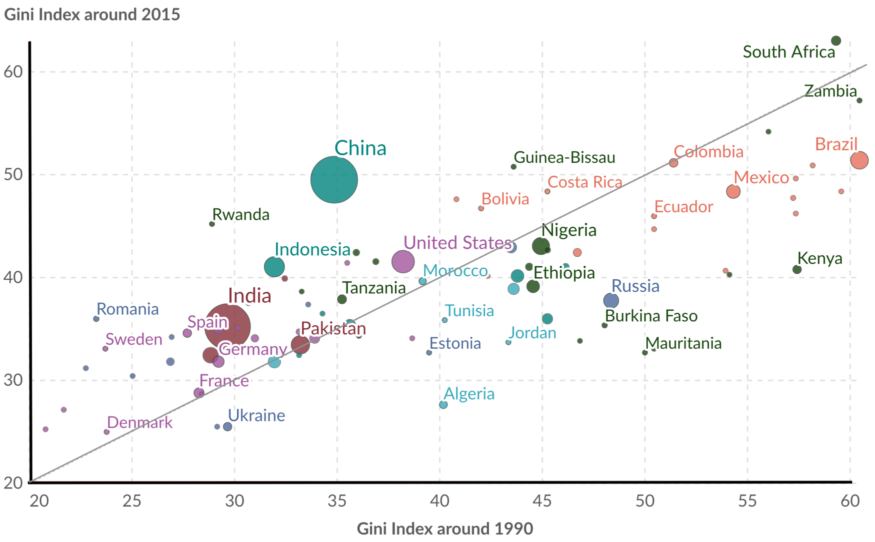
EC9AA Term 3: Lectures on Economic Inequality

Debraj Ray, University of Warwick

- Slides 4: Inequality and Mobility, Part 1

"The Defining Issue of our Time" (Obama 2013)

- Inequality 1990-2015:



Sources: Povcal (2018), The Chartbook of Economic Inequality (2017), Kanbur et al (2017) Table 1.B

Another Defining Issue of our Time

- **Mobility:**

- Centrally important in current (?) debates:

  - Intrinsically: E.g., United States vs Europe

    Chetty et al (2017), Alesina et al (2018), Manduca et al (2020)

  - Instrumentally: Connections to growth and development

    Krueger (2012), Genicot and Ray (2017, 2020), Narayan (2018)

Another Defining Issue of our Time

- **Mobility:**
- Centrally important in current (?) debates:
  - Intrinsically: E.g., United States vs Europe

    Chetty et al (2017), Alesina et al (2018), Manduca et al (2020)
  - Instrumentally: Connections to growth and development

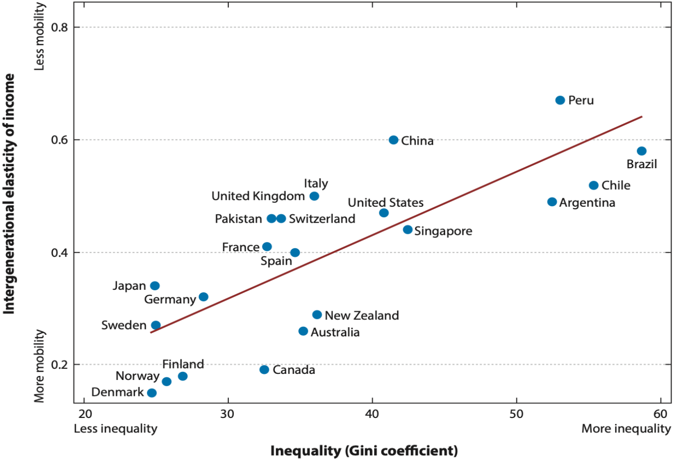
    Krueger (2012), Genicot and Ray (2017, 2020), Narayan (2018)
- A particular connection: **mobility and baseline inequality**.

The Relationship between Inequality and Mobility

- **The Great Gatsby Curve** Krueger (2012), Corak (2013)

The Relationship between Inequality and Mobility

- **The Great Gatsby Curve** Krueger (2012), Corak (2013)



Source: Durlauf-Kourtellos-Tan (2022), based on Corak's data.

