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Storable votes

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Abstract

Motivated by the need for more flexible decision-making mechanisms in the European Union, the paper proposes a simple but novel voting scheme for binary decisions taken by committees that meet regularly over time. At each meeting, committee members are allowed to store their vote for future use; the decision is then taken according to the majority of votes cast. The possibility of shifting votes intertemporally allows agents to concentrate their votes when preferences are more intense, and although the scheme will not achieve full efficiency, storable votes typically lead to ex ante welfare gains over non-storable votes. Welfare gains can be proven rigorously in the case of 2 voters. With more voters, counterexamples can be found, but the analysis suggests that the welfare improvements should continue to hold if one of the following conditions is satisfied: (i) the number of voters is above a minimum threshold; (ii) preferences are not too polarized; (iii) the horizon is long enough.

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1. Introduction

Consider a committee that meets regularly over time to vote up or down proposals that affect all of its members. The committee members are heterogenous and have different

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preferences over the policy to be enacted. Decisions are taken by majority vote, and as always a majority with weak preferences will win over a minority with more strongly held opinions. Think of this simple alternative: although each member continues to accrue one new vote at each meeting, he now has the option of storing his vote for future use. If a member abstains at the first meeting, he will be able to cast either 0, 1 or 2 votes at the second; were he to abstain again, he would have up to 3 votes available for the next meeting. In other words, suppose votes are storable. Would this plausible procedural change improve the efficiency of the committee? If asked at some preliminary constitutional stage, would committee members prefer a system of storable votes? The purpose of this paper is to propose such a mechanism and begin addressing the questions it raises.

Its main results, in the simplified setting the paper studies, are promising. By allowing voters to shift their votes intertemporally, storable votes lead them to concentrate their votes on times when preferences are more intense, and therefore increase the probability of obtaining the desired decision when it is more important. Counterexamples can be found, but under plausible assumptions, *ex ante* welfare is higher than with standard majority voting with non-storable votes. In addition, storable votes appear to behave well even if voters follow plausible rules-of-thumb, as opposed to fully rational strategies. Finally, at least in the examples analyzed in the paper, storable votes have better welfare properties than tradable votes, besides being more transparent and procedurally simpler and thus less objectionable both ethically and practically.

The research project was motivated by concerns with the mechanisms through which the European Union coordinates (or attempts to coordinate) the policies of its members. The problem of achieving a unified policy while respecting the sovereignty of heterogeneous countries is very difficult, and all reforms of European Union's institutions are caught between the need for the faster decision-making that majority voting provides and the importance of respecting each country's priorities lest the whole process of integration comes to an end. Intuitively, a country should be able to weigh more heavily when a fundamental interest is threatened, but at a price: as in the case of money, the choice to obtain control over one item should come at the cost of smaller disposable resources available in the future. Storable votes fulfill this function. Other mechanisms may do so too, but storable votes have the advantage of being extremely simple: the mechanism is very natural, can be explained in a few words and induces very intuitive behavior. Storable votes are not an optimal mechanism, but they are so simple that they could realistically be implemented.

Of course the importance of preserving strongly felt minority preferences extends much beyond the immediate challenges of the European Union, to the design of most democratic institutions. The paper refers to the specific example of the European Central Bank because it provides a concrete example of a repeated binary voting game with fixed agenda, and it is this simple setting that this first model studies. But it should be clear that there is no reason why storable votes should not be studied eventually for potential applications to generic committees.

The idea of using more resources, here more votes, when a decision is valued more is very natural, but storable votes have no clear precedent in the literature. The two closest relations are vote trading (see, for example, Buchanan and Tullock, 1962; Coleman, 1966; Brams and Riker, 1973; Ferejohn, 1974; Philipson and Snyder, 1996; Piketty, 1994) and cumulative voting (Dodgson, 1884; Sawyer and MacRae, 1962; Brams, 1975; Cox, 1990;

Guinier, 1994; Gerber et al., 1998, among others). But storable votes are different from both.

Storable votes differ from vote trading (without monetary exchanges) on two main grounds. First, because they rely on individuals acting alone, storable votes are a simpler and more transparent institution. When votes are exchanged interpersonally, traders must find each other, competitors must outbid one another and future promises must be enforced. How these steps take place depends on auxiliary, but essential, institutions;¹ different alternatives are possible and affect the outcome, creating potential sources of manipulation. These complications do not exist in the case of storable votes. Second, vote trading results in coalitions, and individuals unable to trade votes find their influence reduced both today and in the future. In the absence of side-payments, each voter faces a non-negligible probability of being rationed out of the market—a very costly outcome. With storable votes, on the other hand, when an individual chooses to cast more than 1 vote in any period, the other committee members are automatically compensated by their increased influence in the other periods. At least in the simple example studied in the paper, storable votes are unambiguously welfare superior.

Cumulative voting is a system by which voters who are called to elect a subset of candidates in an election are free to allocate a given stock of votes among them as they see fit. It is a static mechanism where all voters choose simultaneously how to cast all their votes in a single election with multiple options. Storable votes on the contrary are a dynamic mechanism that applies to a series of binary choices taking place over time. As time passes, uncertainty is resolved both with respect to the voters' evolving preferences, and to the stock of votes still available to one's opponents.

Other voting schemes have some of the flavor of storable votes, but again all are different. The observation that introducing a cost to voting selects voters with more intense preferences and thus may be efficiency-enhancing has been made before (for example, Börgers, 2001). But storable votes make that cost endogenous by allowing voters to choose how much future influence to renounce, and at the same time grant voters a richer set of options than the simple vote/abstain choice. Voting by successive veto (Mueller, 1978; Moulin, 1982) and voting by successive pair-wise elimination (Moulin, 1979) are schemes where one of several possible alternatives is selected through a dynamic process of elimination. All information is known at the beginning, and the dynamic aspect allows to select subgame perfect equilibria (and hence restrict the set of possible outcomes). The main concern is the theoretical design of desirable schemes when voters must choose among more than two alternatives. Here instead, information is acquired over time, and the decision is binary each period.

The paper proceeds as follows. Section 2 describes the assumptions of the model. Section 3 presents the main intuition behind storable votes in the simplest setting, when there are only 2 voters and 2 periods. Section 4 characterizes the equilibrium strategies. Section 5 derives welfare results in the case of 2 voters, for arbitrary length of the horizon, while Section 6 extends the analysis to N voters and considers in detail the case of 3 vot-

¹ See, for example, the discussion in Philipson and Snyder, 1996.

ers. Section 7 compares storable and tradable votes, and Section 8 concludes. Appendix A presents the longer proofs.

2. The model

A committee of n individuals meets regularly to take a common decision d that can assume two values: $d \in \{0, 1\}$, where we can think of $d = 0$ as maintaining the status quo, and $d = 1$ as change. In each period, each member's preferences are indexed by a parameter v_{it} drawn from a continuous distribution $F(v)$, defined over the support $[-1, 1]$ and symmetric around zero. In period t , individual i 's utility equals $v_{it}d_t$: if v_{it} is negative, i prefers $d_t = 0$; if v_{it} is positive, i prefers $d_t = 1$, and the absolute value of v_{it} measures the intensity of i 's preferences. The distribution $F(v)$ is common across all committee members and all periods, and v_{it} is independently distributed both across individuals and across time. The committee takes the decision every period for a total of T periods, where T is finite.

For concreteness, think of the committee as the Governing Council of the European Central Bank, meeting each month to decide whether to maintain current interest rates ($d = 0$), or to change them ($d = 1$), under the assumption that both the direction of the possible change and, more controversially, its size are known before the meeting. Each member of the Council has preferences over European monetary policy and these preferences need not be homogenous, reflecting different needs of the national economies. Each member's preferences are summarized by v_{it} .²

Every period each committee member is given one vote. He can cast it in favor of the option he prefers, or store it for use at a later time. Thus in period 1, a member can cast either 0 or 1 votes; if he decides to save his vote, in period 2 he will have a total of 2 votes at his disposal and will decide how many of these, if any, to use; and so on in all successive periods until time T when the game ends. We assume that votes can be stored but not borrowed to avoid the difficult problems that could arise in practice if one member were to run out of votes, but we will show later that the assumption is unimportant.³ Subject to the budget constraint that the votes cast cannot exceed the number of votes available, each member is asked to indicate his preferred decision and the number of votes he is willing to spend to support his choice. When individuals vote, they know the realization of their current $v_{i,t}$ and the probability distribution F , but cannot observe the preferences of the other members and do not know their own future valuations. On the other hand, because the initial allocation of votes and the history of the game are known, the number of votes that each player has at his disposal is common knowledge.

² The assumption of i.i.d. shocks is not ideal in this context.

³ The constraint on borrowing is common to existing policy mechanisms that rely on market-type behavior (for example, environmental regulation through tradable pollution licences), because it reduces the costs of mistakes and inexperience, and increases the credibility of the rules. In addition, when members are subject to appointments or elections, the inability to borrow from the future limits the extent to which current members can expropriate the power of their successors.

The committee selects d according to which of the two alternatives has received more votes. If the votes are equal, the preferences of members who have cast zero votes are considered; if the tie is still not broken, the decision is taken with a coin toss. The tie-breaking mechanism seems plausible and has some advantages in deriving analytical solutions, but does not affect the substance of the results.⁴

The individuals' objective is to announce a policy preference and choose a number of votes each period so as to maximize the expected flow of utility over the whole time horizon. Given a common discount factor δ , the problem amounts to maximizing $EU^i = E(\sum_t \delta^t v_{it} d_t)$, where E is the expectations operator, subject to the constraint that for each committee member the stock of available votes k_{it} equals the votes stored the previous period plus the allocation of 1 new vote ($k_{it} = k_{it-1} - x_{it-1} + 1$, where x_{it-1} are votes cast by i at $t - 1$). Call X_t the vector of strategies, i.e. the number of votes cast by the voters at time t . The state of the game is given by the profile of available votes among all members and calendar time: (K_t, t) . We restrict attention to strategies that depend only on the current state: $x_t(K_t, t)$. The ex ante value of the game to individual i when all players follow optimal strategies is denoted $EV_t^i(K_t, t)$.

The goal of the paper is to compare the storable votes scheme to the more traditional case where votes are not transferable over time, and thus each individual always casts one vote in favor of his preferred alternative. The two games are identical if the time horizon reduces to a single period, but differ otherwise. The storable votes game requires some thought: the choice of how many votes to cast reflects not only the current intensity of preferences, relative to expected future preferences, but also the probability that a vote be pivotal, today or in the future, and thus the expectation of the other players' voting behavior over time. It is a non-stationary dynamic game, where each individual's optimal strategy will be conditioned on the realization of his preference shock, on the distribution of available votes among all players and on calendar time. A simple example builds intuition for the results that follow.

3. An example

Consider the simplest case where two voters i and j must take decision d in two consecutive periods. In period $T - 1$, they are endowed with 1 vote each; they will both receive an additional vote in period T , but the game will then end. At T they will both spend all available votes on their preferred alternative⁵; thus the only problem each player must solve is what preference to announce and whether to cast 1 or 0 votes in its support at $T - 1$.

