

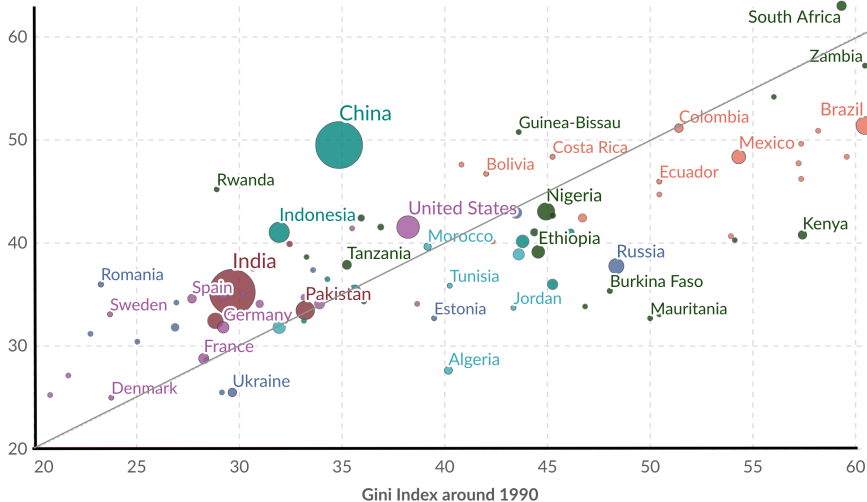
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- [Slides 4](#): Inequality and Mobility, Part 1

“The Defining Issue of our Time” (Obama 2013)

■ Inequality 1990-2015:

Gini Index around 2015



Sources: Povcal (2018). The Chartbook of Economic Inequality (2017). Kanbur et al (2017) Table 1.B

Another Defining Issue of our Time

■ **Mobility:**

- Centrally important in current (?) debates:

- **Intrinsically:** E.g., United States vs Europe

Chetty et al (2017), Alesina et al (2018), Manduca et al (2020)

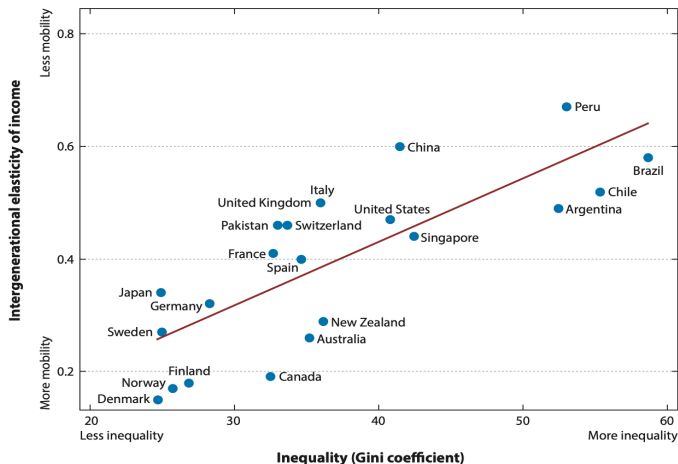
- **Instrumentally:** Connections to growth and development

Krueger (2012), Genicot and Ray (2017, 2020), Narayan (2018)

- A particular connection: **mobility and baseline inequality.**

The Relationship between Inequality and Mobility

■ The Great Gatsby Curve Krueger (2012), Corak (2013)



Source: Durlauf-Kourtellos-Tan (2022), based on Corak's data.

See also: Durlauf et al. (2021), Van der Weide et al (2024, 2025), Fan et al. (2021) for China; Guell et al. (2015) for Italy; Branden (2019) for Sweden, Chetty et al. (2014a) for the US.

The Relationship between Inequality and Mobility

- Mobility is a **multifaceted notion**:
 - We often take measures off the shelf
 - But the conceptual foundations do matter.
- At least **three distinct questions** about the inequality-mobility relationship
 - Each question calls for a different measure.

The Relationship between Inequality and Mobility

1. Advantage or Monotonicity:

- Does inequality strengthen the expected advantage that richer parents pass to their children?

2. Predictability or Movement:

- Does inequality sharpen the persistence across parental and offspring positions on the economic spectrum?

3. Growth Progressivity:

- Does inequality affect the extent to which economic gains are tilted toward those who start poorer?
- These are related but not identical notions.

