aspirations on the basis of what is attainable regardless of the number of individuals actually earning those incomes. For obvious reasons, these aspirations are less sensitive to the ambient distribution of income.

Once we posit a specific description of the aspirations formation process, and know the current income distribution, we are in a position to fully identify the intervals I over which the aspirations ratio is monotonic in income. With that in hand, we may apply Proposition 3 to predict how growth rates will vary across the cross-section of incomes. For instance, in the model of common aspirations formation, the aspirations ratio must be strictly increasing in income, so that by Proposition 3 (and Condition W), we must have a *single* inverted-U of growth rates over the entire cross-section of incomes. In Section 8, we carry this exercise for the model of upward looking aspirations on a sample of Latin American countries in the nineties.

6. Equilibrium

A specific process of aspirations formation yields an aspirations ratio of $\gamma(y) = y/a(y, F)$. Armed with this aspirations ratio, we can tag on the description of individual behavior described in Section 4. The initial income distribution F and the combined behavior of all individuals yields a mapping $\chi(y)$ of next-generation incomes. In equilibrium, this mapping should coincide with the beliefs $z(y)$ held by the parents, and used in (4). An *equilibrium* is therefore a particular sequence of aspirations, and joint income distributions which is "consistently linked" over neighboring pairs of dates (or generations) in this fashion.

Some special sequences of income distributions that interest us are the *steady state* sequences of income distributions, in which all incomes grow at some time-invariant rate of g^*.

7. Steady State Distributions

In a steady state, all incomes grow at some common rate g^*, so that the distribution of cross-individual *relative* income remains unchanged over time. It follows that we can express all incomes as a multiple of average income; the resulting marginal income distribution F^* is time-stationary. As a result, in steady state, aspirations are growing at the rate of g^* as well, so that expressed similarly as a multiple of mean income aspirations remain time-stationary.

7.1. Steady States and Polarized Distributions.

Suppose that F^* is a steady state distribution of (normalized) incomes. For each relative income R, let $\gamma(R) \equiv R/a(R, F^*)$ be the aspirations ratio at R. Suppose that our steady state has the property that aspirations gaps are strictly decreasing in income, i.e. $\gamma(R)$ is strictly monotonic over various values of R in the support of steady state. This has striking consequences for the shape of the steady state distribution:

Proposition 4. *Suppose that γ is strictly monotonic in R in some steady state. Then the support of the distribution is generically finite.*

Specifically, the support above the mean is finite if $w'(\sigma)\sigma$ is nowhere constant, and has at most $m+1$ points, where m is the number of turning points of the function $w'(\sigma)\sigma$.

Assume [W], *then the steady state can have at most two mass points, one below or at the mean and one above it.*

To prove this proposition, consider the necessary first-order condition (3) evaluated at $g = g^*$:

$$\frac{1}{r - g^*} - \frac{\rho}{1 + g^*} = w'\left([1 + g^*]\gamma(R)\right)\gamma(R).$$

That is, for every R in the support of steady-state ratios F^*,

$$\frac{1 + g^*}{r - g^*} - \rho = w'\left(\sigma(R)\right)\sigma(R), \tag{5}$$

where $\sigma(R) \equiv \gamma(R)(1 + g^*)$.

The proposition follows directly from an examination of equation (5). There must be at least one value of R no larger than 1 (the normalized mean) in the support of the steady state distribution, say $\bar{R}$. Define $D \equiv w'\left(\sigma(\bar{R})\right)\sigma(\bar{R})$. For every other R in the support of the steady state distribution, we must have $w'\left(\sigma(R)\right)\sigma(R) = D$. Moreover, if γ is strictly monotonic in R then $\sigma(R) \neq \sigma(R')$ for $R \neq R'$. Hence, the number of points at which this can happen is clearly bounded above by $m + 2$, where m is the number of turning points of the function $w'(\sigma)\sigma$ for $\sigma \geq 1$. This completes the proof.

If $w'(\sigma)\sigma$ declines in σ for $\sigma \geq 1$, $m = 0$ and the steady state can have at most two mass points, one below or at the mean and one above it.

It is worth noting that γ is always strictly monotonic in the case of common aspirations. In that case, aspirations equal the mean of next generation's distribution, so that

$$\gamma(R) = R/(1 + g^*),$$

which is strictly increasing in R. Therefore Proposition 4 applies without qualification to the case of common aspirations.