⁴ Other tie-break rules—no weight on zero voters; status quo wins when votes are tied (with or without considering zero voters)—always yielded the same qualitative results. In the application of the game to the European Central Bank, we could assume that the decision is between a cut ($d = -1$) and an increase ($d = 1$) in interest rates, with the status quo ($d = 0$) prevailing in case of ties. However, this set-up minimizes the role of the status quo, which in fact is often the preferred option for most central bankers. In any case, the results of the two models are identical up to a factor of proportionality in expected payoffs.

⁵ The only possible exception is the voter left with a single vote if state $(2, 1)$ is realized at T . We can appeal to undominated strategies, but note also that in any case his voting decision is always irrelevant, both for his opponent's strategy and for payoffs.

Because preferences and votes are announced simultaneously, and preferences shocks are i.i.d., a voter will always announce preferences truthfully: he cannot manipulate his opponent’s strategy and with 2 voters each always has a positive probability of affecting the outcome. The choice reduces to the number of votes to cast.

Consider individual i , and suppose $v_{iT-1} > 0$ (thus i prefers $d = 1$). His expected utility from casting 0 or 1 vote is given by:⁶

$$Eu_{T-1}^i(x_i = 0) = v_{iT-1} \left(\frac{3}{4} p_{j0} + \frac{1}{2} p_{j1} \right) + \delta (p_{j0} EV_T^i(2, 2) + p_{j1} EV_T^i(2, 1)), \quad (1)$$

$$Eu_{T-1}^i(x_i = 1) = v_{iT-1} \left(p_{j0} + \frac{3}{4} p_{j1} \right) + \delta (p_{j0} EV_T^i(1, 2) + p_{j1} EV_T^i(1, 1)), \quad (2)$$

where p_{jv} is the probability that j casts v votes at $T - 1$, and $EV_T^i(s, k)$ is i ’s expected value of the game in the next and final period, given stocks of available votes s (for player i) and k (for player j). Comparing (1) and (2), we see that i will cast 1 vote at $T - 1$ if and only if:

$$v_{iT-1}/4 \geq \delta (p_{j0} (EV_T^i(2, 2) - EV_T^i(1, 2)) + p_{j1} (EV_T^i(2, 1) - EV_T^i(1, 1))). \quad (3)$$

Solving next period’s expected values, we can obtain an explicit solution for the optimal strategy. In period T , both players cast all votes they have, and the one with most votes wins with probability 1. Thus:

$$EV_T^i(2, 1) = \int_{-1}^0 v dF(v)(0) + \int_0^1 v dF(v)(1) = \int_0^1 v dF(v) \quad (4)$$

since whenever v_{iT} is negative, i will be able to impose $d = 0$, and whenever v_{iT} is positive, d will be equal 1. The player with fewer votes will not be able to influence the choice of d , but half of the times his opponent’s preferred choice matches his own. Hence:

$$EV_T^i(1, 2) = 1/2 \int_{-1}^0 v dF(v) + 1/2 \int_0^1 v dF(v) = 0. \quad (5)$$

Finally, when the two players have the same number of votes, the value of the game at period T is identical to the value of the one-period non-storable votes game (with equal votes). Call the value of this latter game W , noticing that it is time independent and that any number of equal votes is equivalent. For any realization of v_{iT} , player i expects to obtain his preferred value of d with probability 3/4. That is,

$$EV_T^i(1, 1) = EV_T^i(2, 2) = \int_{-1}^0 v dF(v) \left(\frac{3}{4} (0) + \frac{1}{4} (1) \right) + \int_0^1 v dF(v) \left(\frac{3}{4} (1) + \frac{1}{4} (0) \right)$$

⁶ Voter i obtains his desired outcome with probability 1 if his opponent casts fewer votes; with probability 3/4 if j casts the same number of votes (either because i and j agree—with probability 1/2—or because they disagree but i wins the coin toss—with probability $(1/2)(1/2) = 1/4$), and with probability 1/2 if j casts more votes (because 1/2 is the probability that i and j agree).

or

$$EV_T^i(1, 1) = EV_T^i(2, 2) = \frac{1}{2} \int_0^1 v \, dF(v) = W. \tag{6}$$

Substituting (4)–(6) into (3), we obtain:

$$v_{iT-1}/4 \geq \delta(p_{j0} + p_{j1})W$$

or

$$v_{iT-1} \geq 4\delta W \quad \text{if } v_{iT-1} > 0. \tag{7}$$

It is easy to verify that if v_{iT-1} is negative the same logic leads voter i to cast his vote if and only if $-v_{iT-1} \geq 4\delta W$.⁷ Thus i 's optimal strategy is to identify a threshold value $\alpha \equiv 4\delta W > 0$ and cast 1 vote whenever $|v_{iT-1}|$ is larger or equal to α , and cast 0 votes whenever $|v_{iT-1}|$ is smaller than α . Notice that in this two-period example the equilibrium is unique—in fact it is an equilibrium in dominant strategies. The threshold α equals the average intensity of preferences (discounted), and is strictly smaller than 1 as long as there is any probability mass outside the extreme values -1 and 1 . In the simple case where $F(v)$ is uniform and $\delta = 1$, $\alpha = 1/2$.

The conclusion was expected: if i 's policy preference is particularly strong today, he will be willing to sacrifice some of his possible future power to increase his chances of obtaining the desired outcome; vice versa, if his policy preference is weak, he will prefer to abstain today and increase his influence tomorrow. It was this intuition that motivated the paper.

To evaluate the welfare effect of storable votes, consider their impact on ex ante utility. Before the preference shock is realized, the expected value of the game for player i equals:

$$\begin{aligned} &EV_{T-1}(1, 1) \\ &= \int_0^\alpha v \, dF(v) \left(\frac{3}{4} p_{j0} + \frac{1}{2} p_{j1} \right) + \int_{-\alpha}^0 v \, dF(v) \left(1 - \frac{3}{4} p_{j0} - \frac{1}{2} p_{j1} \right) \\ &\quad + (2F(\alpha) - 1)\delta W(p_{j0} + 2p_{j1}) + \int_\alpha^1 v \, dF(v) \left(p_{j0} + \frac{3}{4} p_{j1} \right) \\ &\quad + \int_{-1}^{-\alpha} v \, dF(v) \left(1 - p_{j0} - \frac{3}{4} p_{j1} \right) + 2(1 - F(\alpha))\delta W p_{j1}. \end{aligned} \tag{8}$$

⁷ If v_{iT-1} is negative, i 's expected utility from playing 1 or 0 is analogous to Eqs. (1) and (2) above, but the negative preference shock now multiplies the corresponding probability of losing, as opposed to winning (since instantaneous utility is then different from zero only if i does not succeed in imposing his preference for $d = 0$). The probability of losing when casting 1 or 0 votes is the complement to 1 of the probability of winning we derived earlier, and the two expressions for expected utility are then immediately calculated.

Voter j faces an identical problem and conditions his voting behavior on the same threshold α : he will vote 0 with probability $2[F(\alpha) - 1/2]$, and 1 with probability $2[1 - F(\alpha)]$ (using, as in (8), the symmetry of the probability distribution). Substituting these values for p_{j0} and p_{j1} , the expected value of the two-period game for either player becomes:

$$EV_{T-1}(1, 1) = \int_0^\alpha v dF(v) \left(F(\alpha) - \frac{1}{2} \right) + \int_\alpha^1 v dF(v) F(\alpha) + \delta W. \tag{9}$$

Compare (9) to the ex ante value of the two-period non-storable votes game W_{T-1} , where the finite horizon implies $W_{T-1} = W + \delta W$ and W is given by (6). $EV_{T-1}(1, 1) = W_{T-1}$ at $\alpha = 0$ or 1, and:

$$\frac{\partial EV_{T-1}}{\partial \alpha} = f(\alpha) \left(\int_0^1 v dF(v) - \frac{\alpha}{2} \right) \tag{10}$$

where $f(\alpha)$ is the density $f(v)$ evaluated at α , and thus is positive. The derivative (10) is positive at $\alpha = 0$ and has a single root; with $EV_{T-1}(1, 1) = W_{T-1}$ at $\alpha = 0$ and 1, it follows that $EV_{T-1}(1, 1) > W_{T-1}$ for all $\alpha \in (0, 1)$. And since α is strictly positive and, for any non-degenerate $F(v)$, smaller than 1, we conclude that ex ante utility must indeed be strictly higher with storable votes.

The result is again intuitive and is clearly visible in expression (9). As long as α is strictly interior, $F(\alpha) \in (1/2, 1)$ and $(F(\alpha) - 1/2) \in (0, 1/2)$: the positive threshold α shifts probability mass from payoffs with relatively low value (when $|v|$ is smaller than α) to payoffs with relatively higher value (when $|v|$ is larger than α): the possibility to store votes increases the likelihood that a player will win when his preference is stronger, and thus raises ex ante utility.

Notice that the argument does not rely on the equilibrium value of α —any threshold strictly between 0 and 1 would lead to welfare gains. Indeed, we can say something more about the robustness of the welfare results if voters choose incorrect thresholds, an important consideration in practical applications. When the two thresholds are equal, as they must be in equilibrium, the welfare gain always holds. When they differ, call α voter i 's threshold, and β voter j 's. Then:

$$EV_{T-1}^i(1, 1) = \int_0^\alpha v dF(v) \left(F(\beta) - \frac{1}{2} \right) + \int_\alpha^1 v dF(v) F(\beta) + \delta W (1 + 2(F(\alpha) - F(\beta))). \tag{11}$$

It is not difficult to verify that if $\delta = 1$, $EV_{T-1}^i(1, 1) > W_{T-1}$ for all $\alpha \in (0, 1)$, independently of β (and $EV_{T-1}^i(1, 1) = W_{T-1}$ at $\alpha = 0$ or 1).⁸ If the second period is discounted,

⁸ When $\delta = 1$, $EV_{T-1}^i(1, 1) \geq W_{T-1}$ if $1/2 \int_\alpha^1 v dF(v) \geq \int_0^1 v dF(v)[F(\alpha) - 1]$, a condition that does not depend on β .

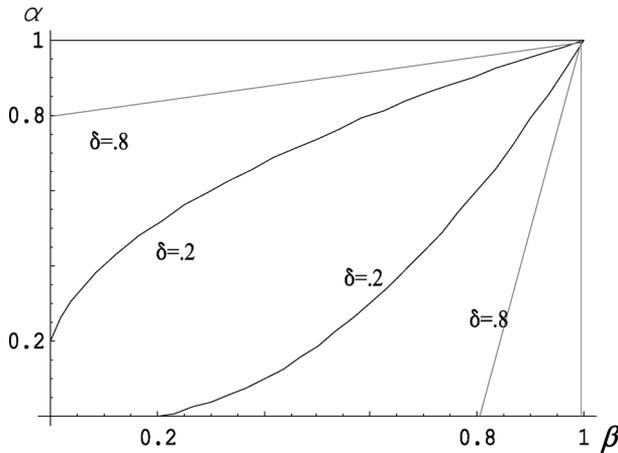


Fig. 1. Two players, two periods. Acceptable thresholds range; $F(v)$ uniform.

limits begin to appear as to how different the thresholds can be, but even for very small δ there is a sizable range of acceptable thresholds values, i.e. values consistent with welfare gains. In Fig. 1, the area between two curves labeled with the same δ value corresponds to the acceptable area at that δ when $F(v)$ is Uniform.