The (Im)Mobility Measure in the Great Gatsby Curve


- The intergenerational elasticity of earnings (IGE):

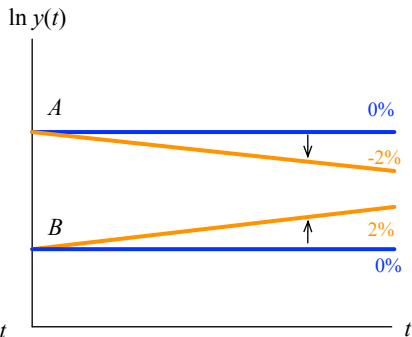
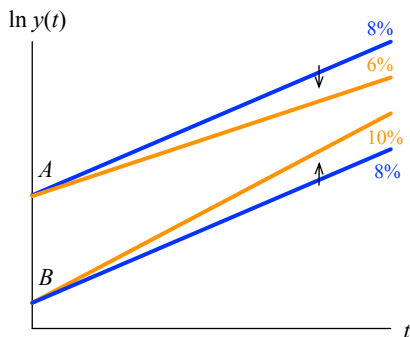
$$\log y(t+1) = \alpha + \beta \log y(t) + u(t).$$

$$\text{IGE} = \beta = \frac{\text{Cov}(\log y(t), \log y(t+1))}{\sigma^2(\log y(t))} \approx \text{“absence of mobility”}.$$

- The IGE is a predictor of **expected advantage**.
- Is it also a good measure of non-progressivity? We will come back to this.

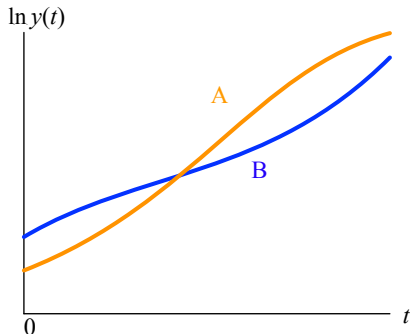
Upward Mobility

- **Upward mobility**: a measure that rewards **progressivity** [Ray  Genicot 2023]:
- Upward mobility is higher if the relatively poor grow faster.



Upward Mobility

- Be careful with crossings, though:



- A was poorer than B, but is now richer
- Do we want to value A's growth more, once she's richer? (No, we don't.)

Snapshots and Trajectories

- Given the possibility of crossings, our approach is in two parts:
 - A “snapshot” or **upward mobility kernel** m ;
intermediate step
 - A **measure on trajectories** M built from these kernels.
what we're after

Snapshots

- Core variable y_i : permanent income (or wealth or consumption).
- $g_i = \dot{y}_i/y_i$ instantaneous growth rate.
- **Upward mobility kernel:** $m(\mathbf{y}, \mathbf{g})$.
- Anonymous, continuous.
- Zero-growth normalization:
 $g_i = 0$ all $i \mapsto m(\mathbf{y}, \mathbf{g}) = 0$.
- Consistency under population mergers.

■ Growth Progressivity

- For $y_i < y_j$, send g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $m(\mathbf{y}, \mathbf{g}) \uparrow$.

■ Notes:

- Measure tolerates lower growth if poor can grow faster.
- Upward mobility \neq overall welfare.

Theorem 1

An upward mobility kernel is growth progressive if and only if it can be written as

$$m(\mathbf{y}, \mathbf{g}) = \sum_{i=1}^n \phi_i(\mathbf{y}) g_i$$

for continuous permutation-invariant $\{\phi_i\}$, with $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$ when $y_i < y_j$.

Sharpening the Kernel

- **Income Neutrality.** $m(\mathbf{y}, \mathbf{g}) = m(\lambda\mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.
- **Growth Alignment.** $\mathbf{g} > \mathbf{g}' \Rightarrow m(\mathbf{y}, \mathbf{g}) > m(\mathbf{y}, \mathbf{g}')$ all \mathbf{y} .
- **Independent Pairwise Growth Tradeoffs:**

Is $m((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij})) \geq m((y_i, g'_i), (y_j, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$?

Answer insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$.