Notice that the proposition does *not* rule out the possibility that the steady state is perfectly equal: that there is just one mass point. In general, though, full equality will not be attainable in steady state. To examine this idea, suppose that we do have an equal steady state. Then all incomes are the the same and grow at some time-invariant rate of g^*. In such a steady state, all relative incomes are just 1 and therefor $a(1, F^*) = (1 + g^*)$. Therefore the ratio of income to aspirations $\gamma(1)$ is simply $1/(1 + g^*)$. The next proposition tells us that if the w-function is steep enough at the "crossover point" $\sigma = 1$, this state of affairs cannot be maintained.

Proposition 5. *If $w'(1)$ is steep enough while w is still bounded, an equal steady state cannot exist.*

Indeed invoking the first-order condition (3), we see that a particular individual will choose g to satisfy

$$\frac{1}{r - g} - \frac{\rho}{1 + g} = w'\left(\frac{1 + g}{1 + g^*}\right)\frac{1}{1 + g^*}, \tag{6}$$

as well as the appropriate second-order conditions. In equilibrium, it must be the case that $g = g^*$, so that we can immediately solve out for the steady-state growth

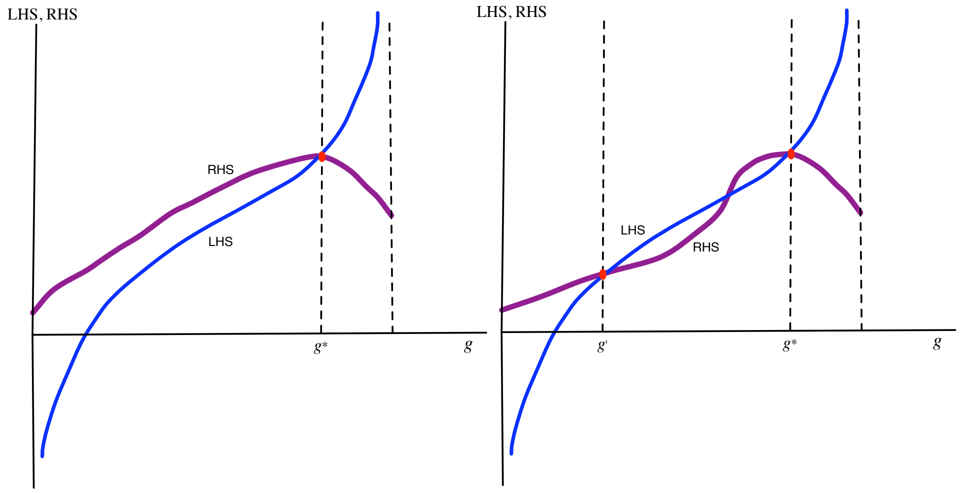

FIGURE 3. AN EQUAL STEADY STATE WITH COMMON ASPIRATIONS

rate from (6); it is given by the solution to

$$\frac{1+g^*}{r-g^*} - \rho = w'(1). \tag{7}$$

Figure 3 plots the solution to (6) and (7). Concentrate on the first panel to begin with. Just as in Figure 2, we draw in the left-hand and right-hand sides of equation (6), with the value of g^* fully pinned down by equation (7). An equilibrium necessitates that the right-hand side intersect the left-hand side precisely at the value g^*.

By our assumptions on w, we know that the right-hand side, which is just proportional to w', is rising as long as g lies below g^*, and falls thereafter. At g^*, the function w' is therefore locally flat. Not only do we obtain the desired intersection at g^*, we also see that w' (being flat there) must cut the left-hand side from above, so that a local second-order condition is also satisfied.

It appears from the discussion thus far that a steady state with perfect equality must always exist. However, such a conclusion would be erroneous. To see this, suppose that the w function becomes progressively steeper at 1; then g^* as defined by (7) must converge to r. This means that first-period consumption is converging to zero (with attendant utility becoming large and negative). That cannot happen if the w function is bounded overall: no one would sacrifice all their consumption for a bounded return. An equal steady state can only exist if $w'(1)$ is not "too large".