4. Equilibrium

Having verified the intuitive appeal of storable votes in the simplest setting, we need to extend the analysis to more general cases, and the first step is to characterize the equilibrium strategies. We restrict ourselves to undominated strategies (ensuring that voters will vote sincerely) and define a strategy as *monotonic* if, at a given state, the number of votes cast is monotonically increasing in $|v_{it}|$, the voter’s intensity of preferences. Then the following results must hold:

Lemma 1. *At any given state, all best response strategies are monotonic.*

Proposition 1.

- (i) *There exists a perfect Bayesian equilibrium in pure strategies.*
- (ii) *Equilibrium strategies are monotone cutpoint strategies.*

(The proofs are in Appendix A.)

Proposition 1 confirms that the intuitive nature of the equilibrium in the two-voter two-period case holds more generally. Because the probability of obtaining the desired outcome is increasing (if possibly weakly) in the number of votes cast, at any given state and taking as given the other voter’s strategies the optimal number of votes cast cannot be decreasing in the intensity of preferences. Once the existence of an equilibrium is established (in the

first part of the proposition), the observation that equilibrium strategies must take the form of monotone cutpoints then follows immediately. The number of votes that an individual has at his disposal is always finite, while the support of $|v_{it}|$, the segment $[0, 1]$, is continuous. At any state of the game, each voter must identify a series of thresholds that divide the segment $[0, 1]$ into a finite number of intervals; for all realizations of $|v_{it}|$ in a given interval, i casts the same number of votes, but higher intervals must correspond to a larger number of votes. The thresholds are functions of the state of the game, including calendar time, and although their number cannot be larger than the number of votes the voter has available, it can well be smaller—some feasible number of votes may never be cast in equilibrium. Note that Proposition 1 does not state that the equilibrium is unique, and uniqueness is not required for what follows.

Our goal is to identify the welfare properties of the storable votes game, but for arbitrary n and T this is very difficult. The problem is that the number of possible states grows very rapidly with T . Consider for example a 2-voter T -period game. Starting from state (k^i, s^j) at t , we need to evaluate $(k^i + 1)(s^j + 1)$ possible states at time $t + 1$, and the ex ante value of the game in each of these states must be solved backwards from all the possible options it itself can give rise to, and so on at all times, using as anchor the expected values of all possible different states in the terminal periods. The only possible solution method must be recursive. But here we encounter another problem: the game is non-stationary, and the equilibrium strategies depend both on the current state and on calendar time. To calculate the expected values of future states, we need to weigh them by their probability of realization, and hence by the probabilities of the voters' alternative strategies in equilibrium. And these change over time, even for given states.

In the case of two voters, it is nevertheless possible to obtain analytical results, and we proceed to describe them in the next section.

5. Welfare—two voters

A useful implication of Proposition 1 is that we can now characterize each voter's expected instantaneous utility in equilibrium. Consider for example the symmetrical state (k_t, k_t) , where both voters enter the period with identical stocks of votes, and focus on symmetrical equilibria where the players select the same strategy at equal valuations and equal state (and calendar time). Call $Eg_t^i(k_t, k_t)$ i 's expected one period equilibrium utility (or payoff) before the realization of the preference shock when both players play optimal strategies, and $\alpha_{x-1}(t, K) \leq \alpha_x(t, K)$ the equilibrium thresholds such that i will cast $x - 1$ votes for all $|v_{it}| \in [\alpha_{x-1}(t, K), \alpha_x(t, K))$. Then in a symmetrical equilibrium:

$$Eg_t^i(k_t, k_t) = \int_0^{\alpha_1(t,k)} v dF(v) \left(F(\alpha_1(t, k)) - \frac{1}{2} \right) + \int_{\alpha_1(t,k)}^{\alpha_2(t,k)} v dF(v) (F(\alpha_1(t, k)) + F(\alpha_2(t, k)) - 1) + \dots$$

$$\begin{aligned}
 & + \int_{\alpha_{k-1}(t,k)}^{\alpha_k(t,k)} v \, dF(v) (F(\alpha_k(t,k)) + F(\alpha_{k-1}(t,k)) - 1) \\
 & + \int_{\alpha_k(t,k)}^1 v \, dF(v) F(\alpha_k(t,k))
 \end{aligned} \tag{12}$$

where $0 \leq \alpha_x(k, t) \leq \alpha_{x+1}(k, t) \leq 1, \forall t, \forall x \in \{1, \dots, k - 1\}$.⁹ A more cumbersome but analogous expression describes expected one-period equilibrium payoffs in asymmetrical states.

Voter i 's expected value of the game at state (k_t^i, k_t^j) before the realization of the preference shock, is given by:

$$EV_t^i(s_t^i, k_t^j) = Eg_t^i(s_t^i, k_t^j) + \delta EV_{t+1}^i(s_t^i - x_t^{i*} + 1, k_t^j - x_t^{j*} + 1) \tag{13}$$

where the asterisk indicates the equilibrium strategy, with abuse of notation, we use a single expectations operator although EV_{t+1} must be calculated by taking expectations over both x_t^j (j 's current strategy) and (x_{t+1}^i, x_{t+1}^j) .

Expressions (12) and its analogue in asymmetrical states, and expression (13) allow us to establish:

Proposition 2. *For any distribution $F(v)$ continuous and atomless, and any $T > 1$, $EV_1(1, 1) > W_1$, with $EV_1(1, 1)/W_1$ monotonically increasing in T .*

Proof. Intuitively, the objective is to reduce the ex ante value of the game at the initial period 1 to the sum of the expected one-period equilibrium payoffs corresponding to each possible state in all future periods. Exactly as in the 2-period case, in symmetric states the possibility of storing votes when preferences are weak results in higher expected one-period payoffs than in the game with non-storable votes. The problem comes in non-symmetrical states: it is the prospect of being the weaker player in these states, possibly protracted over time and absent by assumption from the game with non storable votes, that creates concerns. But notice that in any symmetrical equilibrium and from any symmetrical state (k_t, k_t) , the probability of reaching state $(s_{t+\tau}^i, k_{t+\tau}^j)$ is identical to the probability of reaching state $(k_{t+\tau}^i, s_{t+\tau}^j)$. Thus when evaluating possible future states, we should give the same weight to the two opposite asymmetrical states and in effect consider their mean expected payoff. All we require then is that this mean payoff be higher, or at least not smaller, than the expected payoff with non-storable votes. It is this observation that allows us to establish the Proposition.

The intuition is formalized in the following two results:

⁹ When i casts x_i votes, he obtains the decision he prefers with probability $1 * \Pr(x_j < x_i) + 3/4 * \Pr(x_j = x_i) + 1/2 * \Pr(x_j > x_i)$ or, exploiting Proposition 1, $1/2[F(\alpha_i) + F(\alpha_{i+1})]$. For each interval of v_i values corresponding to a given strategy, i 's expected valuation is weighted by the probability of the decision he prefers minus the probability of the decision he opposes (to account for negative realizations of v_i), or $[F(\alpha_i) + F(\alpha_{i+1}) - 1]$.

Lemma 2.

- (i) $Eg_t^i(k_t, k_t) \geq W \forall t$, with strict inequality at $T - 1$.
 (ii) $Eg_t^i(s_t^i, k_t^j) + Eg_t^i(k_t^i, s_t^j) \geq 2W \forall t$.

Lemma 3. Suppose the following inequalities hold at $t + 1$:

- (i) $EV_{t+1}^i(k_{t+1}, k_{t+1}) > W_{t+1}$,
 (ii) $EV_{t+1}^i(s_{t+1}^i, k_{t+1}^j) + EV_{t+1}^i(k_{t+1}^i, s_{t+1}^j) \geq 2W_{t+1}$.

Then they must hold at t .

The proofs of the two lemmas amount to manipulating expected equilibrium payoffs (expression (12) and its counterpart in asymmetrical states) and the dynamic programming equation (13). They can be found in Appendix A.

Once the two lemmas are established, Proposition 2 follows immediately. Because at T all votes are cast, $EV_T(s_T^i, k_T^j) + EV_T(k_T^i, s_T^j) = 2W$; in addition in all symmetrical equilibria at $T - 1$, $EV_{T-1}(k_T, k_T) = Eg_{T-1}(k_T, k_T) + \delta W > W_{T-1}$ by Lemma 1. By induction, the inequalities hold at all previous times t , and in particular $EV_1(1, 1) > W_1$ for all $T > 1$. Notice that $EV_1(1, 1)/W_1$ cannot be decreasing in T because a larger number of periods means a larger number of states, each of which is associated with mean expected payoffs that are not smaller than the corresponding expected utilities with non-storable votes. \square

The result confirms that the intuition that emerges so clearly in the 2-period example extends to a longer horizon. As in the 2-period case, the proof of Proposition 2 makes no use of the exact values of the equilibrium thresholds, but holds for all monotone symmetrical thresholds. For example, some positive welfare gains would still be realized if a voter holding k votes followed this simple rule of thumb: at any t , divide the interval $[0, 1]$ in $k + 1$ subintervals of equal size, and once v_i is realized cast x_i votes, where x_i satisfies: $x_i/(k + 1) \leq |v_i| < (x_i + 1)/(k + 1)$, $x_i \in \{0, 1, \dots, k\}$. If instead off equilibrium voters' strategies are not symmetric, we can use the proof of Lemma 2(ii) to show that the average ex ante one-period payoff (averaged over the two voters) cannot be inferior to the expected payoff under non-storable votes. By induction this will hold for the average ex ante value of the full game (although not necessarily for each individual player).

Finally, we have assumed so far that votes accrue to voters over time and future allocations cannot be borrowed. Relaxing this constraint would increase the set of possible states, but at any state equilibrium strategies would still take the form of monotone thresholds (see the proof of Lemma 1). Because this is all is needed in Proposition 2, both the proof and the proposition would remain identical.¹⁰

¹⁰ This does not mean that the welfare gain would remain identical. But notice that the second best nature of the problem implies that borrowing need not increase the expected value of the game.

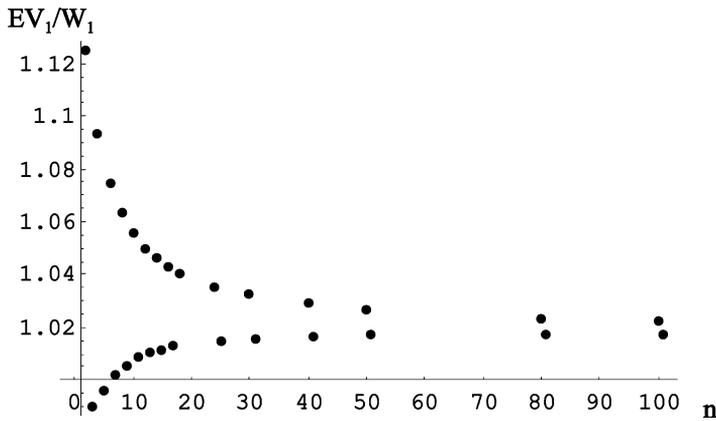


Fig. 2. Two periods, n voters; $F(v)$ uniform; $\delta = 1$.

