EC9AA Term 3: Lectures on Economic Inequality

Debraj Ray, University of Warwick

- Slides 5: Inequality and Mobility, Part 2

Inequality and Upward Mobility

- **Linear specification**:

$$\text{Upward Mobility}_{js}^{\Delta} = \gamma * \textbf{Inequality}_{js} + \phi \mathbf{Z}_{js} + f_j + h_s + \epsilon_{js}$$

- Δ is length of interval $[s, s + \Delta]$ (e.g., 10 or 30 years) starting year s.
- $\text{Upward Mobility}_{js}^{\Delta}$ is M or R for country j over $[s, s + \Delta]$.
- γ: **coefficient of interest**.
- Inequality_{js}: Gini or Atkinson index for country j at baseline year s.
- $\mathbf{Z}_{js}$: time-varying controls including per-capita income for country j at date s.
- f_j and h_s: country and region-year fixed effects.

Inequality and Upward Mobility

- **Flexible specification**:

$$\text{Upward Mobility}_{js}^{\Delta} = \Psi\left(\textbf{Inequality}_{js}\right) + \phi \mathbf{Z}_{js} + f_j + h_s + \epsilon_{js}$$

- Δ is length of interval $[s, s+\Delta]$ (e.g., 10 or 30 years) starting year s.
- $\text{Upward Mobility}_{js}^{\Delta}$ is M or R for country j over $[s, s+\Delta]$.
- Ψ **is a flexible (nonparametric or pre-specified) functional form**.
- Inequality_{js}: Gini or Atkinson index for country j at baseline year s.
- $\mathbf{Z}_{js}$: time-varying controls including per-capita income for country j at date s.
- f_j and h_s: country and region-year fixed effects.

Data

- **Baseline dataset:**
  - World Inequality Database (WID): 133 countries, 1980-2021.
  - Deciles (or percentiles) of the income distribution
  - Lack individual-level data for large subset of country-year observations.
- **Robustness checks** with:
  - Poverty and Inequality Platform (PIP): harmonized global survey data.
  - UNU-WIDER World income Inequality Database: (WIID)

- Panel unit root tests for main variables ▸ Unit Root Tests

Data

- **Baseline dataset:**
  - World Inequality Database (WID): 133 countries, 1980-2021.
  - Deciles (or percentiles) of the income distribution
  - Lack individual-level data for large subset of country-year observations.
- **Robustness checks** with:
  - Poverty and Inequality Platform (PIP): harmonized global survey data.
  - UNU-WIDER World income Inequality Database: (WIID)

- Panel unit root tests for main variables ▸ Unit Root Tests

Data

- **Baseline dataset:**
  - World Inequality Database (WID): 133 countries, 1980-2021.
  - Deciles (or percentiles) of the income distribution
  - Lack individual-level data for large subset of country-year observations.
- **Robustness checks** with:
  - Poverty and Inequality Platform (PIP): harmonized global survey data.
  - UNU-WIDER World income Inequality Database: (WIID)

- Panel unit root tests for main variables ▸ Unit Root Tests

Linear Model

	Upward Mobility[10]							
	Relative [1–4]				Absolute [5–8]			
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Gini	***21.88	***21.87			***21.45	***25.08		
	(2.76)	(2.77)			(2.77)	(4.67)		
Atkinson			***22.44	***22.59			***19.49	***25.41
			(2.37)	(2.37)			(6.49)	(3.91)
Income p.c.		0.02		-0.21		***-7.89		***-8.14
		(0.27)		(0.22)		(0.47)		(0.49)
R^2	0.31	0.31	0.40	0.40	0.32	0.60	0.32	0.62
Obs	4158	4158	4158	4158	4158	4158	4158	4158

Country + region-year fixed effects throughout. Robust standard errors clustered at the country level.

A Contrast

A contrast to the Great Gatsby curve.

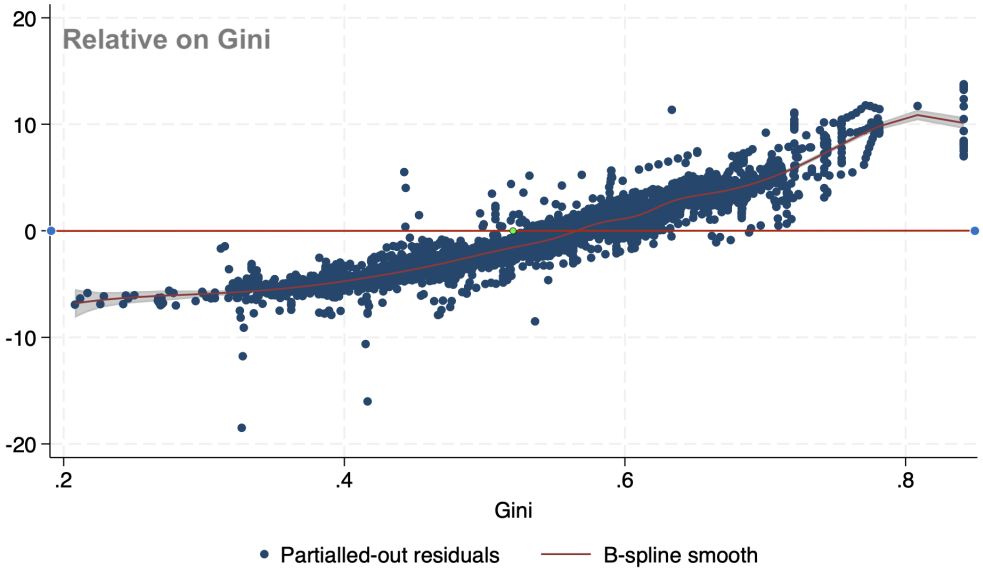
- One SD in the Gini (0.084) associated with:
  - $\uparrow$ in relative upward mobility by 1.8 [$>$ 1SD = 1.56, mean = -0.05]
  - $\uparrow$ in absolute upward mobility by 2.1 [$2/3$ 1SD = 3.4, mean = 1.03]
- One SD in Atkinson (0.085) associated with:
  - $\uparrow$ in relative upward mobility by 1.9
  - $\uparrow$ in absolute upward mobility by 2.2

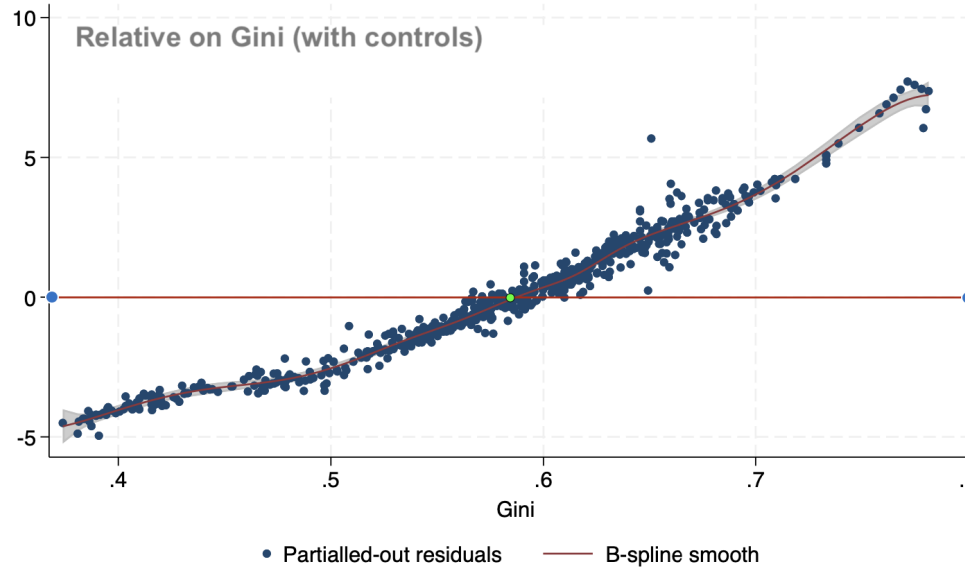
Upward Mobility and Inequality: Controls

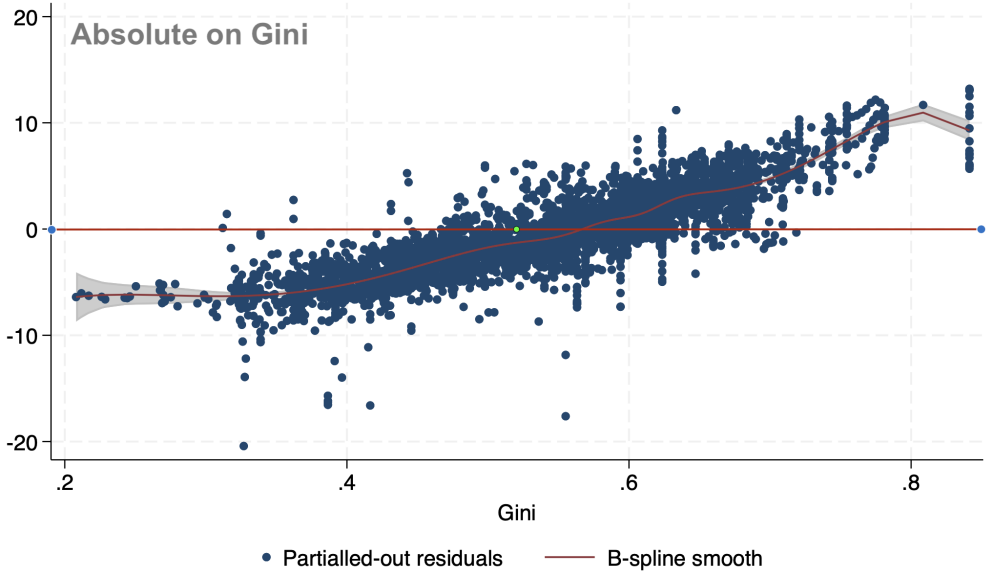
	Upward Mobility[10]					
	Relative			Absolute		
Controls	Gini	SE	R^2	Gini	SE	R^2
None	21.88	(2.76)	0.31	21.46	(6.46)	0.32
+ Income p.c.	21.87	(2.77)	0.31	25.08	(4.67)	0.40
+ Polity2, Gov Exp, Educ	23.30	(3.35)	0.31	29.21	(4.59)	0.51
+ Demographics	30.66	(3.40)	0.42	35.22	(4.32)	0.56
+ Macro Vars	31.62	(4.33)	0.47	36.95	(5.22)	0.54
+ Lagged Mobility	31.75	(4.79)	0.47	35.14	(6.29)	0.54