The second panel in Figure 3 explains the apparent contradiction between this argument and the fact that g^* satisfies both the first and second-order conditions for an optimum. If we perform the thought experiment of progressively steepening w at 1 *and* maintain the uniform boundedness of w throughout, a fresh intersection between the left- and right-hand sides of (6) *must* appear. This intersection also satisfies first- and second-order conditions, and at some point it must dominate g^* as an optimal choice. An equal steady state can no longer exist.

Propositions 4 and 5 tells us that when aspirations are important, there is a tendency for society to get polarized. Such an observation is not far removed from the findings of Quah [13] that there is a tendency for the world income distribution to form a bipolar distribution.

Now notice that lowest income will grow at the rate of g^* as well, and expressed similarly as a multiple of the mean income it will remain time-stationary; call this value $\underline{R}$. In the case of common aspirations and upward looking aspirations, $a(\underline{R}, F^*) = (1+g^*)$. Hence, (5) for the poorest person in the economy tells us that

$$\frac{1+g^*}{r-g^*} - \rho = w'\left(\underline{R}\right)\underline{R}. \tag{8}$$

It follows that, for these aspirations, growth (and mobility) is *negatively* correlated with inequality in the sense of a larger ratio of mean income to minimum income. Indeed consider two distributions of ratios. Since aspirations must be unattained for the poorest person in the economy in steady state, the one with the higher value of $\underline{R}$ will have the higher value of $w'(\underline{R})(\underline{R})$. Using this information in (8), we see that the steady state growth rate in this economy must be higher. Hence, more polarized societies tend to have lower growth and mobility.

The discussion above characterizes steady states when the aspirations gap is strictly decreasing in income. Other steady states can arise when we relax this assumption. In particular, we can have continuous income distributions in steady state if the aspirations gap is constant for all individuals. This clearly cannot be the case with common aspirations, but does arise with upward looking aspirations.

7.2. **Upward Looking Aspirations and the Pareto Distribution.** Recall that with upward looking aspirations, an individual's aspirations are given by the average expected income of the individuals *above* him in the income distribution. The following Proposition tells us that the Pareto distribution is the only income distribution that provides equal incentives to all, thereby resulting in *balanced growth.* Using the mobility measure M_α introduced in Genicot and Ray [6], this growth rate is our measure of mobility.

Proposition 6. *With upward looking aspirations, a sequence of continuous income distributions is an equilibrium steady state if and only if they are Pareto distributions.*

Indeed, assume that F_t and F_{t+1} are Pareto distributions with parameter $\kappa > 1$. It is easy to check that in a steady state with growth g^* this yields an aspirations ratio

$$\gamma(y) = \frac{\kappa - 1}{\kappa(1+g^*)}, \tag{9}$$

that is constant for all y. Moreover, the argument in the Appendix shows that the Pareto distribution is the only one for which this is the case.[5] As a result,

[5] We are grateful to Joan Esteban for this part of the proof.

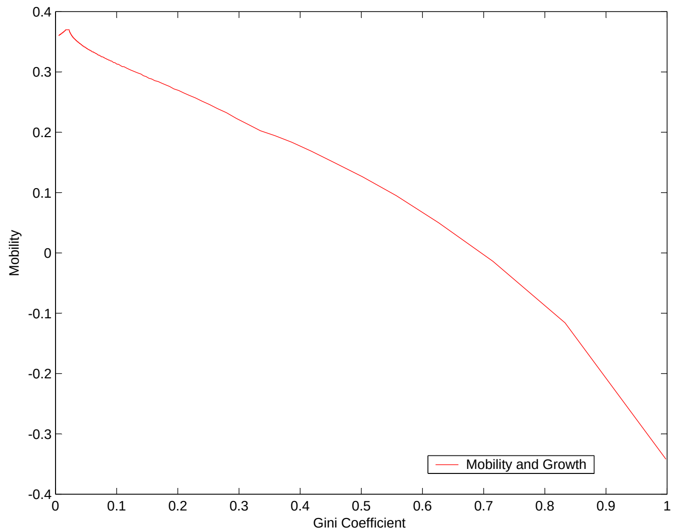

FIGURE 4. INEQUALITY DEPRESSES GROWTH IN STEADY STATE.

investment choices result in the same growth rate for all. Now if incomes at time t followed a Pareto distribution with parameter (m, κ) and they all choose a growth rate g^*, then incomes at time $t+1$ follow a Pareto distribution with parameters $((1+g^*) * m, \kappa)$. Hence, it is a steady state equilibrium.