Theorem 2

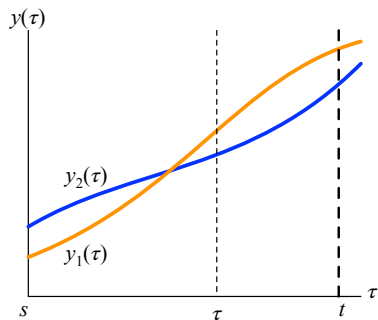
Under additional three axioms and $n \geq 3$, m can be written as:

$$m^\alpha(\mathbf{y}, \mathbf{g}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}}, \text{ for some } \alpha > 0.$$

- Proof employs a substantial extension of Gorman's separability theorem;

see Chatterjee (R) Ray (R) Sen (2021).

Income Trajectories $\vec{y}[s, t]$



- $(\mathbf{y}(\tau), \mathbf{g}(\tau))$ and $m(\mathbf{y}(\tau), \mathbf{g}(\tau))$ well-defined for each $\tau \in [s, t]$.
- $\mathbf{y}[s, t] = \{y_i(\tau)_s^t\}_{i=1}^n$

Extension Axioms

- **Upward mobility measure:** $M(\mathbf{y}[s, t])$.
- Two “extension” axioms:
 1. **Reducibility:** $M(\vec{\mathbf{y}}[s, t]) = \Psi(\{m(\mathbf{y}(\tau), \mathbf{g}(\tau))\}_s^t)$ for some “aggregator” Ψ .
 2. **Averaging:** for every $\tau \in (s, t)$,

$$(t - s)M(\vec{\mathbf{y}}[s, t]) = (\tau - s)M(\vec{\mathbf{y}}[s, \tau]) + (t - \tau)M(\vec{\mathbf{y}}[\tau, t]).$$

Upward Mobility Measure

Theorem 3

Progressivity and the extension axioms hold if and only if

$$M^\alpha(\vec{y}[s, t]) = \frac{1}{t - s} \ln \left[\frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0.$$

■ Remarks:

- The measure is helpfully phrased in “growth units”:
when all growth rates are the same g , $M^\alpha(\vec{y}[s, t]) = g$.
- Can also use income categories and population shares (see paper).

Absolute and Relative

- The formula

$$M^\alpha(\vec{y}[s, t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0$$

measures **absolute** upward mobility.

- **Relative upward mobility** nets out growth:

$$\begin{aligned} R^\alpha(\vec{y}[s, t]) &= M_\alpha(\vec{y}[s, t]) - \frac{1}{t-s} [\ln(\bar{y}(t)) - \ln(\bar{y}(s))] \\ &= \frac{1}{t-s} \ln \left[\frac{\sum_{i=1}^n e_i(t)^{-\alpha}}{\sum_{i=1}^n e_i(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}} \end{aligned}$$

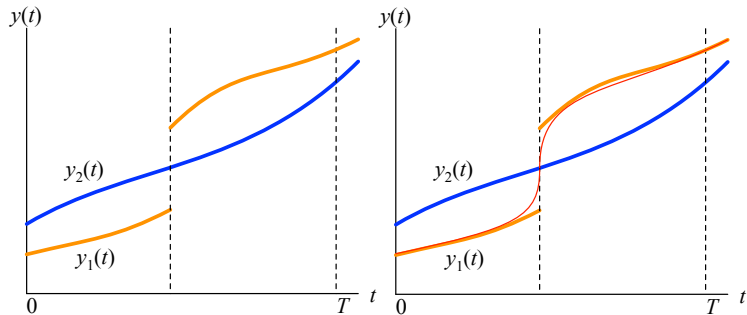
where \bar{y} is per-capita income and $e_i = y_i/\bar{y}$ is relative income

Upward Mobility as Pro-Poor Growth

- **Upward Mobility** = $\frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^n y_j(t)^{-\alpha}}{\sum_{j=1}^m y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$
- **Growth** = $\frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^n y_j(t)}{\sum_{j=1}^m y_j(s)} \right] = M^{(-1)}(\mathbf{y}[s, t])$
- Isn't even on our "boundary" as $\alpha \rightarrow 0$.
- Nevertheless, when all growth rates are the same, $M^\alpha = \text{growth rate}$.

Discontinuous Trajectories

- If there are jumps, then mobility kernels aren't defined at some points.
- Examples: inheritance, job change, promotions ...