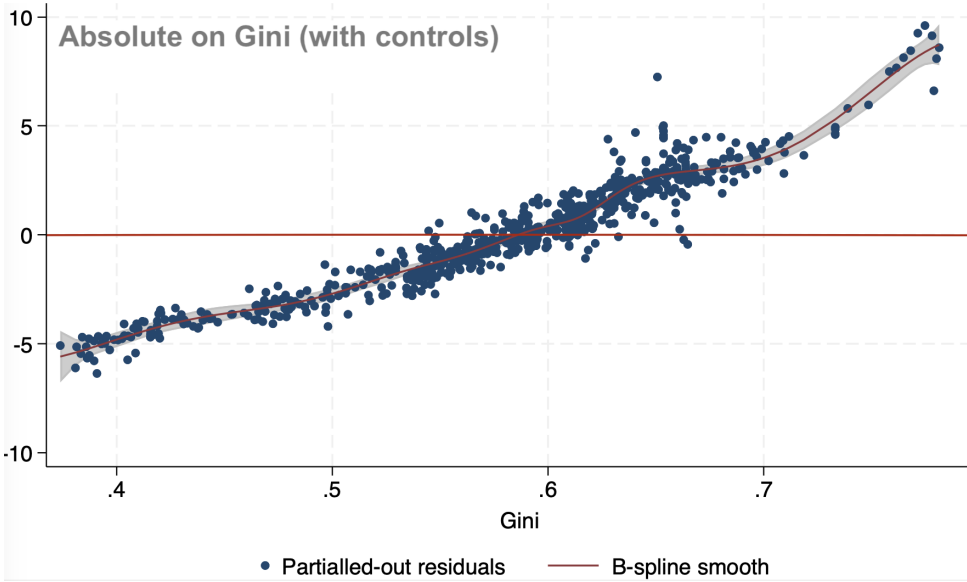
Coefficients on Gini_s with progressively added controls. Country + region-year fixed effects throughout. Robust standard errors clustered at the country level.

Semiparametric Estimation: $\Psi(\text{Ineq})$ Instead of $\gamma * \text{Ineq}$









Robustness Checks

- To averaging incomes over 3 years ▸ Average
- To using wealth data from WID ▸ Wealth
- To different values of the pro-poorness parameter α ▸ α
- To longer horizons ▸ 20/30-Year
- To region-specific effects ▸ Region
- To other datasets ▸ PIP ▸ WIID
- To combining datasets (e.g., inequality from WID, mobility from PIP) ▸ WID-PIP

Upward Mobility and Change in Atkinson Welfare

- Recall that **absolute upward mobility** is given by:

$$M^{\alpha}(\vec{\mathbf{y}}[s,t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{i=1}^{n} y_i^{-\alpha}(t)}{\sum_{i=1}^{n} y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \quad \text{for some } \alpha > 0,$$

- Compare to the famous **Atkinson welfare function**:

$$A^{\alpha}(\mathbf{y}) \equiv \left(\frac{1}{n} \sum_{j=1}^{n} y_j^{-\alpha} \right)^{-\frac{1}{\alpha}} \quad \text{for } \alpha > -1.$$

- What do you see?

- $M^{\alpha}(\vec{\mathbf{y}}[s,t]) =$ average growth of Atkinson welfare on $[s,t]$.

Upward Mobility and Change in Atkinson Welfare

- Likewise, recall that **relative upward mobility** is given by:

$$R^{\alpha}(\vec{\mathbf{y}}[s,t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{n} e_j(t)^{-\alpha}}{\sum_{j=1}^{n} e_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}} \quad \text{for some } \alpha > 0,$$

where $e_i = y_i/\bar{y}$ is income relative to the mean.

- Compare to the **Atkinson equality index**:

$$E^{\alpha}(\mathbf{y}) \equiv A^{\alpha}(\mathbf{y})/\bar{y} = \left(\frac{1}{n} \sum_{j=1}^{n} e_j^{-\alpha} \right)^{-\frac{1}{\alpha}} \quad \text{for } \alpha > -1.$$

- $R^{\alpha}(\vec{\mathbf{y}}[s,t])$ = average growth of Atkinson equality on $[s,t]$.
- $\approx$ average decline in Atkinson inequality $I^{\alpha}(\mathbf{y}) = 1 - E^{\alpha}(\mathbf{y})$ on $[s,t]$.

Connection to the Great Gatsby Curve

- Recall the IGE:

$$\log y(t+1) = \alpha + \beta \log y(t) + u(t).$$

$$\text{IGE} = \beta = \frac{\text{Cov}(\log y(t), \log y(t+1))}{\sigma^2(\log y(t))} \approx \text{“absence of mobility”}.$$

$$\underbrace{\text{IGE}}_{\text{Expected Advantage}} = \underbrace{\rho}_{\text{Predictability}} \times \underbrace{\frac{\sigma(\log y(t+1))}{\sigma(\log y(t))}}_{\text{Change in Dispersion}},$$

where $\rho = \frac{\text{Cov}(\log y(t), \log y(t+1))}{\sigma(\log y(t))\sigma(\log y(t+1))}$ = correlation coefficient.

Connection to the Great Gatsby Curve

$$\text{IGE} = \underbrace{\rho}_{\text{Persistence}} \times \underbrace{\frac{\sigma(\log y(t+1))}{\sigma(\log y(t))}}_{\text{Change in Dispersion}}.$$

- Our approach uses

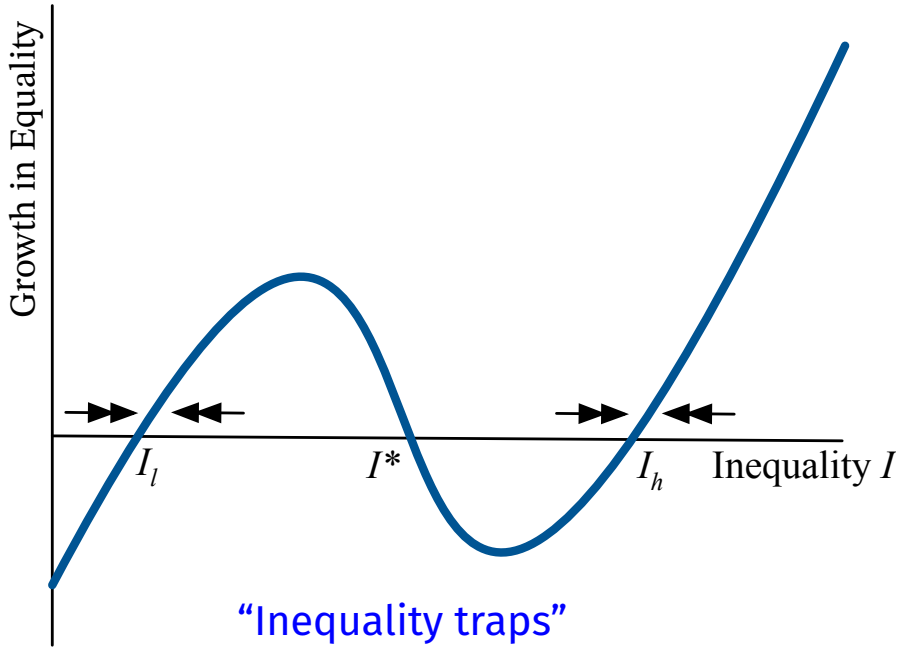
$$R^{\alpha} = \text{Change in Atkinson equality with parameter } \alpha.$$

instead of $\sigma(\log y(t+1))/\sigma(\log y(t))$ but the connection is clear:

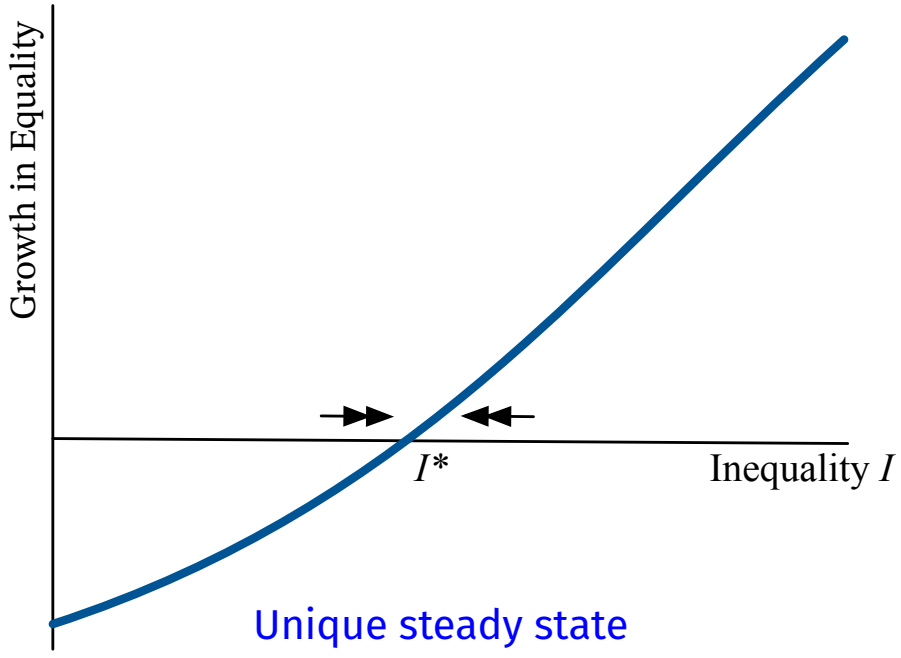
- **IGE goes up** with rising inequality ($\rho \uparrow$), but
- **Upward mobility also goes up** with rising inequality.
- **Implications for ρ?**

The Dynamics of Inequality

- Our equation for relative upward mobility can be written as:
- Growth in [Atkinson] equality $= \Psi\big($Baseline Atkinson Inequality$\big)$,
- so it tells us about the **dynamics of inequality** in society.