As before, we might ask whether a more equal society has higher mobility. In fact, the larger the parameter k of the Pareto distribution, the more equal the distribution (in terms of Lorenz dominance). Hence, a more equal society lowers the aspirations gap for all. Its effect on growth and mobility depends therefore on whether aspirations are met to begin with. Since expected growth rates society-wide are already factored into aspirations, no aspiration will ever be attained in steady state and inequality will always depress growth, by Proposition 2. This is illustrated in Figure 4. We see that starting from very egalitarian societies, as the Gini coefficient – simply equal to $G(y) = 1/(2k-1)$ for the Pareto distribution – rises, mobility and growth decrease.

8. AN ILLUSTRATIVE EXERCISE WITH LOGNORMAL DISTRIBUTIONS

So far we have focus on steady state. However, our model can also being used to study the pattern of growth rates might vary along the cross-section of incomes in the economy. As seen in Proposition 3, our model predicts that in general these growth rates follow an inverted U-shape. With numerical methods, we can make stronger predictions. Approximating the income distribution for a few Latin American countries with lognormal distributions – a commonly used distribution to fit income data – we can assess the growth rates and mobility predicted by our model. This is what we do in this last section with the following illustrative example.

If we take the lognormality of the income distribution seriously, we can estimate the parameters of this distribution — the mean of log income μ and the standard deviation of the log income σ — from existing data on the Gini coefficient and per-capita income. Indeed, Aitchison and Brown [1] show that lognormality of the income distribution implies that the Gini coefficient G is given by

$$G = 2\Phi((\sigma/2)^{1/2}) - 1. \tag{10}$$

where Φ is the standardized normal cumulative distribution. Inverting this expression gives us σ as a function of G.[6] Now, mean log income μ is simply a function of σ and of the income per capita μ_y:

$$\mu = \ln \mu_y - \frac{\sigma^2}{2}. \tag{11}$$

We follow this procedure using the values of the Gini coefficients from the UNU-WIDER World Income Inequality Database and the GDP per capita, PPP (in constant 2005 international \$) from the World Bank's World Development Indicators for the US and for a few Latin American countries in the late eighties. The resulting parameters for our lognormal approximations are given in the last two columns of Table 1.

Country	GDPpc	Gini	μ	σ
Brazil 89	7,692	0.63	7.61	1.64
Costa Rica 86	5,520	0.46	8.33	0.75
Mexico 89	8,900	0.46	8.81	0.75
Chile 89	6,501	0.44	8.55	0.68
Columbia 88	4,742	0.44	8.23	0.68
Argentina 90	7,472	0.43	8.71	0.65
US 89	31,640	0.38	10.24	0.50

TABLE 1. Lognormal Approximations.

We shall assume that aspirations are upward looking. Moreover, for simplicity, we take aspirations to depend on the current income distribution as opposed to the expected income distribution. That is, aspirations are given by

$$a(y, F_t) = \frac{1}{1 - F_t(y)} \int_y^\infty x dF_t(x).$$

Assume that $w(\sigma)$ has a constant-elasticity form:

$$\begin{aligned} w(\sigma) &= K_1 + K_2(\sigma - 1)^b \;\; if \;\; \sigma \geq 1 \\ &= K_1 - K_2(1 - \sigma)^b \;\; if \;\; \sigma < 1 \end{aligned}$$

with coefficients $K_1 = 0$, $K_2 = 20$ and $b = 1/2$, and that $w(\sigma)$ is symmetric around 1.[7] Let $\rho = 0.9$. For each of these distribution, a given return to capital r generates specific individual growth rates as a function of income. These incomes and growth rates imply a specific aggregate growth rate for the country. In this exercise, we

[6] $\sigma = 2(\Phi^{-1}((1 + G)/2))2$.

[7] That is, $w(\sigma) = K_1 + K_2(\sigma - 1)^b$, for $\sigma \geq 1$, while $w(\sigma) = K_1 - K_2(1 - \sigma)^b$ for $\sigma < 1$.