6. Welfare— N voters

With a larger number of voters, the properties of the mechanism are less clear-cut. Figure 2 depicts the ratio $EV_1(1, 1, 1, \dots, 1)/W_1(n)$ of the 2-period game when $F(v)$ is uniform and $\delta = 1$, as function of the number of voters. Three features are particularly noticeable: First, the ratio is larger than 1, i.e. the welfare gains are positive, for all n different from 3 or 5. Second, the ratio behaves differently for n odd and n even: especially when the number of players is small, the welfare gains from the scheme are much higher for n even than for $n - 1$ or $n + 1$. Third, the ratio increases with the number of players if n is odd and decreases if n is even, finally converging to a value larger than 1 for all n large enough.¹¹ The plot is sensitive to the distribution: although the difference between odd and even numbers of voters is preserved, the plot shifts upward if $F(v)$ is unimodal with a peak at 0, and downward if it is bimodal at 1 and -1 (in both cases, more so the more concentrated is the distribution).

The sensitivity of the welfare comparison to n odd or even reflects for the most part the sensitivity of non-storable votes. As expected, non-storable votes do reasonably well when the number of voters is odd, but are very inefficient when the number is even and small: they improve over randomness only because they are able to recognize unanimity, but when voters are equally split, valuations are irrelevant and the tie-break rule determines the outcome.¹² The efficiency of storable votes, on the other hand, is quite stable over different n : the problem posed by an even number of voters is less severe because it does not translate necessarily into a correspondingly even number of votes. Figure 3 plots separately the ex ante value of the 2-period game with storable (the darker dots) and non-storable votes

¹¹ The relevant formulas are in Appendix A. If $F(v)$ is uniform, it can be shown analytically that the ratio $EV_{T-1}(1, 1, 1, \dots, 1)/W_{T-1}(n)$ converges to a number higher than 1 as the number of players becomes very large. (In the limit, if the distribution of valuations is symmetric, a random choice is efficient, and so are non-storable votes (Ledyard and Palfrey, 2002) and storable votes. W_{T-1} converges to expected welfare with random choice from above, EV_{T-1} converges to W_{T-1} from above.)

¹² The problem is mitigated as n increases because the probability of a split into two equally sized group falls.

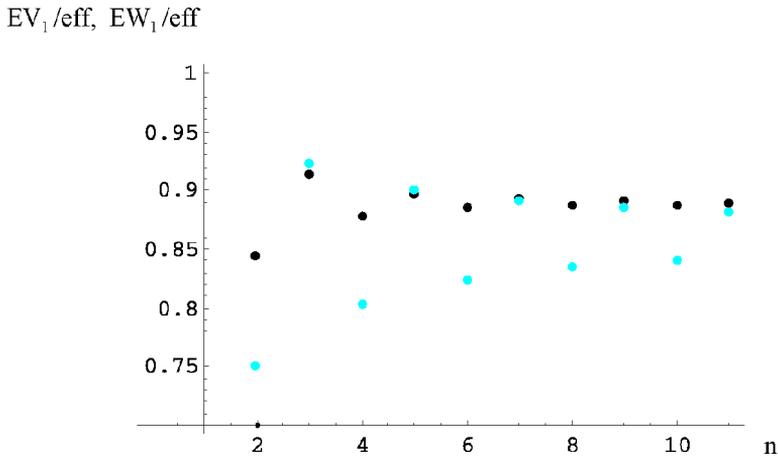


Fig. 3. Expected 2-period payoff as share of efficiency; $F(v)$ uniform; $\delta = 1$.

(the lighter dots), when $F(v)$ is uniform and $\delta = 1$, for n between 2 and 11. Both values are expressed as shares of first best efficiency, defined as expected per capita payoff over the 2 periods if the decision is always taken in favor of the side having higher absolute valuation (see Appendix A for the relevant formulas). The storable votes curve is quite flat while non-storable votes give rise to two very different curves corresponding to even and odd numbers of voters.

The figure explains the sensitivity of the relative welfare gains to n odd or even, but does not explain why storable votes should perform less well than non-storable votes at low, odd n . The intuition for the previous results with $n = 2$ relied on storable votes' ability to elicit and reward strongly held preferences. This should presumably remain true, but we see now that it is possible to find examples where storable votes do not generate welfare gains. Why?

The short answer is that rewarding the intensity of preferences raises efficiency, in our ex ante sense, if the stronger intensity of the minority is more than sufficient to compensate for the higher probability of belonging to the majority—a complication that does not exist in the case of 2 voters. Preferences and the resulting equilibrium strategies must be such that a sufficient wedge exists between the expected valuations when losing and when winning. If the horizon is short, as in our 2-period example, or if the distribution of valuations is very concentrated, difficulties can arise.

This can be seen clearly in the expression for $EV_1(1, 1, 1)$ in the 2-period, 3-voter game:

$$EV_1(1, 1, 1) = \int_0^{\alpha_1} v dF(v) \left(\frac{1}{2} - 4[1 - F(\alpha_1)] \left[F(\alpha_1) - \frac{1}{2} \right] \right) + \int_{\alpha_1}^1 v dF(v) \left(\frac{1}{2} + 2 \left[F(\alpha_1) - \frac{1}{2} \right]^2 \right)$$

$$+\delta W \left(1 - 4[1 - F(\alpha_1)]^2 \left[F(\alpha_1) - \frac{1}{2} \right] \right). \tag{14}$$

By Proposition 1, in the first period each player votes 0 if his draw of $|v|$ is less than a threshold α_1 , and 1 otherwise. $W = 1/2 \int_0^1 v dF(v)$ is the expected one-period payoff with non-storable votes.¹³ With non-storable votes, both terms in parenthesis after the integrals equal 1/2 and the terminal period expected payoff, the last term of (14), is simply δW . With storable votes, each term is modified to capture the new possibility that a single voter be the winner. In line with all previous results, storable votes reduce the probability of obtaining the desired outcome at low valuations, in exchange for an increase in such a probability at high valuations: the term in the first parenthesis is now smaller than 1/2, while the term in the second is larger. However, contrary to the 2-voter case, the decline in the first probability can be larger than the increase in the second, and thus the overall effect of the switch in probabilities will be positive if the difference in expected valuation when abstaining or casting one’s vote is sufficiently large. In other words, the effect of storable votes on one-period expected payoff is now sensitive to the shape of the distribution of valuations. The gain is larger the less polarized is the distribution.

Consider then the expected payoff in the terminal period. The possible states at T are: (1, 1, 1), (2, 2, 2), (1, 2, 2) and its permutations, and (1, 1, 2) and its permutations. In the last period, players always cast all their available votes. In states (1, 1, 1), (2, 2, 2) and (1, 2, 2) (and the latter’s permutations) no voter can win alone, exactly as in the case of non-storable votes, and the expected payoff equals W . If the state is (1, 1, 2), however, the voter controlling 2 votes can win even if the others disagree (if he wins the coin toss), and because votes in the last period are cast independently of the intensity of preferences, the possible victory of the minority voter in this case is unambiguously efficiency reducing—the negative term in the last parenthesis in (14), which indeed reflects the probability of reaching state (1, 1, 2), i.e. the probability that a single voter abstains in period 1.¹⁴ Minority victories that do not mirror more intense preferences are costly, and because intensity of preferences plays no role in the terminal period, we should expect storable votes to perform better over longer horizons.

To evaluate these intuitions, we ran numerical exercises with different time horizons and different distributions of valuations, focusing on $n = 3$. $F(v)$ is modeled as a modified Beta distribution, with support $[-1, 1]$ and constrained to be symmetrical relative to 0 (see the lower panel in Fig. 4). A single parameter b summarizes the curvature of the distribution: $b = 1$ corresponds to the uniform; as b increases, the relative probability mass around zero increases. The results are in Fig. 4, where the expected value of the storable

¹³ Notice that W is unchanged for $n = 3$ and $n = 2$. For arbitrary n :

$$W(n) = \int_0^1 v dF(v)(1/2)^{n-1} \left(\frac{n-1}{n-2+I_n} \right) \quad \text{where } I_n = 1 \text{ if } n \text{ is odd and } 0 \text{ if } n \text{ is even.}$$

¹⁴ In state (1, 1, 2), voters with a single vote obtain their preferred choice with probability 5/8, and the voter with 2 votes with probability 7/8. Thus $E_{gT}(1, 1, 2) = 1/2 W$, $E_{gT}(2, 1, 1) = 3/2 W$, and $1/3(2E_{gT}(1, 1, 2) + E_{gT}(2, 1, 1)) = 5/6 W < W$. The probability of reaching state (1, 1, 2) is $3[(2(1 - F(\alpha_1))^2][2(F(\alpha_1) - 1/2)]$ and EV_T equals $W(1 - 3[(2(1 - F(\alpha_1))^2][2(F(\alpha_1) - 1/2)](1/6))$.

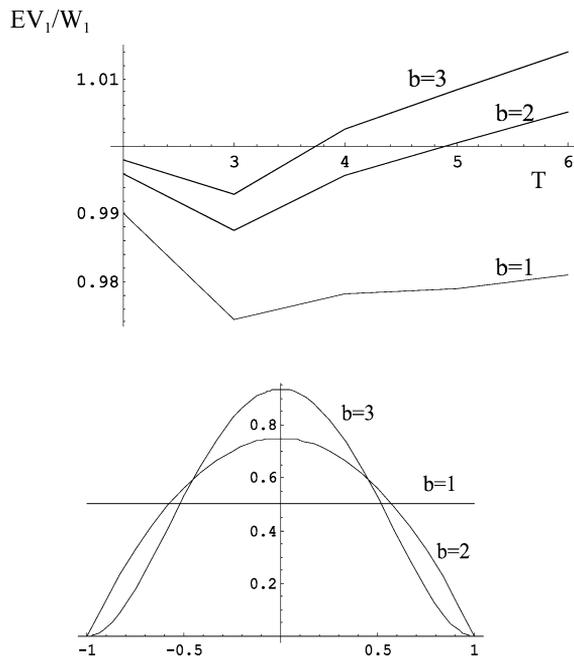


Fig. 4. Three voters, T periods; $F(v)$ beta; $\delta = 1$.
 Lower panel: beta distribution: $F(v) = (1 - v^2)^{b-1} / \int_{-1}^1 (1 - v^2)^{b-1} dv$.

votes game at time 1, relative to the value with non-storable votes, is plotted for different horizons and different b s. Two regularities emerge. First, at any horizon, an increase in b is associated with better ex ante welfare properties for storable relative to non-storable votes. Although the figure does not show it, for b high enough, the ratio EV_1/W_1 is larger than 1 for all T . Second, for $b > 1$ storable votes are associated with ex ante welfare gains if the horizon T is longer than a critical value $\bar{T}(b)$, where \bar{T} is lower the larger is b . For $b = 1$, the uniform case, the conclusion might still hold—for $T > 3$ the ratio EV_1/W_1 is monotonically increasing in T , as in the case of higher b s—but the simulated horizon is not long enough to reflect it.¹⁵

¹⁵ Three further comments. First, reducing δ (increasing future discounting) raises EV_1/W_1 , presumably because the more asymmetrical states expected to arise at the end of the horizon are then discounted more. But the effect is not large because it is countered in part by the reduction in vote saving, and hence the higher likelihood of these same states. Second, as the figure shows, EV_1/W_1 reaches a minimum at $T = 3$ for all b . Although a higher T makes strategies more sensitive to intensities, even in asymmetrical states, it also multiplies the possibility of such states—according to the numerical exercises, the second effect dominates for $T = 3$, but becomes relatively less important as T increases further. Finally, multiple equilibria are possible, but only in one case did we in fact find two equilibria: $b = 1$ and state (5, 4, 3) at $T - 1$ (which requires $T = 6$). In one equilibrium, voter i never plays 1 and player j never plays 1 or 2. In the other equilibrium, player j never plays 3 and player z never plays 1 or 2. Although the first equilibrium leads to slightly better welfare properties, the effect washes out almost completely in the calculation of EV_1/W_1 . The figure uses the second equilibrium.