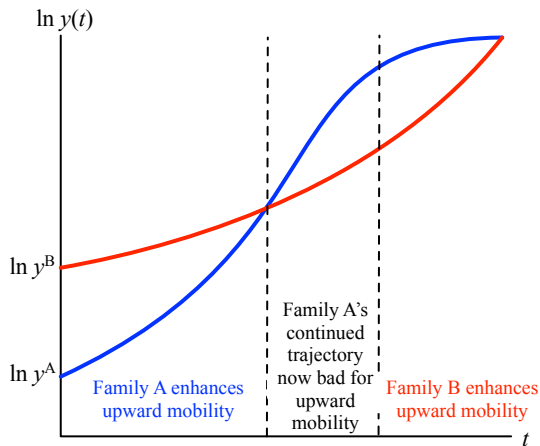


- Approximate by smooth functions and use continuity: **same answer.**

Panel Independence

- $M^\alpha(\vec{y}[s, t])$ and $R^\alpha(\vec{y}[s, t])$ are **panel independent**.
 - To assess a family's changing fortunes, *that* family must be tracked.
 - But to assess upward mobility overall, it is *society* that must be tracked.
 - A family receives **time-varying weights** depending on its relative location.
 - Dynastic effects or pure movements “cancel out.”

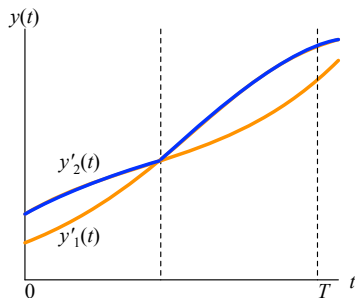
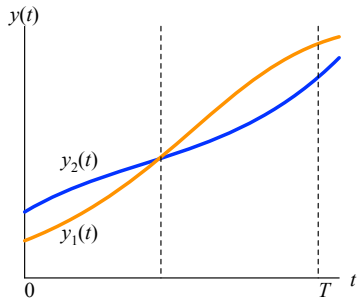
Panel Independence



- Same lineage contributes differently.
- **Caveat:** Variable y must be a “good” sufficient statistic for welfare.

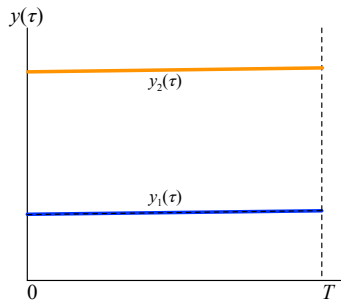
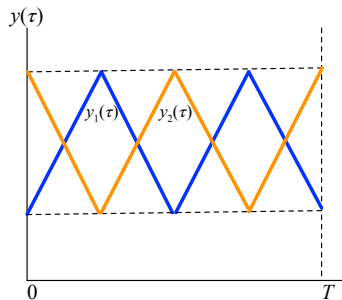
Panel Independence

- But what about mobility as pure movement “back and forth”?



Panel Independence

- But what about mobility as pure movement “back and forth”?



- Different exchange mobility or pure movement. ✓
- Different inequalities. ✓
- But **upward** mobility in both panels is zero.

Panel Independence

- But what about mobility as pure movement “back and forth”?
-

- Upward mobility **subtracts** downward movements from upward movement
- Exchange mobility **adds** them.

- $$M_{\alpha}(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y})g_i = M_{\alpha}^{+}(\mathbf{z}) - M_{\alpha}^{-}(\mathbf{z})$$

- where $M_{\alpha}^{+}(\mathbf{z}) = \sum_{i=1}^n \phi_i^{+}(\mathbf{y}) \max\{g_i, 0\}$ and $M_{\alpha}^{-}(\mathbf{z}) = \sum_{i=1}^n \phi_i^{-}(\mathbf{y}) \max\{-g_i, 0\}$.

- $$E_{\alpha}(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y})|g_i| = M_{\alpha}^{+}(\mathbf{z}) + M_{\alpha}^{-}(\mathbf{z})$$

- Our preferred approach to exchange mobility.
- Such a measure would not be panel-independent.

Panel Independence

- But income may not be a sufficient statistic for lifetime welfare.
-
- **Answer:** That's entirely possible.
 - But imperfection of measurement is not an excuse for changing the measure.
 - It is a reason to use the best data we have.
 - Try **consumption** or **wealth** as proxies for a state variable; Deaton and Zaidi (2002)
 - Or some other measure of permanent income (time-averaged?).
 - Similar recommendations apply to poverty or inequality measurement.