See also: Durlauf et al. (2021), Van der Weide et al (2024, 2025), Fan et al. (2021) for China; Guell et al. (2015) for Italy; Branden (2019) for Sweden, Chetty et al. (2014a) for the US.

The Relationship between Inequality and Mobility

- Mobility is a multifaceted notion:
  - We often take measures off the shelf
  - But the conceptual foundations do matter.

The Relationship between Inequality and Mobility

- Mobility is a multifaceted notion:
  - We often take measures off the shelf
  - But the conceptual foundations do matter.
- At least three distinct questions about the inequality-mobility relationship
  - Each question calls for a different measure.

The Relationship between Inequality and Mobility

1. Advantage or Monotonicity:

- Does inequality strengthen the expected advantage that richer parents pass to their children?

The Relationship between Inequality and Mobility

1. Advantage or Monotonicity:

- Does inequality strengthen the expected advantage that richer parents pass to their children?

2. Predictability or Movement:

- Does inequality sharpen the persistence across parental and offspring positions on the economic spectrum?

The Relationship between Inequality and Mobility

1. Advantage or Monotonicity:

- Does inequality strengthen the expected advantage that richer parents pass to their children?

2. Predictability or Movement:

- Does inequality sharpen the persistence across parental and offspring positions on the economic spectrum?

3. Growth Progressivity:

- Does inequality affect the extent to which economic gains are tilted toward those who start poorer?

The Relationship between Inequality and Mobility

1. Advantage or Monotonicity:

- Does inequality strengthen the expected advantage that richer parents pass to their children?

2. Predictability or Movement:

- Does inequality sharpen the persistence across parental and offspring positions on the economic spectrum?

3. Growth Progressivity:

- Does inequality affect the extent to which economic gains are tilted toward those who start poorer?

- These are related but not identical notions.

The (Im)Mobility Measure in the Great Gatsby Curve

- Recall the intergenerational elasticity of earnings (IGE) from Slides 2:

$$\log y(t+1) = \alpha + \beta \log y(t) + u(t).$$

The (Im)Mobility Measure in the Great Gatsby Curve

- Recall the intergenerational elasticity of earnings (IGE) from Slides 2:

$$\log y(t+1) = \alpha + \beta \log y(t) + u(t).$$

$$\text{IGE} = \beta = \frac{\text{Cov}(\log y(t), \log y(t+1))}{\sigma^2(\log y(t))} \approx \text{"absence of mobility".}$$

The (Im)Mobility Measure in the Great Gatsby Curve

- Recall the intergenerational elasticity of earnings (IGE) from Slides 2:

$$\log y(t+1) = \alpha + \beta \log y(t) + u(t).$$

$$\text{IGE} = \beta = \frac{\text{Cov}(\log y(t), \log y(t+1))}{\sigma^2(\log y(t))} \approx \text{ “absence of mobility”}.$$

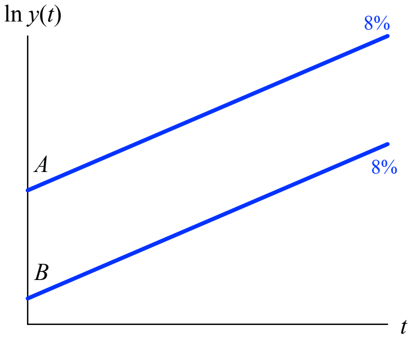
- The IGE is a predictor of expected advantage.
- Is it also a good measure of non-progressivity? We will come back to this.

Upward Mobility

- **Upward mobility**: a measure that rewards **progressivity** [Ray ⓡ Genicot 2023]:
  - Upward mobility is higher if the relatively poor grow faster.

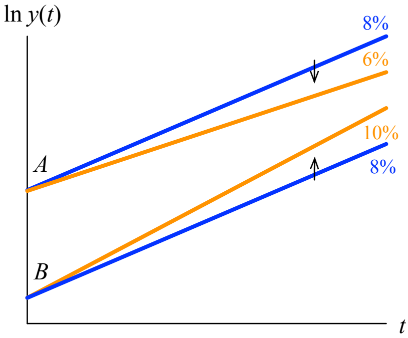
Upward Mobility

- **Upward mobility**: a measure that rewards **progressivity** [Ray ⓡ Genicot 2023]:
  - Upward mobility is higher if the relatively poor grow faster.



