V/C31.0010.002. Fall 1999

Intermediate Microeconomics Midterm 2

Points 70. Time 70 minutes (1.20–2.30pm). The first question carries 30 points; the second 40 points. Questions with parts within them give equal weight to the parts.

Guide for Time Allocation: The questions in part (1) should take no more than 5 minutes each to answer. The questions in part (2) should take you no more than 10 minutes each. This schedule will allow you to finish the exam in 70 minutes. Good luck.

(1) (**30 points, 6 points per part**) Are the following statements true or false? A simple yes-no answer will *not* suffice. Explain, with the use of diagrams where necessary.

[**a**] True. A Cobb-Douglas production function is given by the formula

$$X = AK^{\alpha}L^{\beta},$$

where α and β are both positive. If there are increasing returns to one input — say labor — this means that $\beta > 1$. But then it must be the case that $\alpha + \beta > 1$. It follows that there are increasing returns to scale. [Extra credit if someone also explains why $\alpha + \beta > 1$ means increasing returns to scale.]

[**b**] True. If the demand curve is downward sloping, consider the impact of selling one more unit of the good. First, one gets the market price evaluated at that quantity — call it p. But the impact of selling one more unit is to bring this price down a bit for *every one of the intramarginal units.* This lowers revenue. Therefore the gain in revenue — or marginal revenue — is less than the price.

[**c**] True. If we are minimizing costs subject to an output constraint, then the minimum cost point is given by a tangency of the isocost line with the isoquant. [Draw diagram.] At the tangency, the MRTS equals the factor price ratio. Rewriting this, we see that

$$\frac{\text{MPP}_L}{\text{MPP}_K} = \frac{w}{r},$$

which explains the answer. [Extra credit if a student recognizes this but also argues that there may be corner solutions if both inputs are not used. But this is not needed for full credit.]

[**d**] False. This argument neglects free entry. It is possible for individual supply curves to be upward sloping and for the industry supply curve to be downward sloping. Suppose that an industrial expansion *lowers* the cost of some input, and that all firms are identical. In that case, industrial expansion will lower the shut-down price (because inputs are cheaper). Therefore higher industrial output will be obtained at a lower supply price. At the same time, the *individual* supply curve is upward sloping because a competitive firm takes all prices as given and because it has increasing marginal cost curves.

[**e**] False. It depends on the rate of interest because we need to calculate present value, which in this case is given by

$$100 + \frac{100}{(1+i)} + \frac{100}{(1+i)^2}$$

This may be smaller than the cost — 280 — if the interest rate is high enough. An example is $i = 1/5$ or 20%.

(2) (**40 points, 10 points per part**) Answer the following questions briefly and clearly. You can put in a diagram, and/or a simple example to illustrate. *These are not true-false questions.*

(i) If the (short run) production function is given by $X = \sqrt{L}$, then to *produce* X, you need X^2 labor hours. Each labor hour costs 5. So the total cost of labor is given by $5X^2$. Add to this the fixed cost, which is 50, and you get

$$\text{TC} = 50 + 5X^2.$$

So average total cost is just TC/x, which is

$$\text{ATC} = \frac{50}{x} + 5x$$

Average variable cost is total *variable cost* divided by x, which is

$$\text{AVC} = 5x$$

To do the marginal cost calculation, I just require some thoughts, not anything precise. For example, you could add one unit to total cost and observe that total cost rises by the amount

$$5(x+1)^2 - 5x^2 = 10x + 5$$

which is not the completely correct answer but will get you full credit. Or you can draw a diagram.

This firm should never shut down in the short run, because for any positive price, it can always find an X small anough so that $p >$ AVC (see the formula for average variable cost).

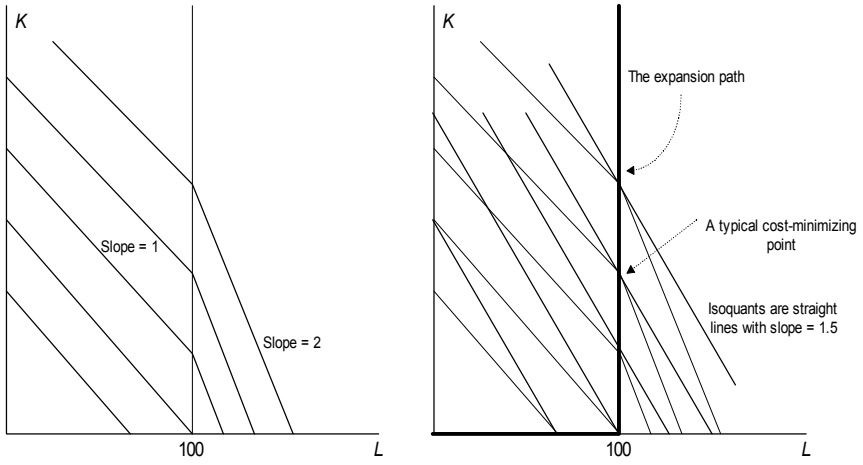

Figure 1: Expansion path for question 2(ii).