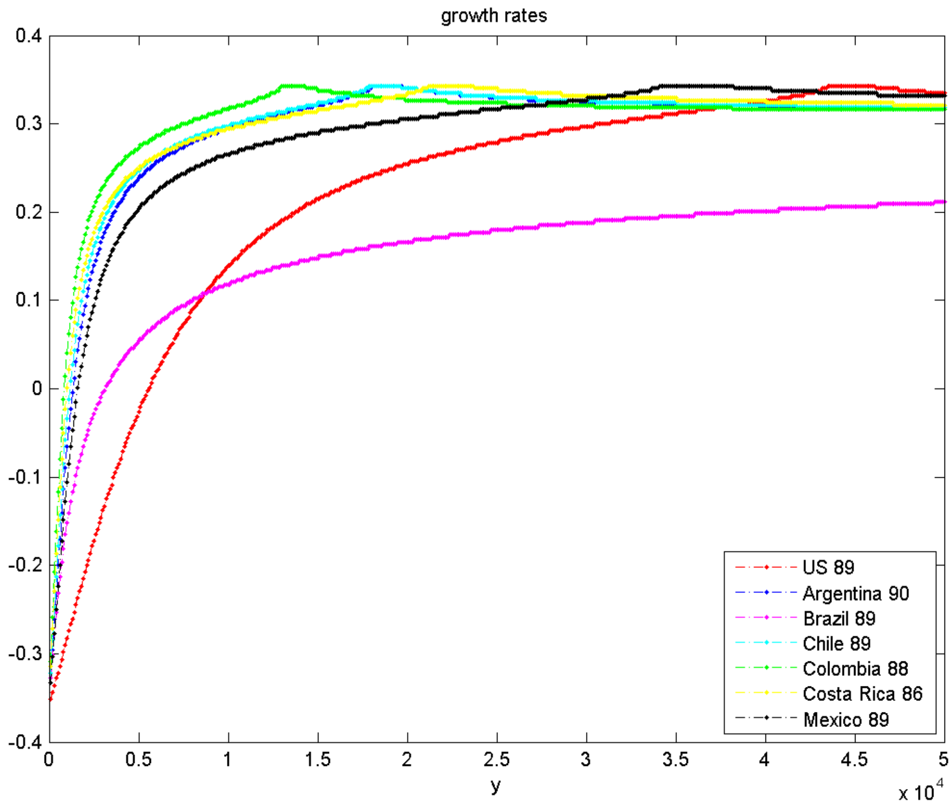

Figure 5. Growth rates with Lognormal approximation.

find the return to capital r that generate in our model the *actual* aggregate growth observed in these countries over a fifteen year period. The fifteen year period has been chosen to represent a generation. The last column of Table 2 shows the actual growth rates that we are matching and its fourth column presents the corresponding return to capital in annual terms: $i = (1+r)^{1/15} - 1$. We see for instance that these returns are high for Chile which has been growing fast over the period.

Now, the interest of this exercise is that once we have fixed the lognormal approximation and the returns to capital, our model offers very strong predictions on the growth rates of specific economic groups in these countries. Figure 5 illustrates the individual intergenerational growth rates as a function of income that our model predicts for these countries. We see that growth rates at the bottom of the distributions are significantly lower than at the top and are actually negative. These predictions could be tested using longitudinal data. It is also interesting to contrast Argentina and Brazil who had very similar GDP per capita in the late eighties but significantly different level of inequality. We see that the growth rates for Brazil are much lower than for Argentina, and particularly so for low incomes. Hence, the predicted mobility should be lower in Brazil than Argentina, especially for high level of the pro-poor weight α.

Country	GDPpc	Gini	i	M_1	M_2	M_5	Growth
US 89	31,640	0.38	2.03%	0.26	0.23	0.14	0.30
Argentina 90	7,472	0.43	2.79%	0.36	0.28	0.07	0.45
Chile 89	6,501	0.44	4.28%	0.72	0.64	0.44	0.80
Colombia 88	4,742	0.44	1.49%	0.07	0.00	−0.18	0.16
Costa Rica 86	5,520	0.46	2.67%	0.27	0.16	−0.05	0.41
Mexico 89	8,900	0.46	0.25%	0.13	0.01	−0.22	0.26
Brazil 89	7,692	0.63	1.63%	−0.32	−0.39	−0.39	0.09

TABLE 2. Growth and Mobility with Lognormal Approximations.