"Inequality traps"

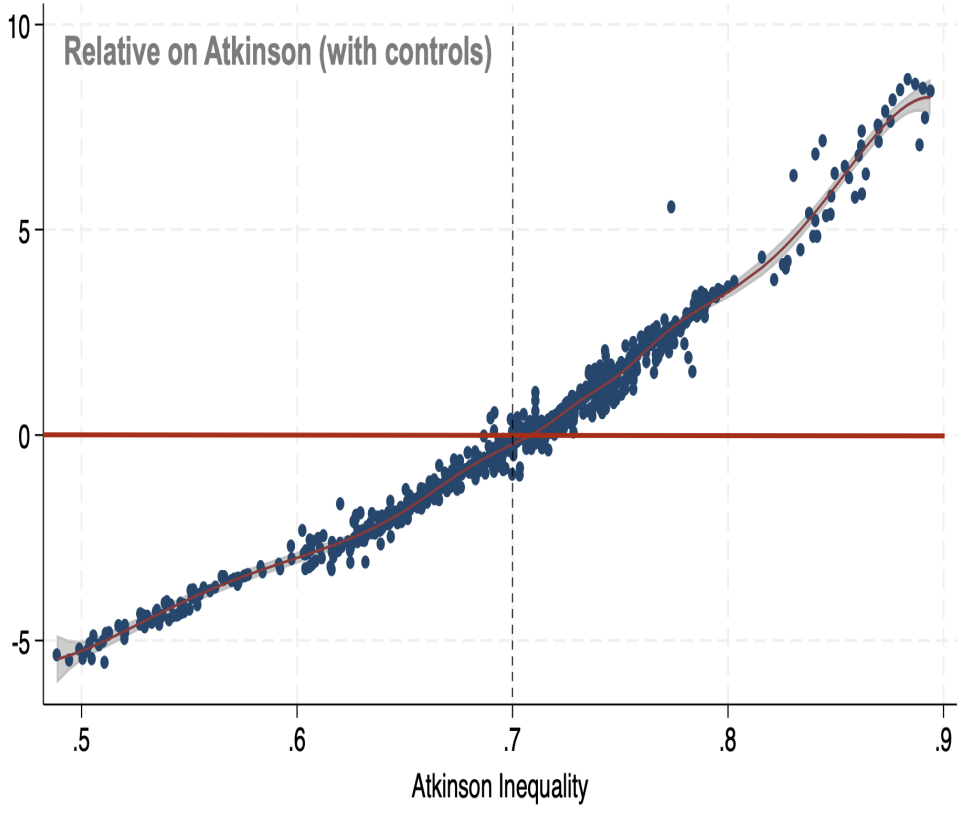


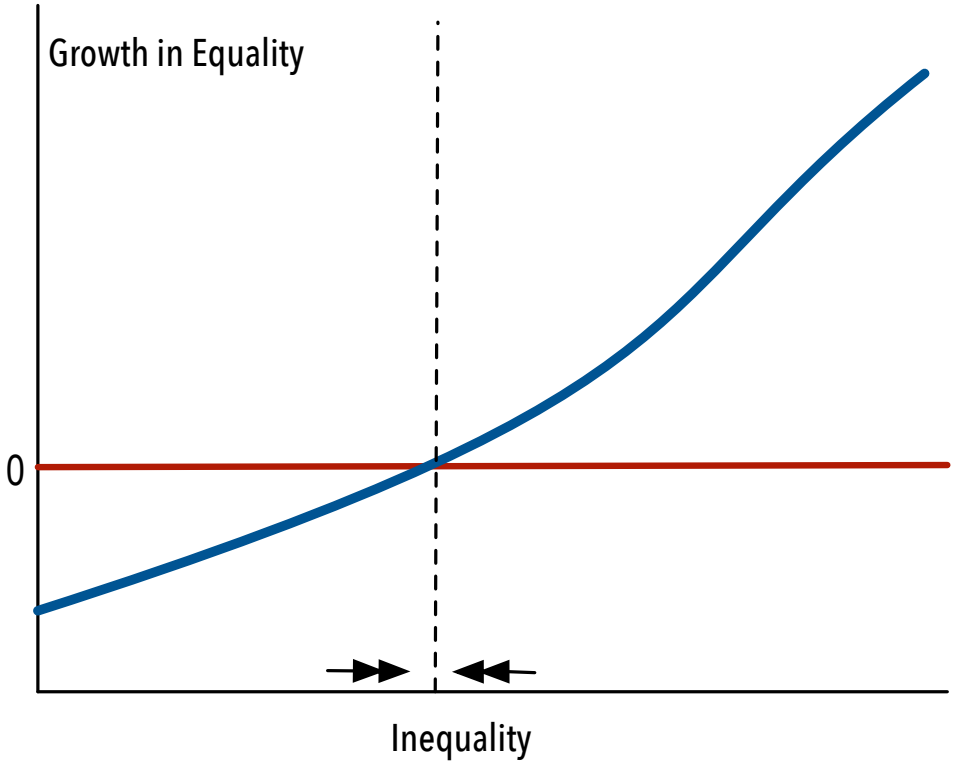
Unique steady state

Which one?

The Dynamics of Inequality

- But relative upward mobility slopes up on Gini or on Atkinson:

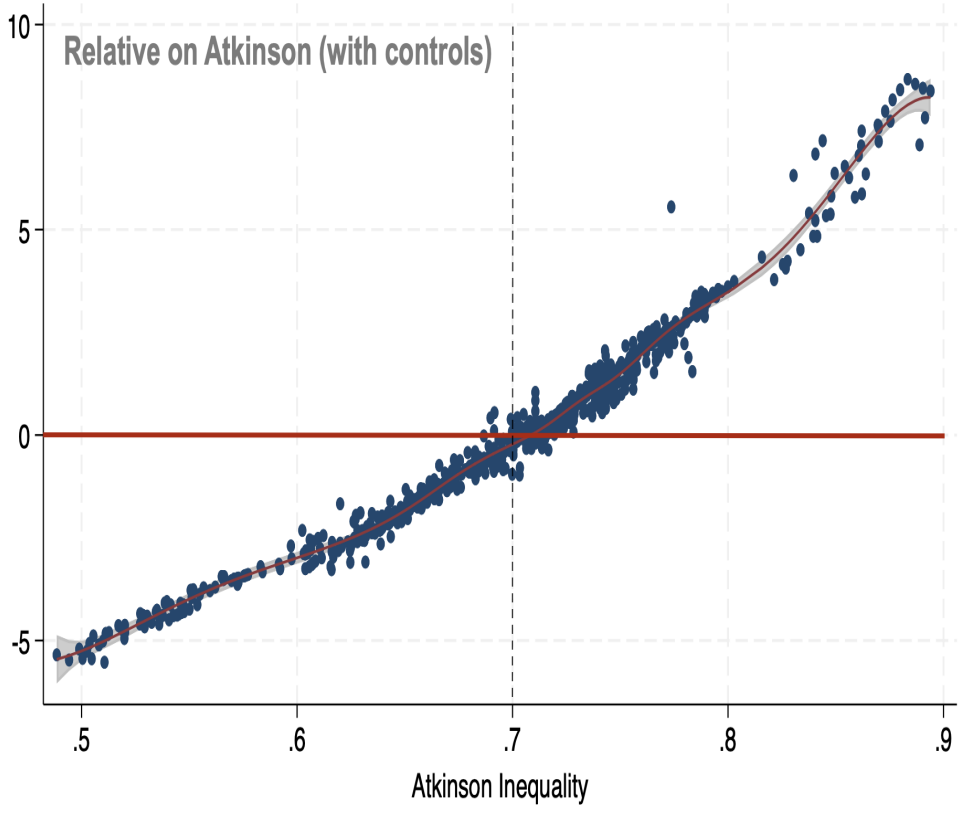


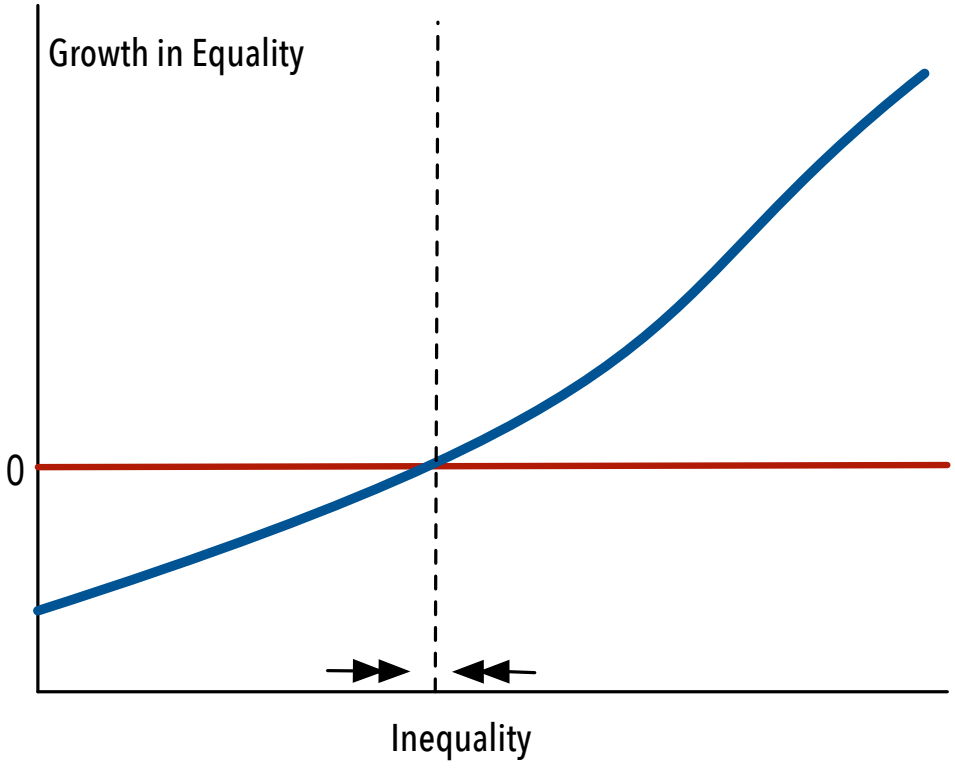


- Comments on **residualization with fixed effects** ▸ Residualization

The Dynamics of Inequality

- But relative upward mobility slopes up on Gini or on Atkinson:

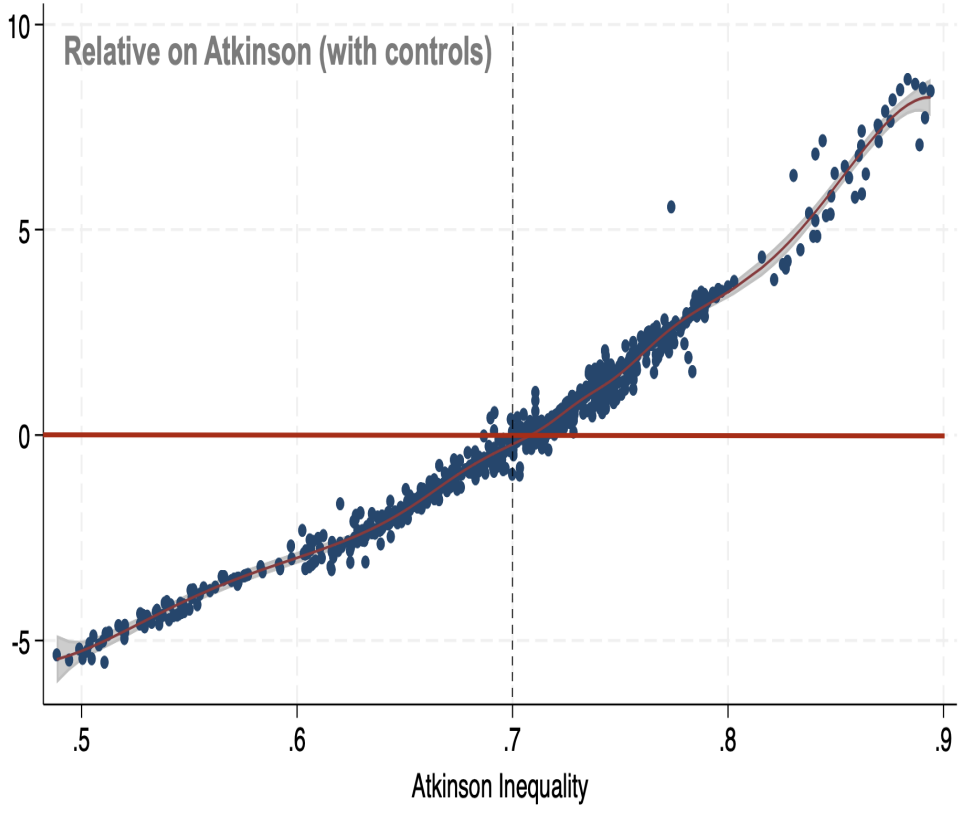


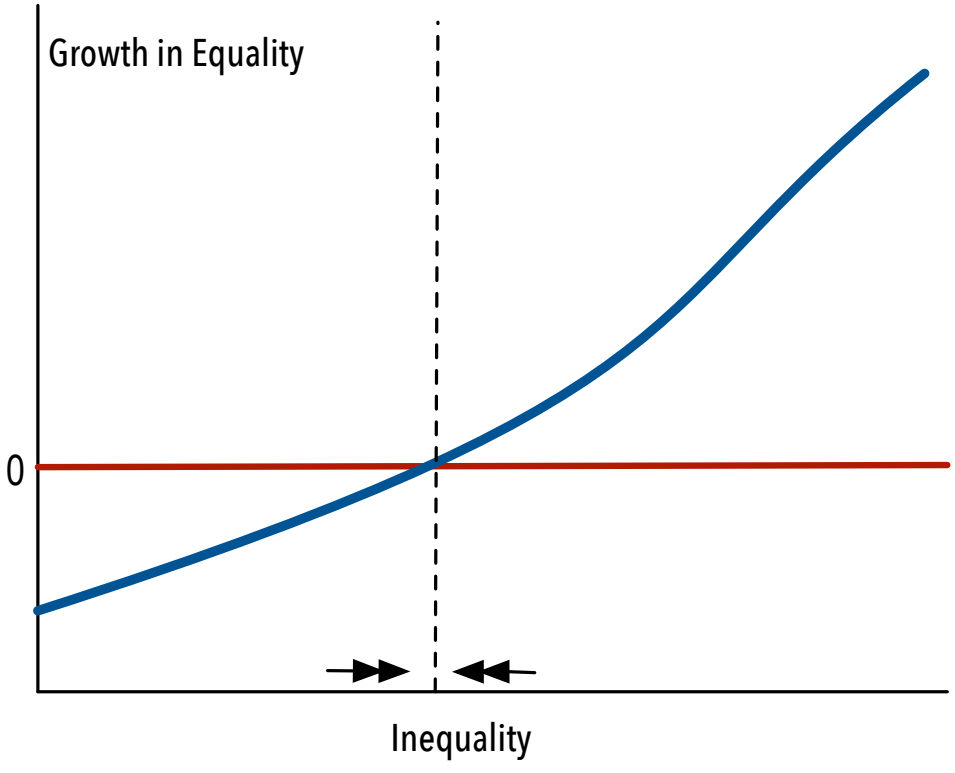


- Comments on **residualization with fixed effects** ▸ Residualization

The Dynamics of Inequality

- But relative upward mobility slopes up on Gini or on Atkinson:





- Comments on **residualization with fixed effects** ▸ Residualization

The Dynamics of Inequality

- As inequality climbs:
- **1.** Persistence sharpens (GGC), but
- **2.** Inequality displays (country-specific) ergodicity.
- **(1)** appears to rule out classical theories of convergence Solow 1956, Brock-Mirman 1972, Becker-Tomes 1979, 1986, Loury 1981… .
- **(2)**, however, appears to also eliminate inequality traps Ray 1990, Banerjee-Newman 1993, Galor-Zeira 1993, Ljungqvist 1993, Freeman 1996, Ghatak-Newman 2025 …
- **Uniqueness without dynastic convergence?** Mookherjee-Ray 2002, 2003, 2010.
- Micro vs macro history-dependence ▸ History

The Dynamics of Inequality

- As inequality climbs:
- **1.** Persistence sharpens (GGC), but
- **2.** Inequality displays (country-specific) ergodicity.
- **(1)** appears to rule out classical theories of convergence Solow 1956, Brock-Mirman 1972, Becker-Tomes 1979, 1986, Loury 1981… .
- **(2)**, however, appears to also eliminate inequality traps Ray 1990, Banerjee-Newman 1993, Galor-Zeira 1993, Ljungqvist 1993, Freeman 1996, Ghatak-Newman 2025 …
- **Uniqueness without dynastic convergence?** Mookherjee-Ray 2002, 2003, 2010.
- Micro vs macro history-dependence ▸ History

The Dynamics of Inequality

- As inequality climbs:
- **1.** Persistence sharpens (GGC), but
- **2.** Inequality displays (country-specific) ergodicity.
- **(1)** appears to rule out classical theories of convergence Solow 1956, Brock-Mirman 1972, Becker-Tomes 1979, 1986, Loury 1981… .
- **(2)**, however, appears to also eliminate inequality traps Ray 1990, Banerjee-Newman 1993, Galor-Zeira 1993, Ljungqvist 1993, Freeman 1996, Ghatak-Newman 2025 …
- **Uniqueness without dynastic convergence?** Mookherjee-Ray 2002, 2003, 2010.
- Micro vs macro history-dependence ▸ History

Epilogue: Ergodic Inequality vs Growing Inequality

- Many authors track the alarming rise of inequality in the US, Europe and Asia.
- In that light, our convergence-to-limit-inequality finding might look . . . weird.
- On the contrary, the two observations are in good agreement.
- Recall structural specification for relative upward mobility:

$$\text{Relative Upward Mobility}_{js}^{\Delta} = \gamma * I_{js} + f_j + h_s + \epsilon_{js}$$

Epilogue: Ergodic Inequality vs Growing Inequality

- Many authors track the alarming rise of inequality in the US, Europe and Asia.
- In that light, our convergence-to-limit-inequality finding might look . . . weird.
- On the contrary, the two observations are in good agreement.
- Recall structural specification for relative upward mobility:

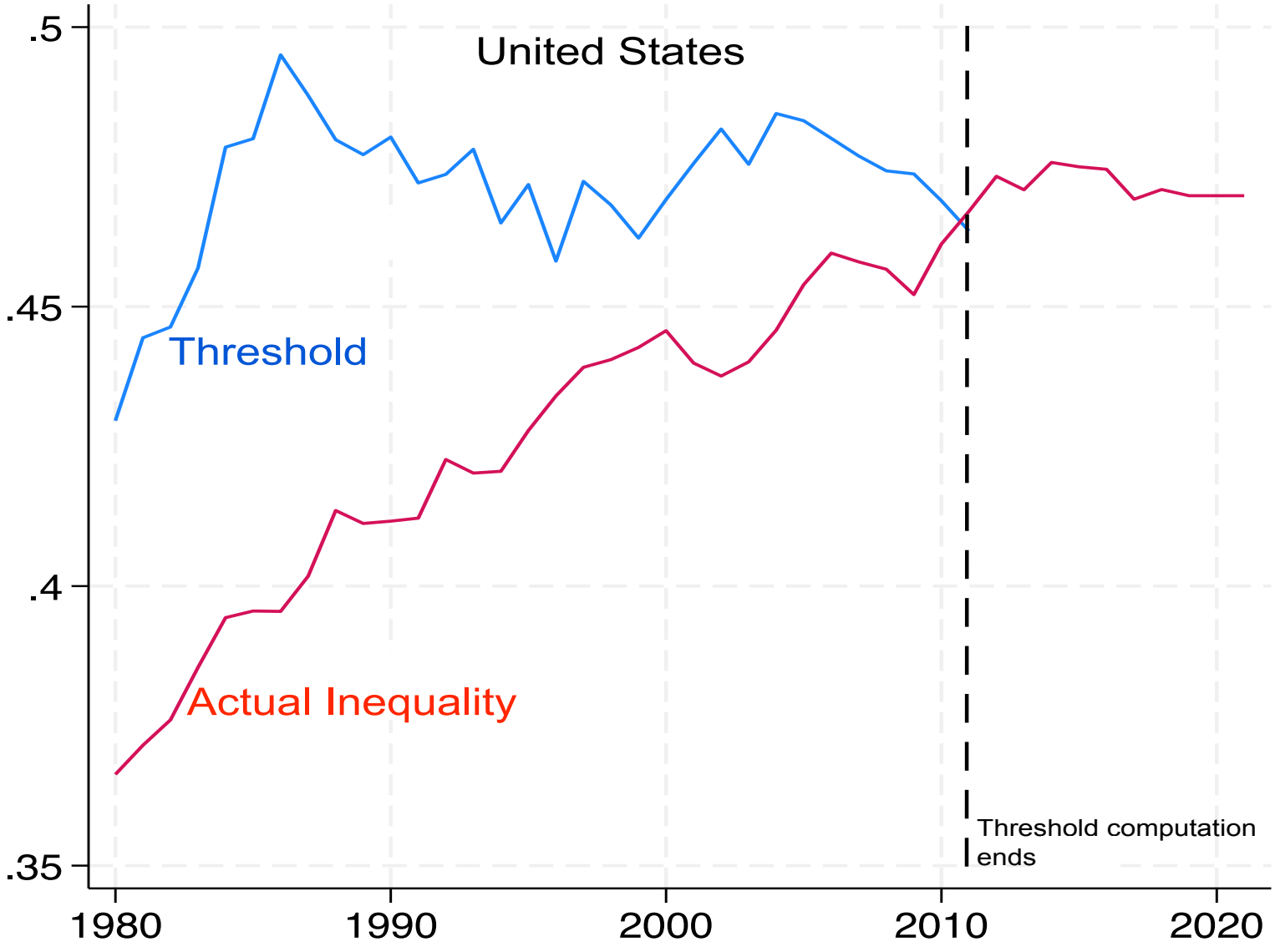
$$-\Delta I_{js} = \gamma * I_{js} + f_j + h_s + \epsilon_{js}$$

- At the same time, $-\Delta I_{js} = \gamma \left[I_{js} - \bar{I}_{js} \right] + \epsilon_{js}$.
- Therefore:

$$\bar{I}_{js} \equiv -\frac{f_j + h_s}{\gamma} = -\frac{\text{country fixed effect + region-year fixed effect}}{\gamma}.$$

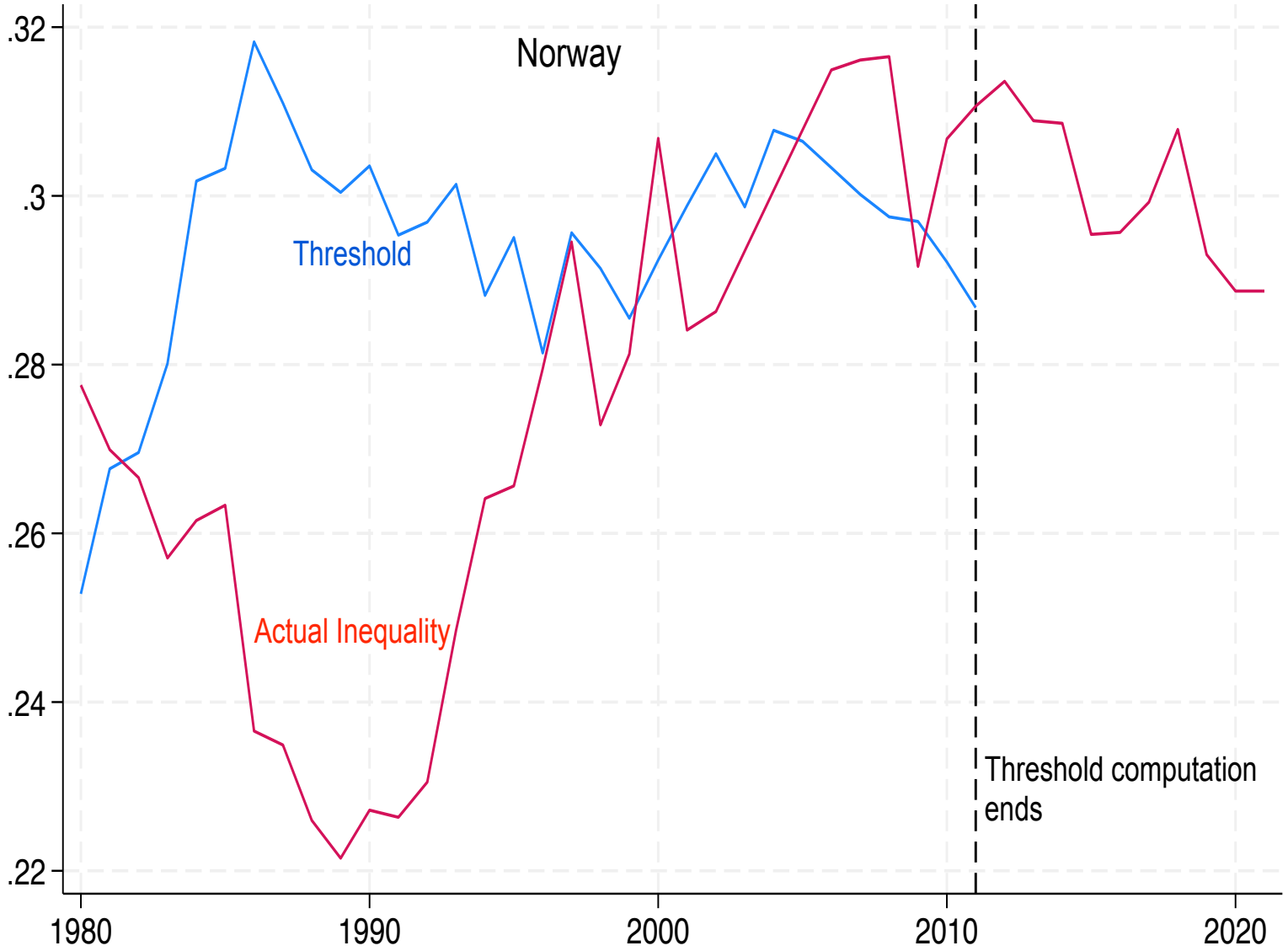
- Let's look at **some of these thresholds**.