Both the importance of the horizon and the role played by the shape of the distribution should remain true at larger n or when n is even, but in these cases the positive features of storable votes, relative to non-storable votes, are stronger. When the number of voters is even, storable votes reduce the reliance on the tie-breaking rule, an important positive effect whose role is larger when n is small. When the number of voters is large, storable votes tend to reward minorities that are not too small, a source of welfare gain that holds whether n is even or odd.

Better welfare properties for longer time horizons pose a commitment problem; as time passes and the end of the game approaches voters may wish to renegotiate. We ignore this aspect here—storable votes are evaluated *ex ante* at some constitutional stage, and we take the possibility of commitment as granted. This said, the conclusion is pleasing; the very idea of storable votes relies on the possibility of intertemporal trades and we should expect it to yield its potential benefits only if there is enough time for these trades to be possible.

7. Storable vs. tradable votes

As discussed in the Introduction, storable votes inevitably bring to mind vote trading. How related are the two mechanisms? We rule out monetary transfers, and compare storable votes to log-rolling: exchanges of current for future votes.¹⁶ To study the two schemes side by side we make them comparable through two assumptions: first, we ignore the problem of enforcement posed by vote trading in our finite horizon setting—debtors would be sure to renege in the last period, with the usual cascading effect. We posit instead the existence of credible outside enforcement. Second, with vote trading players who buy votes are effectively borrowing against their future voting allocation. It seems appropriate then to allow borrowing also in the case of storable votes: we assume that when votes are storable the entire stock of votes is allocated to each player at time 1, for him to distribute over future decisions as he sees fit.

Begin with the simplest case: 2 voters and 2 periods. With storable votes, each player enters the game with 2 votes. Following the usual steps, it is easy to establish that voter i will choose to cast 2 votes if $|v_i| \geq 4\delta W$ and 0 votes otherwise. Although the initial number of votes is different, the outcome is identical to the case studied earlier where each voter was endowed with 1 new vote each period. The expected value of the game is again given by Eq. (9).

Suppose now that votes are tradable, but not storable. At time 1, each voter has 1 vote and three options: he can offer to sell his vote now (S) (and have 2 votes next period, when his opponent will have 0, if the trade takes place); do nothing (N) (and have 1 vote each period, just like his opponent), or offer to buy a vote (B) (and have 0 votes next period, when his opponent will have 2). Call p_B the probability that a player offers to buy, p_S the probability that he offers to sell, and $p_N = 1 - p_B - p_S$ the probability that he does nothing. Given v_i , positive for simplicity, the three alternatives lead to expected utilities:

¹⁶ If, neglecting concerns of practical feasibility, we allow for monetary transfers, then it seems reasonable to focus on the optimal mechanism, which is not a voting mechanism (d'Aspremont and Gérard-Varet, 1979).

$$\begin{aligned}
 Eu_i|S &= p_B(v_i/2 + \delta W) + (1 - p_B)(v_i/4 + \delta W), \\
 Eu_i|N &= v_i/4 + \delta W, \\
 Eu_i|B &= p_S v_i + (1 - p_S)(v_i/4 + \delta W).
 \end{aligned}
 \tag{15}$$

It is possible of course that both voters would want to buy, or both would want to sell. With a longer number of periods, prices would emerge, but in this simple example no voter can offer more (or less) than a one-to-one exchange between a vote today and a vote tomorrow. Thus if both voters find themselves on the same side of the trade no exchange can be concluded, and this is reflected in (15). Given these equations, it is easy to see that the optimal strategy is to offer to buy a vote whenever $|v_i| \geq 4\delta W$, and offer to sell otherwise. It follows that there is a deviation from the reference case of no-trade only when at time 1 one voter’s preference intensities are above the threshold, and the other’s below. But this is exactly what happens with storable votes (in all other cases, the players’ strategies cancel each other). And because the threshold is the same, it is not surprising then that the expected value of the game is given once again by Eq. (9). With 2 players and 2 periods, the two voting mechanisms are identical.¹⁷ Indeed, one can conjecture that with 2 voters the result should continue to hold for any arbitrary time horizon T .

Is this true with more than 2 voters? Consider a 3-voters, 2-period game. With storable votes each player again enters the game with 2 votes. The optimal strategy in the first period is to abstain if $|v_i| < \alpha^{SV}$ and cast 2 votes otherwise, where $\alpha^{SV} \equiv 4\delta W(1 - p_0^2)/(1 - p_2^2)$, and $p_0 = 1 - p_2 = 2[F(\alpha^{SV}) - 1/2]$ (the superscript SV stands for “storable votes”). The expected one period payoff at $T - 1$ and the ex ante value of the game are given by:

$$\begin{aligned}
 Eg_{T-1}^{SV} &= W + \int_{\alpha^{SV}}^1 v dF(v) \frac{p_0^2}{2} - \int_0^{\alpha^{SV}} v dF(v) p_0 p_2, \\
 EV_{T-1}^{SV} &= Eg_{T-1}^{SV} + \delta W(1 - p_0 p_2^2),
 \end{aligned}
 \tag{16}$$

with the probabilities defined above.

Consider now the case of tradable votes. The horizon continues to be 2 periods, implying that again each player can buy or sell at most 1 vote at $T - 1$. As in the case of 2 voters, there are three possible alternatives: each player can offer to sell, offer to buy, or do nothing. When a transaction is proposed, it is concluded only if there is at least one other voter who has made the complementary proposal. But now a new difficulty emerges: a voter willing to transact may be shut out of the market because his 2 opponents trade among themselves. Ruling out side-payments, no price and no bargaining can emerge in the 2-period game, and we assume that if 2 willing buyers, for example, face a single seller, the successful one will be chosen with a coin toss. Taking this into account, expected utilities from any of the

¹⁷ One important caveat. For consistency with the rest of the paper, we are maintaining the information assumption made all along: a voter makes decisions knowing his preferences but not his opponent’s. In the case of tradable votes, this can result in trades between voters on the same side of an issue, albeit with different intensities. Whether a voter would want to reveal his true position is an interesting question not pursued here (see, for example, the discussion in Mueller, 1989).

three actions can be calculated as usual. Consider for example voter i , offering to sell his vote. He succeeds in selling if:

- (i) both other voters offer to buy (with probability p_B^2);
- (ii) one offers to buy and one does nothing (with probability $2p_B p_N$); or
- (iii) one offers to buy, one offers to sell and the coin toss is favorable to i (with probability $p_B p_S$).

If i succeeds, then he is left with 0 votes today in exchange for 2 tomorrow, while his opponents have 2 and 1 votes today and 0 and 1 tomorrow. In both periods, the voter who controls 2 votes alone determines the outcome; thus i 's expected utility, conditional on succeeding in selling, equals $v_i/2 + 2\delta W$ (with v_i positive for simplicity). If i 's offer is not accepted, that may be either because no-one is interested in buying (with probability $(1 - p_B)^2$), in which case no transaction takes place and i 's expected utility equals $v_i 3/4 + \delta W$, or because the other 2 voters trade among themselves (with probability $p_B p_S$), in which case i carries no weight either this period or the next and his expected utility equals $v_i/2$. Thus we can write:

$$Eu_i|S = (p_B^2 + 2p_B p_N + p_B p_S)(v_i/2 + 2\delta W) + (1 - p_B)^2(v_i 3/4 + \delta W) + p_B p_S v_i/2. \tag{17}$$

The expected utilities associates with the two remaining alternatives are calculated analogously:

$$Eu_i|B = (p_S^2 + 2p_S p_N + p_B p_S)v_i + (1 - p_S)^2(v_i 3/4 + \delta W) + p_B p_S v_i/2, \\ Eu_i|N = p_B p_S v_i + (1 - 2p_B p_S)(v_i 3/4 + \delta W). \tag{17'}$$

Given these equations it is easy to establish that it is never optimal to do nothing. There is a single relevant threshold α^M such that voter i offers to sell his vote if $|v_i| < \alpha^M$, and offers to buy otherwise (the superscript M stands for “market”), where once again $\alpha^M = 4\delta W$. Expected one period payoff at $T - 1$ and the ex ante value of the game are given by:

$$Eg_{T-1}^M = W + \int_{\alpha^M}^1 v dF(v) \frac{p_S^2}{2} - \int_0^{\alpha^M} v dF(v) \frac{1 + p_S}{2} p_B, \\ EV_{T-1}^M = Eg_{T-1}^M + \delta W(1 - p_S p_B), \tag{18}$$

where $p_S = 1 - p_B = 2[F(\alpha^M) - 1/2]$.

Comparing Eqs. (16) and (18) is very instructive. It is particularly easy when $F(v)$ is Uniform, because in that case $\alpha^{SV} = \alpha^M = 0.5$, and $p_0 = p_S, p_2 = p_B$. The expected value of the game is unequivocally lower with tradable than with storable votes, a result that arises because expected payoffs are lower in both periods. And the reason is simple: tradable votes require two sides for a trade: with 3 voters, the buyer guarantees himself control over the public decision in the first period, and the seller in the second. The third voter, excluded from the transaction, has no voice in either periods. With storable votes,

on the other hand, each voter decides his allocation of votes on his own. If a voter decides to abstain in the first period, the probability of being pivotal increases for both of his opponents; nor can the abstaining voter be sure of controlling the public decision in period 2. The intuition seems robust: the welfare results are unchanged for all $F(\nu)$ we have tried.¹⁸ The probability of being rationed when votes are tradable remains positive for all finite number of players, and although the game will be more complicated, the logic is unchanged. Similarly, the emergence of prices when trading occurs over a longer horizon should be matched by an equivalent flexibility in the intertemporal program with storable votes. We cannot draw general conclusions at this point, but there is no obvious reason why the welfare results should be reversed.

8. Conclusions

This paper has discussed a very simple—indeed natural—voting mechanism for committees that meet repeatedly over time: voters are allowed to store their votes and shift them intertemporally. As a result, voters cast more votes when their preferences are more intense, and the probability of obtaining their preferred decision shifts from times when preferences are weaker to times when they are stronger. Relative to non-storable votes, ex ante welfare should rise.

This transparent intuition appears clearly, and can be proven rigorously, in the case of two voters. When the number of voters is larger, some complications arise and counterexamples can be found, but the analysis suggests that the conclusion continues to hold if one of the following conditions is satisfied:

- (i) the number of voters is above a minimum threshold;
- (ii) preferences are not too polarized;
- (iii) the horizon is long enough.