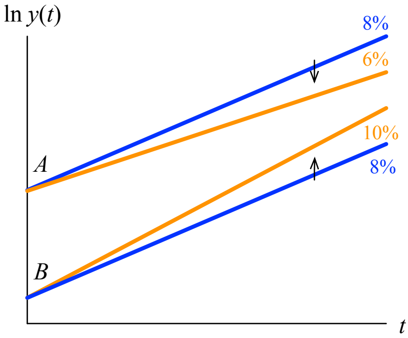
Upward Mobility

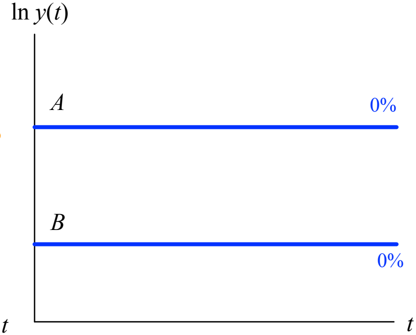
- **Upward mobility**: a measure that rewards **progressivity** [Ray ⓡ Genicot 2023]:
  - Upward mobility is higher if the relatively poor grow faster.



Upward Mobility

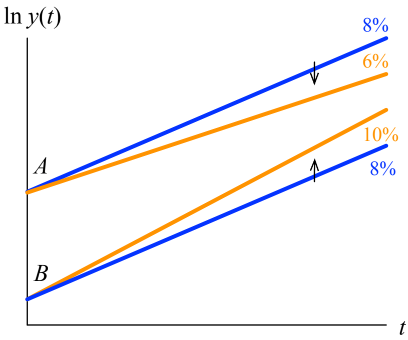
- **Upward mobility**: a measure that rewards **progressivity** [Ray ⓡ Genicot 2023]:
  - Upward mobility is higher if the relatively poor grow faster.





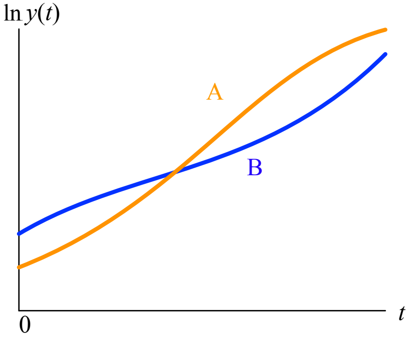
Upward Mobility

- **Upward mobility**: a measure that rewards **progressivity** [Ray ⓡ Genicot 2023]:
  - Upward mobility is higher if the relatively poor grow faster.



Upward Mobility

- Be careful with crossings, though:



- A was poorer than B, but is now richer

- Do we want to value A's growth more, once she's richer? (No, we don't.)

Snapshots and Trajectories

- Given the possibility of crossings, our approach is in two parts:

Snapshots and Trajectories

- Given the possibility of crossings, our approach is in two parts:
  - A "snapshot" or **upward mobility kernel** m;
    intermediate step
  - A **measure on trajectories** M built from these kernels.
    what we're after

Snapshots

- Core variable y_i: permanent income (or wealth or consumption).
    - $g_i = \dot{y}_i / y_i$ instantaneous growth rate.

Snapshots

- Core variable y_i: permanent income (or wealth or consumption).
  - $g_i = \dot{y}_i / y_i$ instantaneous growth rate.
- **Upward mobility kernel**: $m(\mathbf{y}, \mathbf{g})$.
  - Anonymous, continuous.
  - Zero-growth normalization:

$g_i = 0$ all $i \mapsto m(\mathbf{y}, \mathbf{g}) = 0$.

  - Consistency under population mergers.

Details

Core Axiom

- **Growth Progressivity**
  - For $y_i < y_j$, send g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $m(\mathbf{y}, \mathbf{g}) \uparrow$.

Core Axiom

- **Growth Progressivity**
  - For $y_i < y_j$, send g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $m(\mathbf{y}, \mathbf{g}) \uparrow$.
- Notes:
  - Measure tolerates lower growth if poor can grow faster.
  - Upward mobility $\neq$ overall welfare.