Panel Independence

■ But even so, individuals may belong to different **social** groups. How do we take that into account?

- **Answer:** K social groups. Each person i belongs to one $k(i) \in K$.
- Data for kernel: (\mathbf{z}, \mathbf{w}) , with $z_i = (y_i, g_i)$, w_k the mean income of group k .

Social Growth Progressivity. For any \mathbf{z} , i and j with $(y_i, w_{k(i)}) < (y_j, w_{k(j)})$, form \mathbf{z}' by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $M(\mathbf{z}') > M(\mathbf{z})$.

Social Income Neutrality. $M(\lambda \mathbf{y}, \mathbf{g}, \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$ & $M(\mathbf{y}, \mathbf{g}, \lambda \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$.

Social Binary Growth Tradeoffs. For any i, j , any $(y_i, y_j, w_{k(i)}, w_{k(j)})$, comparing $((y_i, w_{k(i)}, g_i), (y_j, w_{k(j)}, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ and $((y_i, w_{k(i)}, g'_i), (y_j, w_{k(j)}, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ is insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)})$.

Panel Independence

Social groups, contd.

Theorem 4

The above axioms hold if and only if for $n \geq 3$ and groupings K ,

$$\mu_{\alpha,\beta}(\mathbf{y}[s, t], K) = \frac{1}{t-s} \left\{ \ln \left[\frac{\sum_{i=1}^n y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^n y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}} \right]^{-1/\alpha} - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau \right\},$$

for some $(\alpha, \beta) \gg 0$, where $a_k(\tau)$ is Atkinson equivalent group income.

- First term on RHS is panel-independent.
- Second term depends on trajectories, but **only at the group level**.
- Can approximate group Atkinson by standard inequality measures (see paper).

Panel Independence

- Anyway, we typically have panel data, don't we?
-

- **Answer:** No.

- For the United States, [Chetty et al \(2017\)](#) estimate:

- % population share: children \succ parents (US birth cohorts, 1940–84).

- Transitions estimated from a [unique panel of tax records](#)

- \oplus marginal income distributions from CPS and Census.

- Generally very hard to get hold of.

- Though similar studies exist for other countries; e.g., [Acciari et al \(2021\)](#).

Upward Mobility: Other Measures

Skip?

- The **Chetty et al** (2017) measure (also Berman 2021, Acciari et al 2021):

$$m^c(\mathbf{y}[0, 1]) = \frac{1}{n} \sum_{i=1}^n I(y_i(0), y_i(1)).$$

- where $I(y_i(0), y_i(1))$ is indicator for $y_i(0) < y_i(1)$.
- Population share for whom future \succ present.

- The **Fields-Ok** (1999) measure:

$$m^{fo}(\mathbf{z}) = \frac{1}{n} \sum_{i=1}^n [\ln(y_i(0)) - \ln(y_i(1))] = \frac{1}{n} \sum_{i=1}^n \left[\int_0^1 g_i(\tau) d\tau \right].$$

- Both must fail growth progressivity.

Upward Mobility: Other Measures

■ Example for m^c :

- Two persons at incomes \$10,000 and \$20,000.
- Growth rates 1% for both. Then $m^c = 1$.
- Transfer 2 points of growth from rich to poor. Then $m^c = 1/2$.
- But growth progressivity asks that mobility must rise.

Upward Mobility: Other Measures

- **Rank-weighted measures:**

- Such measures fail our axioms in a seemingly technical way:
- They are not continuous — and this isn't just a technicality.

- **Tiny changes** in incomes can generate **discrete jumps** in mobility.

- And worse: large changes in *relative* income could go unnoticed.

- **Our measure is indeed correlated with rank-based measures.**

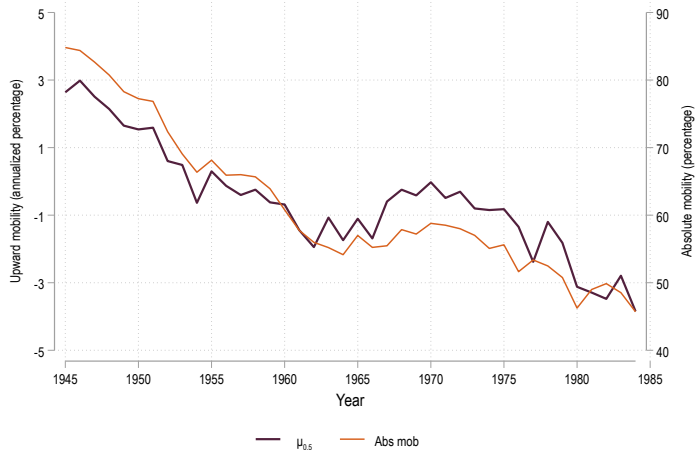
- But is sensitive throughout, without being unduly affected by a rank switch.