(ii) Figure 1 shows you the correct diagram. The isocost lines have a slope of 1 until labor hours cross 100, after that, the slope becomes 2 because of the overtime. In the second panel, we draw in the isoquants which are straight lines with a slope of 1.5 (the reason is that 1.5 units of K must be added to compensate for the loss of 1 unit of L). This means that the solution is at a corner (only labor is used in production) unless labor hours cross 100. Thereafter capital is used. The expansion path is therefore the "inverted L" shown in boldface in the second panel of Figure 1.

(iii) Consult Figure 2 in what follows. In the first panel, we show the competitive equilibrium before a quota is imposed. At the equilibrium price P^* and quantity Q^*, it is easy to see that total consumer surplus is given by the area A, while the total producer surplus is given by the area B. In the second panel, the quota is imposed. Now the equilibrium price will have to be the price that clears the

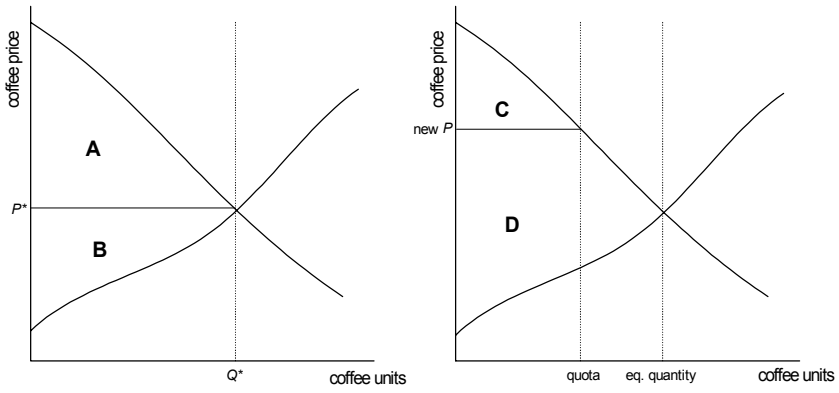

Figure 2: Diagram for question 2(iii).

market with a quota — it is the demand price at the quota. I have drawn it in as "new P". The new consumer surplus is C and the new producer surplus is D.

(a) It should be absolutely obvious that $C < A$ (the price has gone up, so consumer surplus must have fallen). But D and B are *not*, in general, comparable, It is true that there is a quota; this lowers production and tends to lower producer surplus. But the price has gone up. So D may well be bigger than B.

Extra credit will be given if someone argues that D will exceed B when the demand curve for coffee is inelastic.

It follows that consumers will always oppose the quota, while producers may support it (if $D > B$).

(b) Now notice that whatever the ambiguities between B and D, the *sum* $A + B$ must be greater than the sum $C + D$. Therefore a quota must lower the *unweighted* sum of producer and consumer surplus. It follows that the government, in imposing the quota, cannot be placing equal equal weight on consumer and producer surplus. Indeed, it must be placing a higher weight on producer surplus.

(iv) This question is completely standard and its grading will therefore be strict, to make sure that all the points are properly made. The first point is the shutdown rule. You should draw the average variable cost curve, where the notion of what is "variable" depends on whether we are considering the short or the long run. Then mark the minimum point of this curve. State that if the price is lower than this, the firsm will shut down, and explain why.

The second point is the profit-maximizing rule should the firm decide to produce. Here you should state that price equals marginal cost, and you must explain why. The explanation can be done in a variety of ways. (1) you can argue verbally, saying that if this is not the case, then we have a contradiction by either expanding or contracting output. (2) You can argue graphically, drawing the total revenue curve (a straight line) and the total cost curve and arguing that the objective is to make the vertical distance between these two as large as possible.

Students who argue that $P = \text{MC}$ and the shutdown rule are together necessary but still not sufficient will get some extra credit. This is because the "price equals marginal cost" equality may happen at many different points, some of which may be local minima.