This intuition is confirmed for our sample. Indeed, M_1, M_2 and M_3 in Table 2 present the implied levels of intergenerational mobility for three values of the pro-poor index α. It is easy to see that the more pro-poor the measure of mobility, the stronger the negative correlation between inequality and the the lack of mobility.

9. CONCLUSION

This paper develops and analyzes a model of aspirations and mobility. The premise for the model is two-fold: people's aspirations for their own future or for their children's future affects their incentives to invest, and other people's experience helps shape one's aspirations. Hence, this paper marries a model of *reaction* to aspirations with a simple theory of aspirations *formation* to study the relationship between aspirations and the distribution of income. Through it's impact on investments, aspirations affect economic mobility and income distribution, which in turn shape aspirations. Thus aspirations, income (and its distribution), investment and economic mobility evolve jointly, and in many situations in a self-reinforcing way. We study the resulting connections between the shape of the distribution of a society and its degree of mobility. We show that when aspirational effects are important, perfect inequality is not possible in steady state. Moreover polarized societies are shown to be less mobile.

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Appendix: Additional Proofs

Proof of Proposition 1. We want to show that if $\gamma_1 < \gamma_2$ then $\sigma_1 \leq \sigma_2$. Assume not, then there exists $\gamma_1 < \gamma_2$ such that $\sigma_1 > \sigma_2$. This implies the following growth rates $g_1 = \frac{\sigma_1}{\gamma_1} - 1$ and $g_2 = \frac{\sigma_2}{\gamma_2} - 1$ with $g_1 > g_2$. Let

$$L(g) \equiv \ln(r - g) + \rho(1 + g). \tag{12}$$

$L(g)$ is strictly concave and decreasing for $g > \underline{g} \equiv \frac{r\rho - 1}{1+\rho}$.

We know that g_i maximizes $L(g_i) + w(\sigma_i)$ subject to $\sigma_i = (1 + g_i)\gamma_i$ for $i = 1, 2$. Hence, $g_1 > g_2 > \underline{g}$. Moreover, choosing σ_1 under γ_1 implies that

$$w(\sigma_1) + L(\frac{\sigma_1}{\gamma_1} - 1) \geq w(\sigma_2) + L(\frac{\sigma_2}{\gamma_1} - 1)$$

while choosing σ_2 under γ_2 implies that

$$w(\sigma_2) + L(\frac{\sigma_2}{\gamma_2} - 1) \geq w(\sigma_1) + L(\frac{\sigma_1}{\gamma_2} - 1).$$

It follows that

$$L(\frac{\sigma_2}{\gamma_2} - 1) - L(\frac{\sigma_1}{\gamma_2} - 1) \geq w(\sigma_1) - w(\sigma_2) \geq L(\frac{\sigma_2}{\gamma_1} - 1) - L(\frac{\sigma_1}{\gamma_1} - 1).$$

But this is a contradiction given the concavity of L and the fact that $g_2 > \underline{g}$ since $\frac{\sigma_1 - \sigma_2}{\gamma_1} > \frac{\sigma_1 - \sigma_2}{\gamma_2} > 0$ and $\frac{\sigma_2}{\gamma_1} > \frac{\sigma_2}{\gamma_2}$. ■

Proof of Proposition 2. To prove the first part of the claim consider $\gamma_1 < \gamma_2$ so that $\gamma_2(1 + g_1) < 1$ and denote as g_1 and g_2 the two corresponding growth rates. We want to show that $g_2 \geq g_1$. If at g_2 aspirations are attained or exceeded $\gamma_2(1 + g_2) \geq 1$ then the claim trivially holds. Hence, assume that $\gamma_2(1 + g_2) < 1$ and that the claim is false so that $g_2 < g_1$.