Upward Mobility in the Data

- **Chetty et al (2017) estimate m^c** for US birth cohorts, 1940–84.
 - They estimate a copula from a unique panel of tax records.
- *In practice, the dependence on exact copulas seems limited*; Berman (2021)

“Estimating the absolute mobility in the United States with different copulas, some of which are very different from the one characterizing the United States, results in a similar evolution in time.”

μ_α Compared to Chetty et al (2017) for the United States

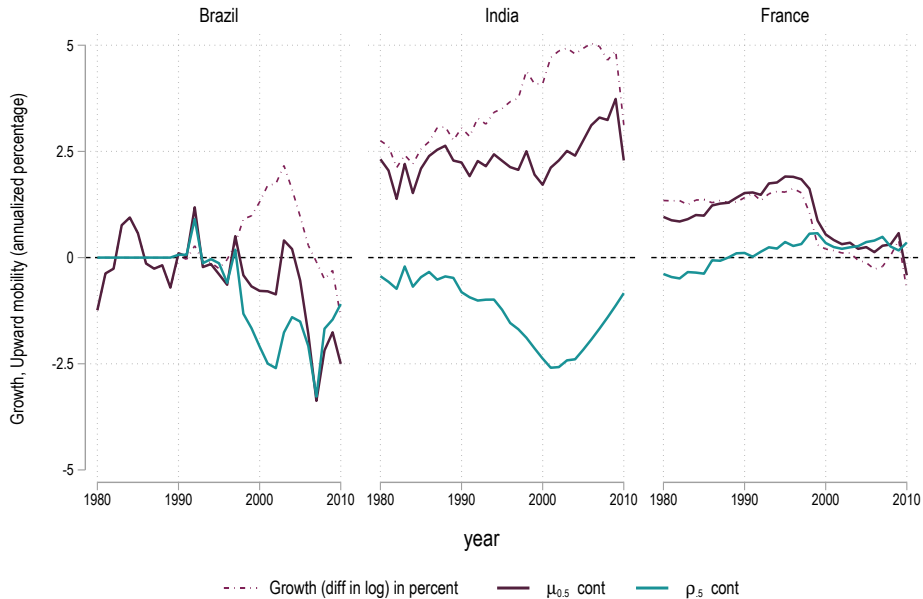


- Robust to different α .
- Robust to using other publicly available databases (e.g., WID).

Upward Mobility in Brazil, India and France

- **Ten-year upward mobility in Brazil, India and France:**
 - Data from the World Inequality Database (repeated cross-sections).
 - Measure $M^{0.5}(\mathbf{y}[t, t + 10])$ and $R^{0.5}(\mathbf{y}[t, t + 10])$.
 - Robust with respect to choice of α (see paper).

Upward Mobility in Brazil, India and France



APPENDIX

Appendix 1: Population Consistency

Given: $\mathbf{z} = (y_1, g_1, \dots, y_k, g_k, \dots, y_n, g_n)$

$$\mathbf{z}' = (y_1, g_1, \dots, y_k, g_k - \epsilon, \dots, y_n, g_n) \quad |$$

$$\mathbf{z}'' = (y_1, g_1, \dots, y_k, g_k + \epsilon, \dots, y_n, g_n)$$

and \mathbf{z}' and \mathbf{z}'' have average mobility distinct from \mathbf{z} : $\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$,

Then: $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$.