Epilogue: Ergodic Inequality vs Growing Inequality



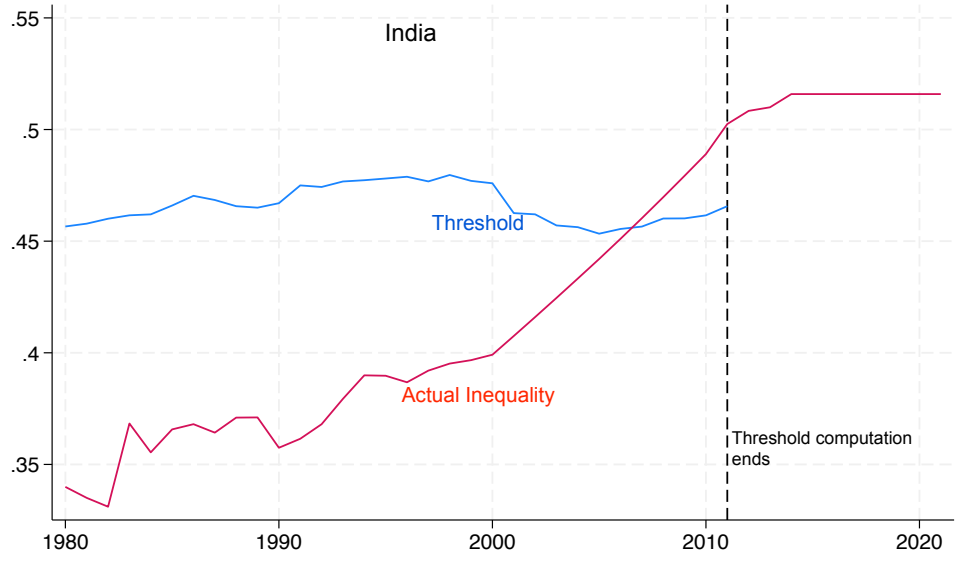
- The US is below its estimated steady state throughout …

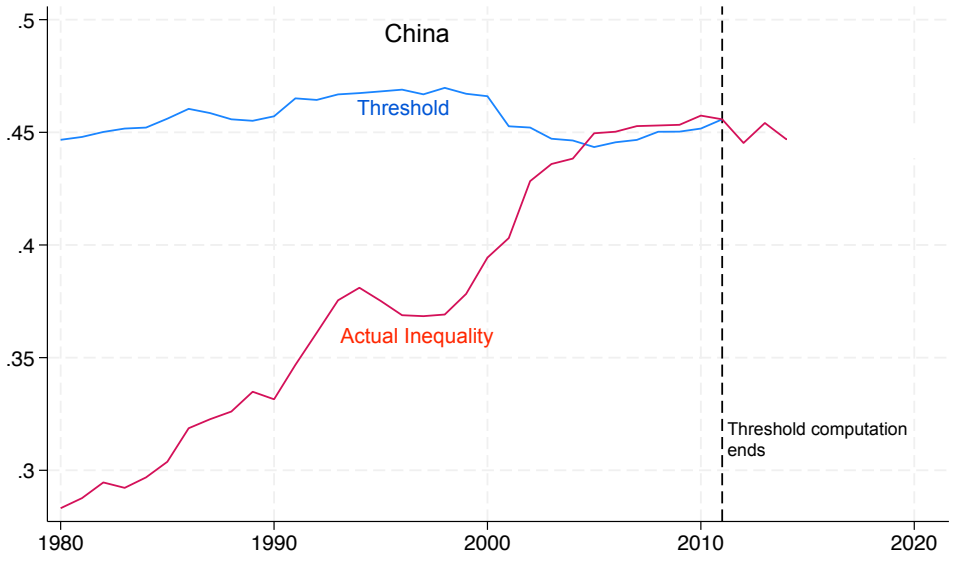
Epilogue: Ergodic Inequality vs Growing Inequality

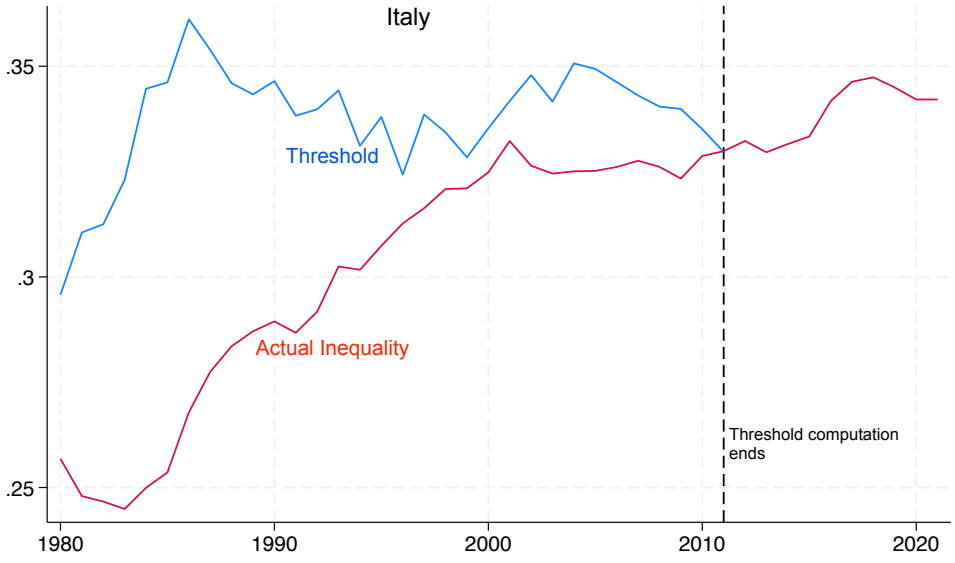


- Norway does the zig-zag.

Epilogue: Ergodic Inequality vs Growing Inequality

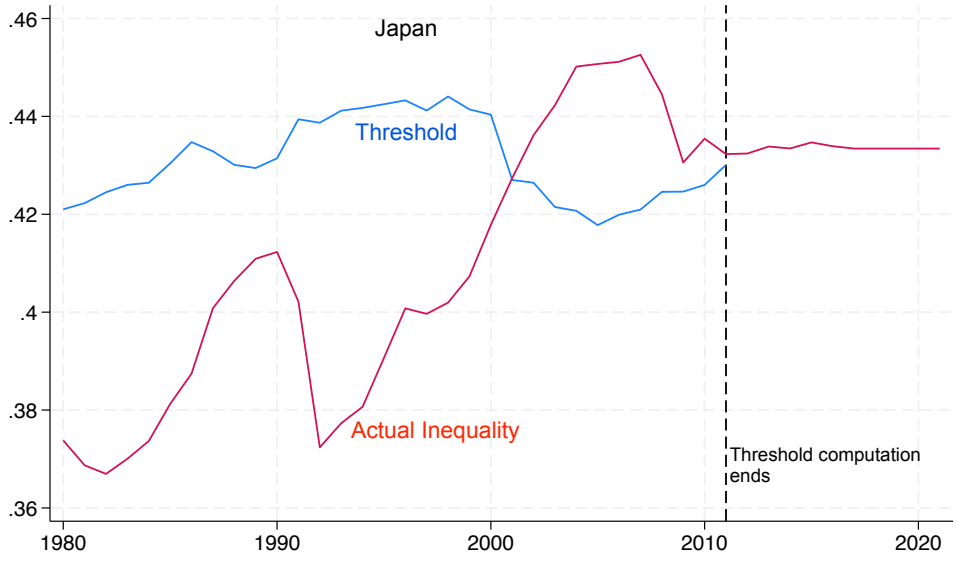


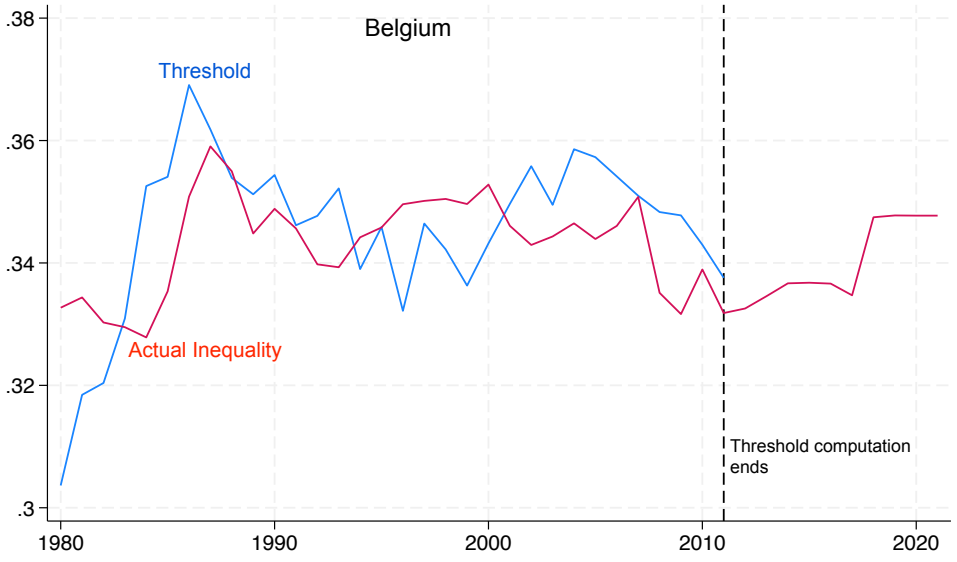




More US-like examples

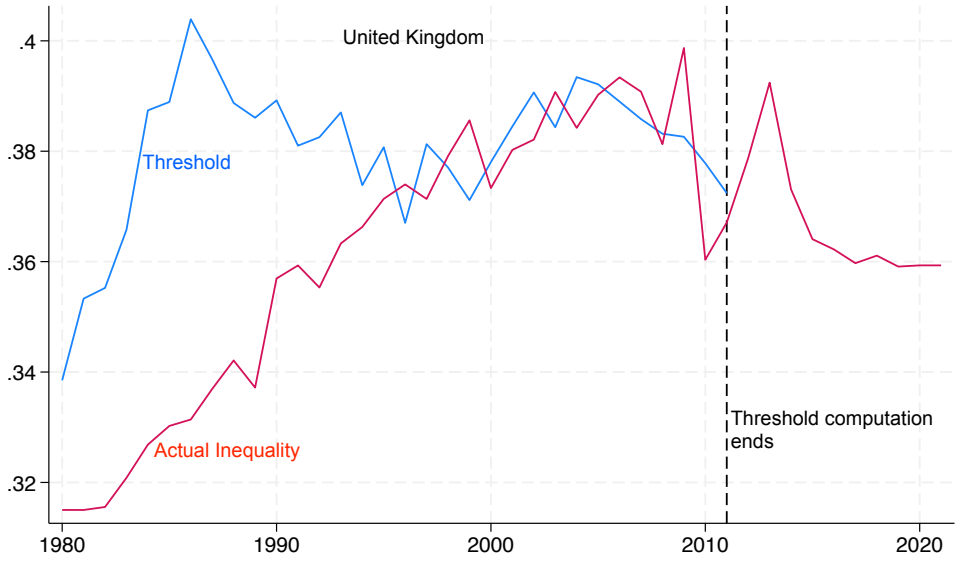
Epilogue: Ergodic Inequality vs Growing Inequality