Although the rationale for the scheme is transparent, the game is in fact complicated, and a natural question is whether in practical applications voters would be able to identify the equilibrium strategies and reap the potential efficiency gains. We address this question in a companion experimental paper (Casella et al., 2003). The experimental subjects did not, for the most part, cast the equilibrium number of votes, but they consistently did cast more votes when intensities were higher. This was enough to achieve efficiency gains that matched almost perfectly the predictions of the theory.

The model studied here is very simple, and some of its restrictive assumptions will have to be relaxed before the promise of the voting scheme can be confirmed. Some needed extensions are immediate generalizations of this initial model. The importance of the horizon length suggests allowing for infinite horizon, keeping the analysis tractable, for example, by having votes expire after a fixed number of periods, or by studying overlapping generations of committee members with fixed terms. Different information assumptions should

¹⁸ As b increases, α^M becomes larger than α^{SV} , implying that the ex ante probability of putting one's vote up for sale when votes are tradable increases more than the probability of abstaining.

be studied—what if opponents' preferences are known? What if at least their signs, if not their intensities, are known? In many applications, correlation among preferences should be allowed, either over time or across voters.

Other extensions require introducing new issues. How robust are the results to endogenous agenda? Because storable votes derive their value from intertemporal planning, influencing the order in which votes will be called could be important. An individual or a group controlling the agenda might be able, for example, to exhaust opponents' votes before an issue he considers crucial is decided. But the opposite can be argued too—the ability to shift votes intertemporally provides everybody with more flexibility and might in fact neutralize the advantage enjoyed by those who set the agenda.

A related if different question is the impact of storable votes on minorities. Advocates of cumulative voting, the static multi-candidate counterpart of storable votes, have stressed their potential for increasing the power of minorities (Guinier, 1994), an observation confirmed at least partially by formal and experimental analyses (Cox, 1990; Gerber et al., 1998). Others have expressed concern that when voting is costly the voters most likely to express their votes might be those with most extreme preferences (Campbell, 1999; Osborne et al., 2000). Would decisions be dominated by extremists? What would the welfare implications be then? Notice that once again the outcome is not obvious: when votes are storable, the cost of voting is endogenous and the majority can control a small minority at relatively low cost, if the coordination problem is not too severe.

Finally, we have maintained the assumption that aggregating voters' preferences is made difficult by their divergence. Alternatively, we could model the voting problem as a common value problem: voters have the same preferences but receive different signals about the optimal choice (for example, Feddersen and Pesendorfer, 1997; Piketty, 1999). Piketty (1994) has argued that in this case market-type mechanisms applied to voting, in particular spot markets for votes, are less efficient than simple majority voting, because they induce abstentions and thus reduce the amount of information transmitted through voting. To what extent would this argument apply to storable votes? What if both private and common values are present?

These questions are important and difficult, and will need to be addressed. For now, we conclude that storable votes, although not the most efficient mechanism theoretically possible, are very simple, could realistically be implemented and appear to take us part of the way towards efficiency without violating our ethical priors.

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Appendix A

Proof of Lemma 1. Monotonicity. Suppose $v_{it} > 0$ and call $\Pr(w|x)$ the probability that i obtains the desired decision over the current proposal (“wins”) when casting x votes, i.e. $\Pr(w|x) \equiv \Pr(d_t = 1 | x_t^i = x)$. For any number of voters n , $\Pr(w|x)$ must be monotonically (if possibly weakly) increasing in x . Given the valuation v_{it} , i ’s expected utility from casting x votes equals $v_{it}\Pr(w|x) + \delta EV_{t+1}(k_{t+1}^i, EK_{t+1}^{-i})$ where $k_{t+1}^i = k_t^i - x + 1$. Call x' (x'') the equilibrium number of votes cast by voter i when $v_{it} = v'$ (v'') (with $v' > v'' > 0$). By definition of equilibrium, the following two inequalities must hold:

$$\begin{aligned} &v'\Pr(w|x') + \delta EV_{t+1}(k_t^i - x' + 1, EK_{t+1}^{-i}) \\ &\geq v'\Pr(w|x'') + \delta EV_{t+1}(k_t^i - x'' + 1, EK_{t+1}^{-i}), \\ &v''\Pr(w|x'') + \delta EV_{t+1}(k_t^i - x'' + 1, EK_{t+1}^{-i}) \\ &\geq v''\Pr(w|x') + \delta EV_{t+1}(k_t^i - x' + 1, EK_{t+1}^{-i}). \end{aligned}$$

Adding the two inequalities, we obtain:

$$(v' - v'')(\Pr(w|x') - \Pr(w|x'')) \geq 0.$$

But with $v' > v''$ and $\Pr(w|x)$ monotonically increasing in x , this implies $x' \geq x''$, establishing the result. The logic is identical, with the appropriate sign changes, for $v_{it} < 0$. Notice that the proof holds for any strategies chosen by the other voters, implying that all best response functions must be monotonically increasing. □

Proof of Proposition 1. (i) Existence of equilibrium in pure strategies. Formally, we are looking for a perfect Bayesian equilibrium of a multi-stage game. An important simplification is that players’ types are i.i.d. across different periods: the game has no updating of information on players’ types, and if we restrict our focus to Markov strategies the only intertemporal link across periods is the evolution of the state variable—the accumulation or depletion of the votes’ stock (which is common knowledge), and the change in calendar time. It follows that we can find a perfect equilibrium by backward induction. In period T , the dominant strategy is to cast all remaining votes. In period $T - 1$, given the state K_{T-1} the continuation value of the game depends only on the strategies at $T - 1$ (and on the expected value of $|v_T| \equiv 2W$, an exogenous parameter); the one-period payoff depends on the realization of one’s own type v_{iT-1} , and on the strategies at $T - 1$; thus, given the state, the 2-period payoff of the game at $T - 1$, $EU_{T-1}(x_{T-1}; v_{iT-1}, K_{T-1})$, depends only on current strategies and v_{iT-1} . We can study the game at $T - 1$ as a one-stage simultaneous move game. The game satisfies a number of conditions:

- (i) other voters’ types do not enter i ’s payoff directly (their strategies do);
- (ii) players’ types are independently distributed;
- (iii) $F(v)$ is assumed to be continuous and atomless;
- (iv) for each player, the strategies’ space is finite.

By using the notion of distributional strategies—joint distributions on actions and types—Milgrom and Weber (1985) have shown that these conditions guarantee that an equilibrium

exists, and that all equilibrium strategies are empirically indistinguishable from pure strategies. (See also Fudenberg and Tirole, 1992, Section 6.8.) But if the game at $T - 1$ has an equilibrium, then equilibrium strategies at $T - 1$ can be anticipated, as function of the types' realizations and the state at $T - 1$. Again, expected types' realizations are exogenous and K_{T-1} is determined completely by state and strategies at $T - 2$. Hence, given $v_{i,T-2}$ and K_{T-2} , we can study the game at $T - 2$ as a one-stage game and rely once more on Milgrom and Weber's result. With a finite horizon T , the complete game has a finite number of stages and states, and using backward induction we can replicate the procedure for each of them.

(ii) *Monotone cutpoint strategies.* Given existence, the result follows immediately from Lemma 1. \square

Proof of Lemma 2. Begin by proving part (i) for symmetrical states. Recall that expected one-period equilibrium payoff is given by (12), reflecting the optimal thresholds chosen by the voters. Define a function $\Psi(\alpha_x, \dots, \alpha_k)$ representing (fictional) expected payoff when thresholds $\alpha_1, \dots, \alpha_{x-1}$ are set to zero, and all other thresholds are kept at their equilibrium values (and where to simplify notation we ignore the time subscript). By construction $\Psi(\alpha_1, \dots, \alpha_k) = Eg(k, k)$. We can show that the following two conditions hold:

- (a) $\Psi(\alpha_k) \geq W$,
- (b) $\Psi(\alpha_x, \dots, \alpha_k) \geq \Psi(\alpha_{x+1}, \dots, \alpha_k)$.

To establish (a), note that given (12) and the definition of $\Psi(\alpha_k)$, we can write:

$$\Psi(\alpha_k) \equiv \int_0^{\alpha_k} v \, dF(v) \left(F(\alpha_k) - \frac{1}{2} \right) + \int_{\alpha_k}^1 v \, dF(v) F(\alpha_k)$$

where $\alpha_k \in [0, 1]$. At $\alpha_k = 0$ or $\alpha_k = 1$, $\Psi(\alpha_k) = W$; in addition it is easy to verify that $\partial\Psi(\alpha_k)/\partial\alpha_k$ is positive at $\alpha_k = 0$ and has a single root in the interval $\alpha_k \in (0, 1)$. Hence $\Psi(\alpha_k) > W \forall \alpha_k \in (0, 1)$ and $\Psi(\alpha_k) = W$ if $\alpha_k = 0$ or 1 . But from (12) we also know:

$$\begin{aligned} \Psi(\alpha_x, \dots, \alpha_k) \geq \Psi(\alpha_{x+1}, \dots, \alpha_k) &\iff \int_0^{\alpha_x} v \, dF(v) \left(F(\alpha_x) - \frac{1}{2} \right) \\ &+ \int_{\alpha_x}^{\alpha_{x+1}} v \, dF(v) (F(\alpha_x) + F(\alpha_{x+1}) - 1) \geq \int_0^{\alpha_{x+1}} v \, dF(v) \left(F(\alpha_{x+1}) - \frac{1}{2} \right). \end{aligned} \quad (A.1)$$

The left-hand side of (A.1) is identical to the right-hand side if $\alpha_x = 0$ or $\alpha_x = \alpha_{x+1}$. At $\alpha_x = 0$, the left-hand side is increasing in α_x and again it can easily be shown that the derivative has a single root. Hence $\Psi(\alpha_x, \dots, \alpha_k) > \Psi(\alpha_{x+1}, \dots, \alpha_k) \forall \alpha_x \in (0, \alpha_{x+1})$ and $\Psi(\alpha_x, \dots, \alpha_k) = \Psi(\alpha_{x+1}, \dots, \alpha_k)$ for $\alpha_x = 0$ or $\alpha_x = \alpha_{x+1}$, and (b) is established.