We know that g_i maximizes $L(g_i) + w((1+g_i)\gamma_i)$, with $L(g)$ defined in (12). Hence, $g_1 \geq g_2 > \underline{g} \equiv \frac{r\rho - 1}{1+\rho}$ so that $L(g_1) \leq L(g_2)$. Moreover, choosing g_1 under γ_1 implies that

$$w((1 + g_1)\gamma_1) + L(g_1) \geq w((1 + g_2)\gamma_1) + L(g_2)$$

while choosing g_2 under γ_2 implies that

$$w((1 + g_2)\gamma_2) + L(g_2) \geq w((1 + g_1)\gamma_2) + L(g_1)$$

Combining these expressions gives us

$$w((1 + g_1)\gamma_1) - w((1 + g_2)\gamma_1) \geq L(g_2) - L(g_1) \geq w((1 + g_1)\gamma_2) - w((1 + g_2)\gamma_2).$$

This is a contradiction since $g_1 > g_2$ and w is strictly convex for $\sigma < 1$.

To prove the second part of the claim, consider now $\gamma_1 < \gamma_2$ so that $\gamma_2(1 + g_1) > 1$ and denote as g_1 and g_2 the two corresponding growth rates. We want to show that $g_2 < g_1$. If at g_2 aspirations are unattained or just attained $\gamma_2(1 + g_2) \leq 1$ then the claim trivially holds. Hence, assume that $\gamma_2(1 + g_2) > 1$.

For $\sigma > 1$, the maximization problem in (1) is strictly concave. Hence, there can be at most one growth rates that satisfies the first order conditions (3). Hence, everything hangs on what happens to the right-hand side of the first-order condition

(3) as γ changes. If $w'(\sigma)\sigma$ is decreasing, an increase in the right-hand side of (3) must now be associated with a *lowering* of g. Hence, $g_2 < g_1$. ■

Proof of Proposition 6. Before we proceed with the main proof, two Lemmas are useful.

Lemma 7. *If Z follows a Pareto distribution with parameter (m, κ) then $Y = cZ$ for a constant c follows a Pareto distribution with parameters $(c * m, \kappa)$.*

Proof. The cumulative distribution of Y is $P(Y < y) = P(cZ < y) = P(Z < y/c)$. Since the cumulative distribution of Z is $P(Z < z) = 1 - (m/z)^\kappa$, then $P(Z < y/c) = 1 - (m * c/y)^\kappa$. ■

Lemma 8. *$\gamma(y)$ is constant if and only if y follows a Pareto distribution.*

Proof.[8] Let $\phi(y) = [(1 + g^*)\gamma(y)]^{-1}$.

$$\phi(y) = \frac{1}{y(1 - F(y))} \int_y^\infty x dF(x).$$

$\gamma(y)$ constant means that $\phi(y) = k$ and $\phi'(y) = 0$. Hence,

$$\phi'(y) \equiv -h(y) - \frac{1}{y}(1 - yh(y))\phi(y) = 0,$$

where $h(y) = \frac{f(y)}{1-F(y)}$. This condition can be rewritten as

$$\frac{\partial log(1 - F(y))}{\partial y}(1 - \phi(y)) - \frac{\partial log y}{\partial y}\phi(y).$$

Substituting in $\phi(y) = k$ yields

$$\frac{\partial}{\partial y}\left(log(1 - F(y))\right). = \frac{\partial}{\partial y}\left(log y^{\frac{k}{1-k}}\right).$$

Therefore,

$$1 - F(y) = Ay^{\frac{k}{1-k}},$$

a Pareto distribution. ■

Now, from the proof of Proposition 4, we learned that if $\gamma(R)$ is strictly increasing over some interval in the support of the steady state distribution, the distribution must have a finite number of mass point over this interval. Hence, to have a continuous distribution of relative income in steady state, it must be the case that $\gamma(R)$ is constant over the support of the distribution. Lemma 8 tells us that this is the case if and only if F_{t+1} is a Pareto distribution.

Indeed, assume that F_{t+1} is a Pareto distribution with parameter $\kappa > 1$. It is easy to check that in a steady state with growth g^* this yields an aspirations ratio

$$\gamma(y) = \frac{\kappa - 1}{\kappa(1 + g^*)},$$

that is constant for all y. As a result, investment choices result in the same growth rate for all. If incomes grew at a rate g^* from time t to time $t+1$ and resulted in a Pareto distribution with parameter $(m * (1 + g^*), \kappa)$ at time $t+1$, Lemma 7 implies

[8] We are thankful to Joan Esteban for this proof.

that income at time t follow a Pareto distribution with parameters (m, κ). Hence, it is a steady state equilibrium. ■