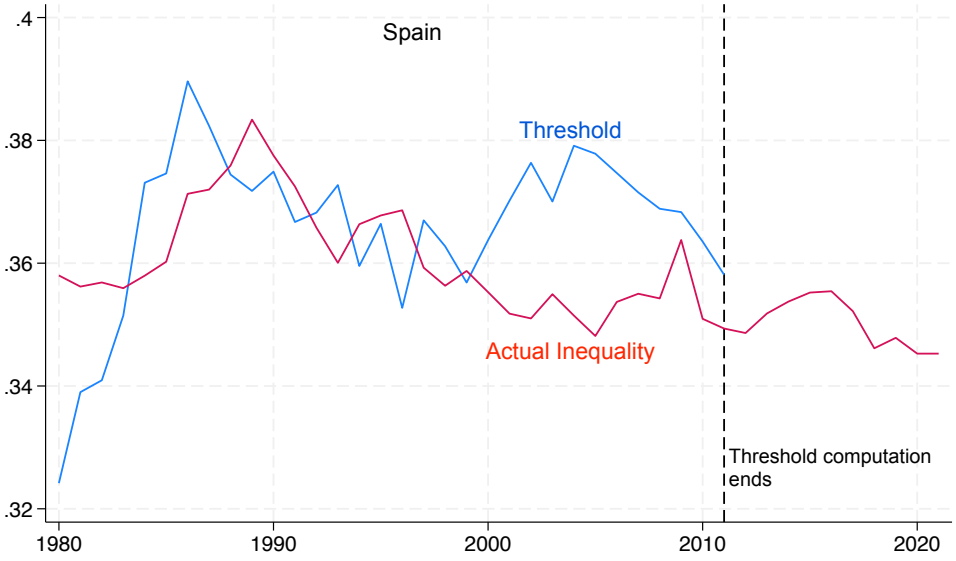


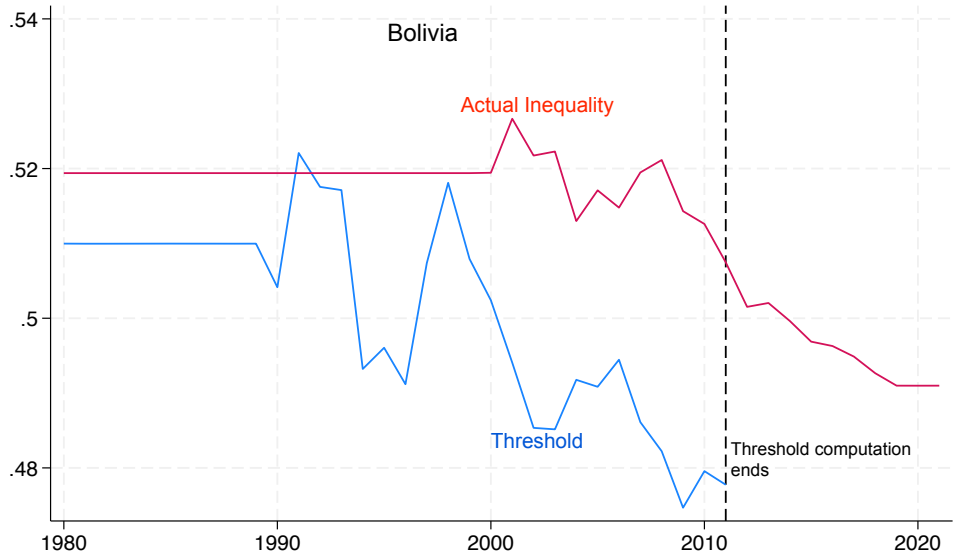


More nuanced examples

Epilogue: Ergodic Inequality vs Growing Inequality







More nuanced examples

Summary

- Recall our questions:

1. Advantage:

- Does inequality strengthen expected advantage?

2. Movement:

- Does inequality sharpen persistence?

3. Progressivity:

- Does inequality affect growth progressivity?

- The GGC studied (1). We studied (3). The results have opposite sign.

- Our exercise suggests a unique (country-specific) steady state for inequality.

APPENDIX

Panel Unit Root Tests

Back

To assess the stationarity of the main variables, we apply:

- **Levin-Lin-Chu (2002) test:** Null hypothesis is the presence of a unit root.
  - Test assumes a common autoregressive root, but:
  - Allows for unequal lag lengths to correct for autocorrelation in residuals.
- **Choi (2001) Fisher-type tests:**
  - Combine p-values from unit root tests on country-specific series to produce an overall test statistic.
- Presence of unit root in all variables decisively rejected.

Panel Unit Root Tests

Back

	Panel Unit Root Tests			
	Adjusted LLC t-stat	LLC p-value	Fisher t-stat	Fisher p-value
Atkinson welfare	-3.70	0.000	654.10	0.00
Atkinson relative welfare	-4.43	0.000	711.85	0.00
Relative mobility	-5.79	0.000	846.51	0.00
Absolute mobility	-4.30	0.000	730.27	0.00
Atkinson inequality	-5.13	0.000	730.27	0.00
Gini	-7.16	0.000	746.92	0.00
Log income	-4.78	0.000	691.50	0.00

Upward Mobility and Inequality (10-yr Horizon, Average Data)

	Upward Mobility[10]			
	Relative		Absolute	
	[1]	[2]	[3]	[4]
Gini	***22.32	***31.70	***25.89	***37.35
	(2.78)	(4.61)	(4.94)	(5.48)
Log income pc	-0.04	-0.82	***-8.15	***-5.97
	(0.25)	(0.72)	(0.51)	(1.067)
Controls	No	Yes	No	Yes
R^2	0.34	0.47	0.64	0.54
Obs	3898	1430	3898	1430

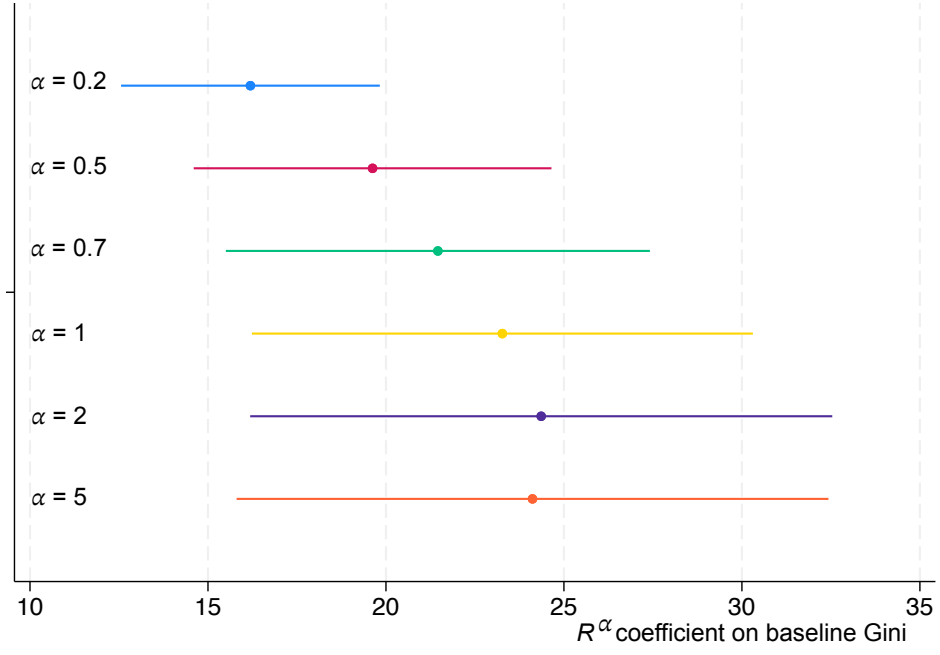
All variables are averaged over $s-1$, s, and $s+1$. Regressions include country and region-year fixed effects. Robust standard errors clustered at the country level. back

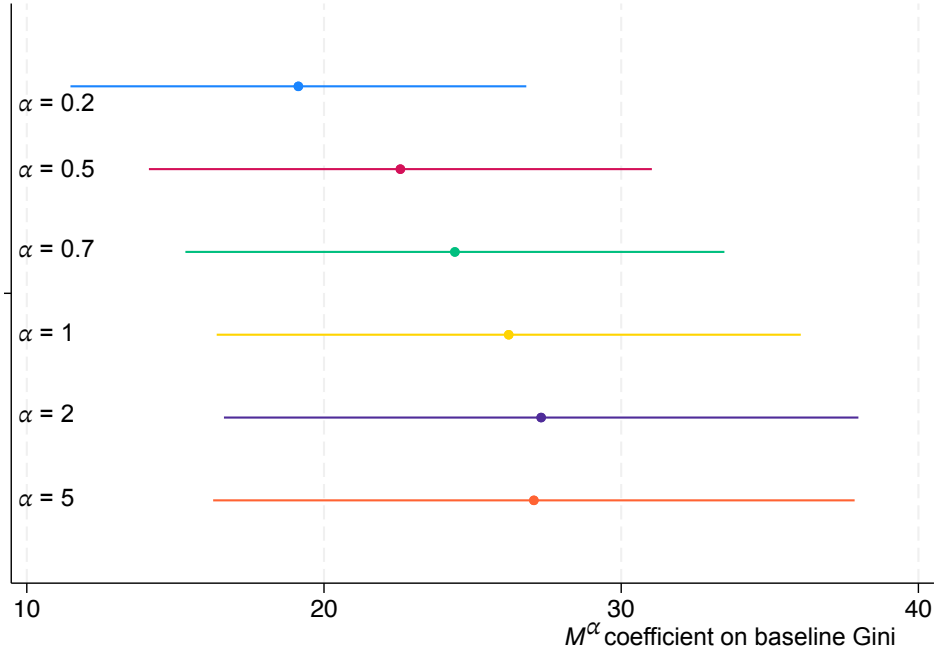
Upward Mobility and Inequality (10-yr Horizon, Wealth)