Finally, it follows that (a) and (b) can both hold with equality only if all thresholds are either 0 or 1 or if there exist an α_x and α_{x+1} such that $\alpha_1 = \dots = \alpha_x = 0$ and $\alpha_{x+1} = \dots = \alpha_k = 1$, with $x \in \{1, \dots, k - 1\}$, i.e. only if the same strategy is followed for all

realizations of v_i . If at least one threshold is strictly between 0 and 1, then the inequality in part (i) of Lemma 2 is strict. We expect that to be the case in all symmetrical states, but it is particularly easy, and sufficient for our purposes, to show that this must be true at $T - 1$. Suppose both players are endowed with k votes at $T - 1$. We show that casting x votes for all realizations of v_i cannot be an equilibrium. Given v_{iT-1} , which we suppose positive for simplicity, the expected utility of voter i casting x votes is given by: $EU_{T-1}^i(x) = v_{iT-1}[\Pr(x_t^j < x) + 3/4\Pr(x_t^j = x) + 1/2\Pr(x_t^j > x)] + \delta W[\Pr(x_t^j = x) + 2\Pr(x_t^j > x)]$. It is easy to establish then that:

$$EU_{T-1}^i(x' + 1) - EU_{T-1}^i(x') = (v_{iT-1}/4 - \delta W)[\Pr(x_t^j = x' + 1) + \Pr(x_t^j = x')],$$

$$EU_{T-1}^i(x') - EU_{T-1}^i(x' - 1) = (v_{iT-1}/4 - \delta W)[\Pr(x_t^j = x') + \Pr(x_t^j = x' - 1)].$$

Recall that $4\delta W \in (0, 1)$. Take any x' such that $\Pr(x_t^j = x') > 0$. It is immediate to show that if $v_i < 4\delta W$, then player i must prefer $x' - 1$ to x' , and if $v_i > 4\delta W$ he must prefer $x' + 1$ to x . If x' equals 0 or k only one direction of deviation is feasible, but in all cases no x can be the equilibrium strategy for all v_i . Part (i) of Lemma 2 is established.

(ii) To establish part (ii) of the lemma for asymmetrical states, we follow the same logic. Suppose $s > k$, and denote $\{\gamma_1, \gamma_2, \dots, \gamma_{k+1}\}$ the equilibrium thresholds for the player holding s votes at t , and $\{\beta_1, \beta_2, \dots, \beta_k\}$ the equilibrium thresholds for the player holding k votes (where, again, to simplify notation time subscripts are omitted). Notice that the player holding s votes can never gain by casting more than $k + 1$. We can write:

$$\begin{aligned}
 Eg^i(k^i, s^j) + Eg^i(s^i, k^j) &= \int_0^{\beta_1} v dF(v)(F(\gamma_1) - 1/2) \\
 &+ \int_{\beta_1}^{\beta_2} v dF(v)(F(\gamma_1) + F(\gamma_2) - 1) + \dots + \int_{\beta_k}^1 v dF(v)(F(\gamma_k) + F(\gamma_{k+1}) - 1) \\
 &+ \int_0^{\gamma_1} v dF(v)(F(\beta_1) - 1/2) + \int_{\gamma_1}^{\gamma_2} v dF(v)(F(\beta_1) + F(\beta_2) - 1) + \dots \\
 &+ \int_{\gamma_x}^{\gamma_{x+1}} v dF(v)(F(\beta_x) + F(\beta_{x+1}) - 1) + \dots \\
 &+ \int_{\gamma_k}^{\gamma_{k+1}} v dF(v)(F(\beta_k)) + \int_{\gamma_{k+1}}^1 v dF(v). \tag{A.2}
 \end{aligned}$$

Define a function $\Psi(\beta_1, \dots, \beta_x, \gamma_1, \dots, \gamma_{x+1})$, representing the (fictional) sum of expected payoffs in (A.2) when thresholds $\beta_{x+1}, \dots, \beta_{k-1}, \gamma_{x+2}, \dots, \gamma_{k+1}$ are set to 1 (which is now more convenient than setting the omitted thresholds to 0), and all other thresholds are kept at their equilibrium values. By construction, $\Psi(\beta_1, \dots, \beta_k, \gamma_1, \dots, \gamma_{k+1}) \equiv Eg^i(s^i, k^j) +$

$Eg^i(k^i, s^j)$. As in the symmetrical case, we proceed by first evaluating the function Ψ at the smallest threshold and then adding successively higher ones. We can show that the following three conditions hold:

- (a) $\Psi(\gamma_1) \geq 2W, \Psi(\beta_1) \geq 2W$;
- (b) $\Psi(\gamma_1, \beta_1) \geq \Psi(\gamma_1), \Psi(\gamma_1, \beta_1) \geq \Psi(\beta_1)$;
- (c) (i) $\Psi(\beta_1, \dots, \beta_{x+1}, \gamma_1, \dots, \gamma_m) \geq \Psi(\beta_1, \dots, \beta_x, \gamma_1, \dots, \gamma_m)$ where either $\beta_{x+1} > \gamma_m$ or $\gamma_m = 1$,
 (ii) $\Psi(\beta_1, \dots, \beta_x, \gamma_1, \dots, \gamma_{m+1}) \geq \Psi(\beta_1, \dots, \beta_x, \gamma_1, \dots, \gamma_m)$ where either $\gamma_{m+1} > \beta_x$ or $\beta_x = 1$.

To verify (a), notice that given (A.2) we can write:

$$\Psi(\gamma_1) \equiv \int_0^1 v \, dF(v)(F(\gamma_1) - 1/2) + \int_0^{\gamma_1} v \, dF(v)1/2 + \int_{\gamma_1}^1 v \, dF(v).$$

Differentiating $\Psi(\gamma_1)$ with respect to γ_1 , it is easy to see that $\Psi(\gamma_1) > 2W \forall \gamma_1 \in (0, 1)$ and $\Psi(\gamma_1) = 2W$ if $\gamma_1 = 0$ or 1 . The same reasoning, and a corresponding equation, establish $\Psi(\beta_1) \geq 2W$. To verify (b), notice that from (A.2) we also know:

$$\Psi(\beta_1, \gamma_1) \geq \Psi(\gamma_1) \iff \int_0^1 v \, dF(v)(F(\beta_1) - 1/2) \geq \int_0^{\beta_1} v \, dF(v)1/2,$$

an inequality that holds strictly for all $\beta_1 \in (0, 1)$, and weakly at $\beta_1 = 0$ or 1 . Again an equivalent condition establishes $\Psi(\beta_1, \gamma_1) \geq \Psi(\beta_1)$.

Finally, to verify (c), consider case (i) first. We can derive from (A.2):

$$\Psi(\beta_1, \dots, \beta_{x+1}, \gamma_1, \dots, \gamma_m) \geq \Psi(\beta_1, \dots, \beta_x, \gamma_1, \dots, \gamma_m) \iff \int_{\beta_{x+1}}^1 v \, dF(v)(F(\gamma_{x+2}) - F(\gamma_x)) + \int_{\gamma_x}^{\gamma_{x+2}} v \, dF(v)(F(\beta_{x+1}) - 1) \geq 0, \tag{A.3}$$

where $\gamma_{x+2} \leq \gamma_m$ and hence either $\gamma_{x+2} \leq \beta_{x+1}$ or $\gamma_{x+2} = 1$ (and similarly, either $\gamma_x \leq \beta_{x+1}$ or $\gamma_x = 1$, where $\gamma_x \leq \gamma_{x+2}$). At $\gamma_{x+2} = 1$, the inequality in (A.3) becomes:

$$\int_{\beta_{x+1}}^1 v \, dF(v)(1 - F(\gamma_x)) - \int_{\gamma_x}^1 v \, dF(v)(1 - F(\beta_{x+1})) \geq 0,$$

a condition that is satisfied for all $\gamma_x \leq \beta_{x+1}$ or at $\gamma_x = 1$. If $\gamma_{x+2} \neq 1$, notice that (A.3) holds with equality at $\gamma_{x+2} = \gamma_x$ and becomes:

$$\int_{\beta_{x+1}}^1 v \, dF(v)(F(\beta_{x+1}) - F(\gamma_x)) - \int_{\gamma_x}^{\beta_{x+1}} v \, dF(v)(1 - F(\beta_{x+1})) \geq 0$$

at $\gamma_{x+2} = \beta_{x+1}$, an inequality that is satisfied for all $\beta_{x+1} \geq \gamma_x$. But differentiating (A.3) with respect to γ_{x+2} , we can easily establish that for all $\beta_{x+1} < 1$ the derivative has a unique root which must be a maximum. Hence if the inequality is satisfied at $\gamma_{x+2} = \gamma_x$ and at $\gamma_{x+2} = \beta_{x+1}$, it must be satisfied everywhere. For $\beta_{x+1} = 1$, (A.3) equals 0 for any γ_{x+2}, γ_x . We conclude that condition (b) (i) is established. The proof of condition (b) (ii) proceeds identically, and we leave it to the reader. \square

Proof of Lemma 3. Once again, we proceed in two steps. First, we consider symmetrical states; then we prove the corresponding result for asymmetrical states. What follows could be written in matrix form, but the expanded notation, though cumbersome, is more transparent and is maintained here.

Consider state (k_t, k_t) . We can write:

$$\begin{aligned}
 EV_t^i(k_t, k_t) &= EG_t^i(k_t, k_t) \\
 &+ \delta \left[p_{i0}(p_{j0}EV_{t+1}^i(k_t + 1, k_t + 1) + p_{j1}EV_{t+1}^i(k_t + 1, k_t) + \dots \right. \\
 &+ p_{jk}EV_{t+1}^i(k_t + 1, 1)) + p_{i1}(p_{j0}EV_{t+1}^i(k_t, k_t + 1) \\
 &+ p_{j1}EV_{t+1}^i(k_t, k_t) + \dots + p_{jk}EV_{t+1}^i(k_t, 1)) + \dots \\
 &\left. + p_{ik}(p_{j0}EV_{t+1}^i(1, k_t + 1) + p_{j1}EV_{t+1}^i(1, k_t) + \dots + p_{jk}EV_{t+1}^i(1, 1)) \right]
 \end{aligned}$$

or, more compactly:

$$\begin{aligned}
 EV_t^i(k_t, k_t) &= EG_t^i(k_t, k_t) + \delta \left[\sum_{x_i=0}^k p_{ix_i} \left(\sum_{x_j=0}^k p_{jx_j} EV_{t+1}^i(k_t - x^i + 1, k_t - x^j + 1) \right) \right] \tag{A.4}
 \end{aligned}$$

where p_{jx} is the probability that $|v_{jt}|$ falls into the interval that corresponds to j 's optimal strategy x . But the game is symmetric, and starting from the symmetrical state (k_t, k_t) , $p_{ix} = p_{jx}$ for all x . We can thus collect terms and rewrite (A.4) as:

$$\begin{aligned}
 EV_t^i(k_t^i, k_t^j) &= EG_t^i(k_t, k_t) + \delta \left[\sum_{x=0}^{k-1} p_{ix} \sum_{r=x+1}^k p_{jr} (EV_{t+1}^i(k - x + 1, k - r + 1) \right. \\
 &\left. + EV_{t+1}^i(k - r + 1, k - x + 1)) \right] \\
 &+ \sum_{x=0}^k p_{ix} p_{jx} EV_{t+1}^i(k - x + 1, k - x + 1) \tag{A.5}
 \end{aligned}$$

Substituting the conditions stated in Lemma 3, we then obtain:

$$EV_t^i(k_t, k_t) > EG_t^i(k_t, k_t) + \delta W_{t+1} \left[2 \sum_{x=0}^{k-1} \sum_{r=x+1}^k p_{ix} p_{jr} + \sum_{x=0}^k p_{ix} p_{jx} \right].$$

Once again using $p_{ix}p_{jr} = p_{ir}p_{jx}$, it is not difficult to verify that the probabilities, which span all possible equilibrium strategies, sum up to 1. Hence:

$$EV_t^i(k_t, k_t) > Eg_t^i(k_t, k_t) + \delta W_{t+1}.$$

But we know by Lemma 2 that $Eg_t^i(k_t, k_t) \geq W$. Hence $EV_t^i(k_t, k_t) > W_t(k, k)$, and the first part of Lemma 3 is established.