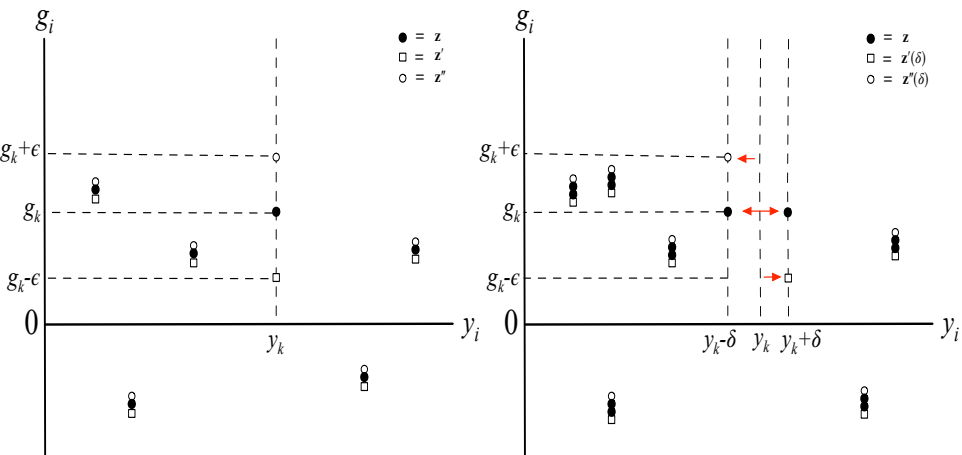
Appendix 2: Proof of Theorem 1

- **Step 1. For every k , $m(g_k) \equiv M(g_k|\mathbf{y}, \mathbf{g}_{-k})$ is affine in g_k , or equivalently:**

$$m(g_k) = \frac{1}{2} [m(g_k - \epsilon) + m(g_k + \epsilon)] \text{ for every } \epsilon > 0.$$

- Suppose false for some g_k and ϵ .
- Define $\mathbf{z} = (\mathbf{y}, \mathbf{g}_{-k}, g_k)$, $\mathbf{z}' = (\mathbf{y}, \mathbf{g}_{-k}, g_k - \epsilon)$, and $\mathbf{z}'' = (\mathbf{y}, \mathbf{g}_{-k}, g_k + \epsilon)$.
- Then $M(\mathbf{z}') + M(\mathbf{z}'') \neq M(\mathbf{z}) + M(\mathbf{z})$.
- By Local Merge, $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$.
- Say $M(\mathbf{z}' \oplus \mathbf{z}'') > M(\mathbf{z} \oplus \mathbf{z})$.

Appendix 2: Proof of Theorem 1



Appendix 2: Proof of Theorem 1

- **Step 2. (Gallier 1999)** $m(\mathbf{y}, \mathbf{g})$ **multiaffine so can be written as:**

$$m(\mathbf{y}, \mathbf{g}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right].$$

for a collection $\{\phi_S\}$ defined for every $\emptyset \neq S \subset \{1, \dots, n\}$.

- **Step 3. All nontrivial product terms above *must have zero coefficients*.**

Suppose $\{ij\} \subset S$ for some S with $\phi_S(\mathbf{y}) \neq 0$. We will only move g_i and g_j but with $g_i + g_j = G$, so hold all else fixed and write

$$\begin{aligned} m(\mathbf{y}, \mathbf{g}) &= \alpha g_i(G - g_i) + \beta g_i + \gamma(G - g_i) + \delta. \\ \Rightarrow \frac{\partial m(\mathbf{y}, \mathbf{g})}{\partial g_i} - \frac{\partial m(\mathbf{y}, \mathbf{g})}{\partial g_j} &= \alpha G - 2\alpha g_i + \beta - \gamma. \end{aligned}$$

Choose G and g_i to violate Growth Progressivity. [back](#)

Details for Additional Axioms

1. Income Scaling. $m(\mathbf{y}, \mathbf{g}) = m(\lambda \mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.

2. Population Scaling. Given $\mathbf{z} \equiv (y_1, g_1, \dots, y_k, g_k, \dots, y_n, g_n)$, form

$$\mathbf{z}' \equiv (y_1, g_1, \dots, y_k, g_k - \epsilon, \dots, y_n, g_n), \mathbf{z}'' \equiv (y_1, g_1, \dots, y_k, g_k + \epsilon, \dots, y_n, g_n).$$

Then $[\frac{1}{2}[m(\mathbf{z}') + m(\mathbf{z}'')] \neq m(\mathbf{z})] \Rightarrow [m(\mathbf{z}' \oplus \mathbf{z}'') \neq m(\mathbf{z} \oplus \mathbf{z})]$.

3. IIA: Is $m((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij})) \geq m((y_i, g'_i), (y_j, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$?

Answer insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$. [back](#)