	Relative Upward Mobility				Absolute Upward Mobility			
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
$Gini_s$	***22.747	***33.912			***26.581	***40.252		
	(2.922)	(4.907)			(4.669)	(5.864)		
Atk. $Ineq_s$			***22.877	***29.717			***25.847	***35.364
			(2.407)	(3.535)			(3.774)	(4.716)
Log wealth $p.c._s$	0.028	-0.389	-0.177	-0.589	***-7.786	*** -6.706	***-8.012	***-6.947
	(0.242)	(0.535)	(0.207)	(0.613)	(0.478)	(0.855)	(0.498)	(0.840)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
R^2	0.325	0.484	0.418	0.523	0.589	0.505	0.607	0.525
Obs	4763	1667	4763	1667	4763	1667	4763	1667

The use of wealth instead of incomes from WID. Regressions include country and region-year fixed effects. Robust standard errors clustered at the country level. back

Robustness to different values of α





RELATIVE UPWARD MOBILITY

ABSOLUTE UPWARD MOBILITY

back

Upward Mobility and Inequality (20/30-Year Horizon)

	Relative Upward Mobility				Absolute Upward Mobility			
	$\Delta = 20$		$\Delta = 30$		$\Delta = 20$		$\Delta = 30$	
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Gini	***17.81	***19.74			***20.47	***23.83		
	(1.68)	(1.53)			(1.90)	(2.09)		
Gini			***9.09	***10.78			***11.15	***10.07
			(1.16)	(0.62)			(1.55)	(2.72)
Log p.c. y	0.07	0.33			***-5.35	***-3.49		
	(0.13)	(0.31)			(0.24)	(0.58)		
Log Income p.c. y			0.10	-0.03			***-3.53	***-3.03
			(0.15)	(0.14)			(0.22)	(0.38)
Controls	No	Yes	No	Yes	No	Yes	No	Yes
R^2	0.49	0.75	0.23	0.92	0.78	0.66	0.57	0.90
Obs	2858	693	1558	105	2858	693	1558	105

Controls include: Polity2, government expenditure, capital formation, population growth and structure, education levels, labor force participation, inflation, and credit to the financial sector. Robust standard errors clustered at the country level. All regressions include country fixed effects

Upward Mobility and Inequality: Regional Heterogeneity

	Relative Upward Mobility		Absolute Upward Mobility	
	[1]	[2]	[3]	[4]
Gini× Africa	***22.65	***31.78	***25.39	***36.30
	(4.44)	(5.75)	(5.91)	(6.88)
Gini× Asia	***11.88	*11.85	**19.30	9.39
	(4.46)	(6.75)	(9.01)	(8.36)
Gini× Latin America	8.81	***37.89	17.88	***48.18
	(15.14)	(11.24)	(16.29)	(14.17)
Gini× Neo Europe	***20.01	***20.52	***24.10	**14.82
	(4.08)	(2.83)	(7.73)	(5.89)
Gini× Middle East	***16.20	***35.57	-7.88	**25.98
	(4.72)	(6.38)	(14.06)	(11.17)
Controls	No	Yes	No	Yes
R^2	0.30	0.49	0.60	0.45
Observations	4,158	1,430	4,158	1,430

Robust standard errors clustered at the country level. All regressions include country and region-year fixed effects; columns (2) and (4) also include controls.

back

Upward Mobility and Inequality: PIP ($\Delta = 10$)

	Relative Upward Mobility [1-5]					Absolute Upward Mobility [6-10]				
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
Gini	***19.93	***19.72	***24.69				28.30	***27.91		
	(2.22)	(2.16)	(3.50)				(5.03)	(6.62)		
Atk. Ineq				***15.19	***15.18	***16.60			***21.38	18.21
				(1.52)	(1.54)	(1.55)			(2.70)	(2.83)
log p.c.y		-0.01	-0.37		0.04	0.34	***-8.91	***-10.00	***-8.83	***-9.19
		(0.30)	(0.74)		(0.24)	(0.39)	(0.61)	(1.09)	(0.49)	(0.77)
Controls	No	No	Yes	No	No	Yes	No	Yes	No	Yes
R^2	0.55	0.54	0.78	0.70	0.70	0.87	0.66	0.82	0.71	0.86
Obs	1238	872	442	872	872	442	872	442	872	442

Robust standard errors clustered at the country level are reported in parentheses. All regressions include country and region-year fixed effects.

back

Upward Mobility and Inequality: WIID ($\Delta = 10$)

	Relative Upward Mobility [1-5]					Absolute Upward Mobility [6-10]				
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
Gini	***19.56	***20.65	***31.62				***30.09	***43.30		
	(3.38)	(4.23)	(6.67)				(5.68)	(9.61)		
Atk. Ineq				***16.89	***17.05	***21.55			***22.99	***28.33
				(2.84)	(2.84)	(2.99)			(3.28)	(4.16)
log p.c.y		**0.87	-0.33		**0.83	0.17	***-5.84	**-7.39	***-5.94	**-6.75
		(0.37)	(1.33)		(0.36)	(1.26)	(0.97)	(2.93)	(1.00)	(2.85)
Controls	No	No	Yes	No	No	Yes	No	Yes	No	Yes
R^2	0.46	0.50	0.69	0.62	0.63	0.79	0.61	0.67	0.65	0.71
Obs	1373	868	353	868	868	353	868	353	868	353

Robust standard errors clustered at the country level are reported in parentheses. All regressions include country and region-year fixed effects.

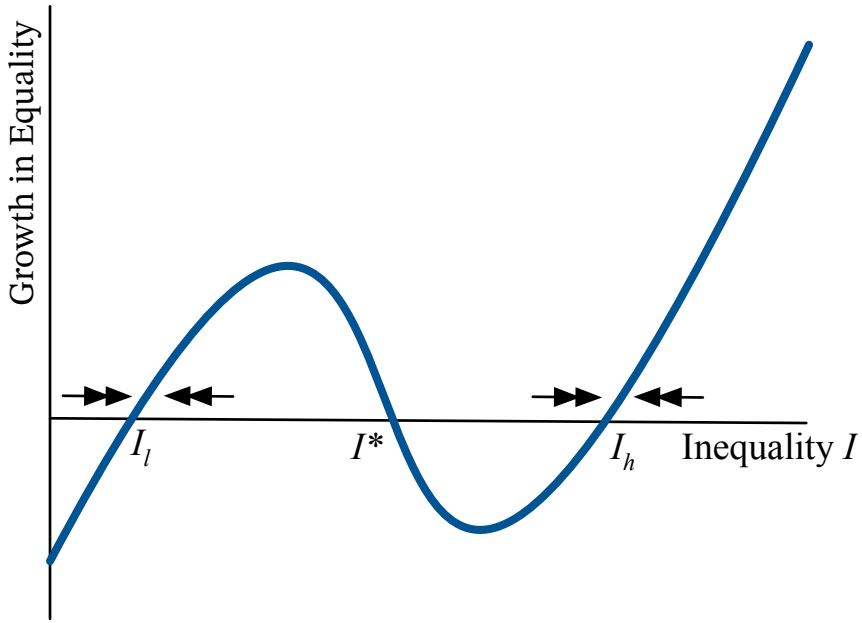
Back

Upward Mobility and Inequality: WID Meets PIP ($\Delta = 10$)

	Relative Upward Mobility [1-5]					Absolute Upward Mobility [6-9]			
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
Gini (WID)	**6.533*** **	**7.246*** **	**8.707*** **	**7.848*** **	**8.353*** **	**20.796*** **	**19.882*** **	**15.171** **	**15.503*** **
	(2.368)	(2.324)	(2.785)	(2.864)	(2.816)	(7.628)	(6.756)	(5.760)	(5.685)
log income p.c.		0.419	0.365	0.331	0.577	-8.186***	-8.175***	-7.702***	-8.081***
		(0.344)	(0.391)	(0.527)	(0.475)	(0.660)	(0.763)	(0.971)	(0.932)
Polity 2			-0.007	-0.045	-0.036		0.027	-0.057	-0.024
			(0.026)	(0.037)	(0.037)		(0.112)	(0.060)	(0.059)
Gov. exp/gdp			-0.007	-0.010	-0.012		-0.003	-0.019	-0.015
			(0.010)	(0.011)	(0.010)		(0.015)	(0.013)	(0.015)
Gross Cap. Formation (% GDP)				0.007	0.007			0.003	0.001
				(0.022)	(0.023)			(0.034)	(0.035)
Pop. Growth				-0.085	-0.120			-0.384	-0.216
				(0.160)	(0.145)			(0.383)	(0.302)
% Pop at most primary				0.036	0.040			0.072	0.080
				(0.036)	(0.040)			(0.049)	(0.054)
% Pop at most secondary				0.022	0.025			0.071**	0.067*
				(0.024)	(0.029)			(0.034)	(0.038)
Labor force part.					-0.061				-0.131*
					(0.048)				(0.072)
% Pop. below 14					0.125				-0.209
					(0.129)				(0.269)
% Pop. between (15-64)					0.034				-0.162
					(0.135)				(0.311)

Residualization and Multiple Steady States

Back



Residualization and Multiple Steady States

Back