The logic of the proof is identical in the asymmetrical state (s_t^i, k_t^j) . We can write:

$$\begin{aligned} &EV_t^i(s_t^i, k_t^j) + EV_t^i(k_t^i, s_t^j) \\ &= Eg_t^i(s_t^i, k_t^j) + Eg_t^i(k_t^i, s_t^j) + \delta \left[p_{i0}(s_t^i, k_t^j) \left(p_{j0}(s_t^i, k_t^j) EV_{t+1}^i(s_t^i + 1, k_t^j + 1) \right. \right. \\ &\quad \left. \left. + \dots + p_{jk}(s_t^i, k_t^j) EV_{t+1}^i(s_t^i + 1, 1) \right) + \dots \right. \\ &\quad \left. + p_{is}(s_t^i, k_t^j) \left(p_{j0}(s_t^i, k_t^j) EV_{t+1}^i(1, k_t^j + 1) + \dots + p_{jk}(s_t^i, k_t^j) EV_{t+1}^i(1, 1) \right) \right] \\ &\quad + \delta \left[p_{i0}(k_t^i, s_t^j) \left(p_{j0}(k_t^i, s_t^j) EV_{t+1}^i(k_t^i + 1, s_t^j + 1) + \dots \right. \right. \\ &\quad \left. \left. + p_{js}(k_t^i, s_t^j) EV_{t+1}^i(k_t^i + 1, 1) \right) + \dots \right. \\ &\quad \left. \left. + p_{ik}(k_t^i, s_t^j) \left(p_{j0}(k_t^i, s_t^j) EV_{t+1}^i(1, s_t^j + 1) + \dots + p_{js}(k_t^i, s_t^j) EV_{t+1}^i(1, 1) \right) \right]. \end{aligned} \tag{A.6}$$

As always, the probability that a given strategy is chosen by either player is a function of the state; and since we are considering two different states this dependence is recognized explicitly. Using $p_{ix}(s_t^i, k_t^j) = p_{jx}(k_t^i, s_t^j) \forall x, s, k, t$, we can simplify (A.6):

$$\begin{aligned} &EV_t^i(s_t^i, k_t^j) + EV_t^i(k_t^i, s_t^j) \\ &= Eg_t^i(s_t^i, k_t^j) + Eg_t^i(k_t^i, s_t^j) + \delta \left[p_{i0}(s_t^i, k_t^j) \sum_{x=0}^k p_{jx}(s_t^i, k_t^j) \right. \\ &\quad \left. \times \left(EV_{t+1}^i(s_t^i + 1, k_t^j - x + 1) + EV_{t+1}^i(k_t^i - x + 1, s_t^j + 1) \right) + \dots \right. \\ &\quad \left. + p_{is}(s_t^i, k_t^j) \sum_{x=0}^k p_{jx}(s_t^i, k_t^j) \left(EV_{t+1}^i(1, k_t^j - x + 1) \right. \right. \\ &\quad \left. \left. + EV_{t+1}^i(k_t^i - x + 1, 1) \right) \right]. \end{aligned} \tag{A.7}$$

More compactly, we can write:

$$\begin{aligned} &EV_t^i(s_t^i, k_t^j) + EV_t^i(k_t^i, s_t^j) \\ &= Eg_t^i(s_t^i, k_t^j) + Eg_t^i(k_t^i, s_t^j) + \delta \sum_{r=0}^s p_{ir} \sum_{x=0}^k p_{jx} \left(EV_{t+1}^i(s_t^i - r + 1, k_t^j - x + 1) \right. \\ &\quad \left. + EV_{t+1}^i(k_t^i - x + 1, s_t^j - r + 1) \right). \end{aligned} \tag{A.8}$$

(All probabilities are now conditional on the same state, and in absence of ambiguity the notation is simplified.) Substituting the conditions in Lemma 3, we then derive:

$$EV_t^i(s_t^i, k_t^j) + EV_t^i(k_t^i, s_t^j) \geq Eg_t^i(s_t^i, k_t^j) + Eg_t^i(k_t^i, s_t^j) + 2\delta W_{t+1} \sum_{r=0}^s \sum_{x=0}^k p_{ir} p_{jx}.$$

It is easy to verify that the probabilities sum up to 1. Hence:

$$EV_t^i(s_t^i, k_t^j) + EV_t^i(k_t^i, s_t^j) \geq (Eg_t^i(s_t^i, k_t^j) + \delta W_{t+1}) + (Eg_t^i(k_t^i, s_t^j) + \delta W_{t+1}).$$

But by Lemma 2, the second part of Lemma 3 is then established. \square

Derivation of Fig. 2. When votes are storable and all players can either cast 1 vote or abstain, the probability of obtaining one’s preferred outcome when voting equals:

$$\sum_{z=0}^{n-1} p_1^z p_0^{n-z-1} \binom{n-1}{z} \left[\frac{1}{2} \left(1 + \left(\frac{1}{2} \right)^z \binom{z}{\frac{z-I_z}{2}} \right) \right],$$

where $I_z \equiv 1$ for z odd, and 0 for z even.

The corresponding probability when abstaining equals:

$$\sum_{z=0}^{n-1} p_1^z p_0^{n-z-1} \binom{n-1}{z} \left[\frac{1}{2} \left(1 + (1 - I_z) \left(\frac{1}{2} \right)^{n-1} \binom{z}{\frac{z}{2}} \binom{n-z-1}{\frac{n-z-2+I_n}{2}} \right) \right],$$

where $I_n \equiv 1$ for n odd, and 0 for n even.

From Proposition 1, the probabilities $p_1(n)$ and $p_0(n)$ continue to depend on a threshold $\alpha(n)$ such that $p_1(n) = 2[1 - F(\alpha(n))]$ and $p_0(n) = 2[F(\alpha(n)) - 1/2]$. On the basis of these equations, it is possible to derive expected payoffs and the expected value of the game.

Derivation of Fig. 3. We plot the ex ante payoff of storable and non-storable votes, as proportion of the expected efficient payoff, which we define as the expected payoff if the decision were always resolved in favor of the size with larger total valuation (in absolute value). $F(v)$ is uniform. Consider for example the case of 2 voters. Then:

$$EU^*(2) = \frac{1}{2} \int_0^1 \frac{v}{2} dv + \frac{1}{2} \left[\int_0^1 \left(\int_{v_j}^1 \frac{v_i}{2} dv_i - \int_0^{v_j} \frac{v_i}{2} dv_i \right) dv_j \right],$$

where the first integral captures expected payoff when both agree (note that a positive payoff is expected only if both voters have positive valuations) and the second when they disagree, again taking into account that when the larger absolute valuation is negative, d is set to 0, and when it is positive, the voter with negative valuation suffers a loss.

The expected efficient payoff can be calculated in a similar manner for different numbers of voters, keeping in mind that the characteristic function of a sum w of n random variables, each independently distributed uniformly over $[0, 1]$ is given by:

$$P_n(w) = \frac{1}{2(n-1)!} \sum_{k=0}^n (-1)^k \binom{n}{k} (w-k)^{n-1} \text{sgn}(w-k).$$

We then derive, for arbitrary n :

$$EU^*(n+1) = \left(\frac{1}{2}\right)^{n+2} + \sum_{k=0}^{n-1} \binom{n}{k} \left(\frac{1}{2}\right)^{n+1} \\ \times \left[\frac{1}{2} \left(\int_0^k P_n(w) dw - \int_{k+1}^n P_n(w) dw \right) + \int_k^{k+1} \left(\int_{w-k}^1 v dv - \int_0^{w-k} v dv \right) P_n(w) dw \right].$$

The expected efficient payoff has no temporal dimension: given that valuations are i.i.d. the expected efficient payoff in the 2-period game is simply $EU^*(n) + \delta EU^*(n)$.

References

- Börgers, T., 2001. Costly voting. Mimeo. University College London.
- Brams, S., 1975. *Game Theory and Politics*. Free Press, New York.
- Brams, S., Riker, W.H., 1973. The paradox of vote trading. *Amer. Polit. Sci. Rev.* 67, 1235–1247.
- Buchanan, J.M., Tullock, G., 1962. *The Calculus of Consent*. Univ. Michigan Press, Ann Arbor.
- Casella, A., Gelman, A., Palfrey, T.R., 2003. An experimental study of storable votes. Working paper No. 9982, NBER. September.
- Campbell, C.M., 1999. Large electorates and decisive majorities. *J. Polit. Economy* 107, 1199–1217.
- Coleman, J., 1966. The possibility of a social welfare function. *Amer. Econ. Rev.* 56, 1105–1122.
- Cox, G.W., 1990. Centripetal and centrifugal incentives in electoral systems. *Amer. J. Polit. Sci.* 34, 903–935.
- d'Aspremont, C., Gérard-Varet, L.-A., 1979. Incentives and incomplete information. *J. Public Econ.* 11, 25–45.
- Dodgson, C., 1884. *The Principles of Parliamentary Representation*. Harrison and Sons, London (Supplement, 1885).
- Feddersen, T., Pesendorfer, W., 1997. Voting behavior and information aggregation in elections with private information. *Econometrica* 65, 1029–1058.
- Ferejohn, J., 1974. Sour notes on the theory of vote trading. Mimeo. California Institute of Technology.
- Fudenberg, D., Tirole, J., 1992. *Game Theory*. The MIT Press, Cambridge, MA.
- Gerber, E., Morton, R., Rietz, T., 1998. Minority representation in multimember districts. *Amer. Polit. Sci. Rev.* 92, 127–144.
- Guinier, L., 1994. *The Tyranny of the Majority*. Free Press, New York.
- Ledyard, J.O., Palfrey, T.R., 2002. The approximation of efficient public good mechanisms by simple voting schemes. *J. Public Econ.* 83, 153–171.
- Milgrom, P.R., Weber, R.J., 1985. Distributional strategies for games with incomplete information. *Math. Operations Res.* 10, 619–632.
- Moulin, H., 1979. Dominance solvable voting schemes. *Econometrica* 47, 1337–1352.
- Moulin, H., 1982. Voting with proportional veto power. *Econometrica* 50, 145–162.
- Mueller, D.C., 1978. Voting by veto. *J. Public Econ.* 10, 57–75.
- Mueller, D.C., 1989. *Public Choice II*. Cambridge Univ. Press, Cambridge.
- Osborne, M., Rosenthal, J., Turner, M., 2000. Meetings with costly participation. *Amer. Econ. Rev.* 90, 927–943.
- Philipson, T., Snyder, J., 1996. Equilibrium and efficiency in an organized vote market. *Public Choice* 89, 245–265.
- Piketty, T., 1994. Information aggregation through voting and vote-trading. Mimeo. MIT. April.
- Piketty, T., 1999. The information aggregation approach to political institutions. *Europ. Econ. Rev.* 43, 791–800.
- Sawyer, J., MacRae, D., 1962. Game theory and cumulative voting in Illinois: 1902–1954. *Amer. Polit. Sci. Rev.* 56, 936–946.