Snapshots

Theorem 1

An upward mobility kernel is growth progressive if and only if it can be written as

$$m(\mathbf{y}, \mathbf{g}) = \sum_{i=1}^{n} \phi_i(\mathbf{y}) g_i$$

for continuous permutation-invariant $\{\phi_i\}$*, with* $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$ *when* $y_i < y_j$*.*

Proof Outline

Sharpening the Kernel

- **Income Neutrality.** $m(\mathbf{y}, \mathbf{g}) = m(\lambda\mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.

Sharpening the Kernel

- **Income Neutrality.** $m(\mathbf{y}, \mathbf{g}) = m(\lambda \mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.
- **Growth Alignment.** $\mathbf{g} > \mathbf{g}' \Rightarrow m(\mathbf{y}, \mathbf{g}) > m(\mathbf{y}, \mathbf{g}')$ all $\mathbf{y}$.

Sharpening the Kernel

- **Income Neutrality.** $m(\mathbf{y}, \mathbf{g}) = m(\lambda\mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.

- **Growth Alignment.** $\mathbf{g} > \mathbf{g}' \Rightarrow m(\mathbf{y}, \mathbf{g}) > m(\mathbf{y}, \mathbf{g}')$ all $\mathbf{y}$.

- **Independent Pairwise Growth Tradeoffs:**

  Is $m((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij})) \geq m((y_i, g_i'), (y_j, g_j'), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$?

  Answer insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$.

Sharpening the Kernel

Theorem 2

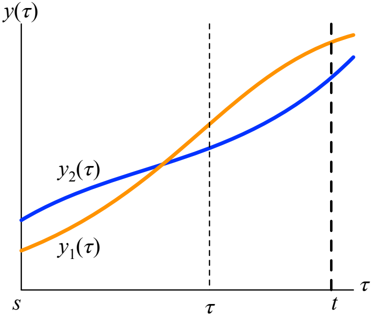
Under additional three axioms and $n \geq 3$, m can be written as:

$$m^{\alpha}(\mathbf{y}, \mathbf{g}) = \frac{\sum_{i=1}^{n} y_i^{-\alpha} g_i}{\sum_{i=1}^{n} y_i^{-\alpha}}, \text{ for some } \alpha > 0.$$

- Proof employs a substantial extension of Gorman's separability theorem;

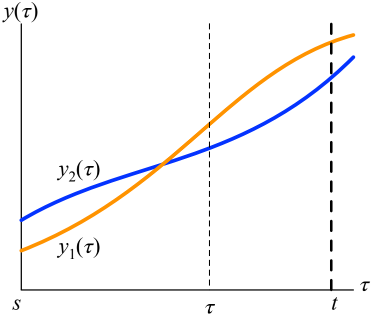
  see Chatterjee ⓡ Ray ⓡ Sen (2021).

Income Trajectories $\vec{\mathbf{y}}[s,t]$



- $(\mathbf{y}(\tau), \mathbf{g}(\tau))$ and $m(\mathbf{y}(\tau), \mathbf{g}(\tau))$ well-defined for each $\tau \in [s,t]$.

Income Trajectories $\vec{\mathbf{y}}[s,t]$



- $(\mathbf{y}(\tau), \mathbf{g}(\tau))$ and $m(\mathbf{y}(\tau), \mathbf{g}(\tau))$ well-defined for each $\tau \in [s,t]$.
- $\mathbf{y}[s,t] = \{y_i(\tau)_s^t\}_{i=1}^n$

Extension Axioms

- **Upward mobility measure**: $M(\mathbf{y}[s,t])$.

Extension Axioms

- **Upward mobility measure**: $M(\mathbf{y}[s,t])$.

- Two "extension" axioms:

1. Reducibility: $M(\vec{\mathbf{y}}[s,t]) = \Psi(\{m(\mathbf{y}(\tau),\mathbf{g}(\tau))\}_s^t)$ for some "aggregator" Ψ.

2. Averaging: for every $\tau \in (s,t)$,

$$(t-s)M(\vec{\mathbf{y}}[s,t]) = (\tau - s)M(\vec{\mathbf{y}}[s,\tau]) + (t-\tau)M(\vec{\mathbf{y}}[\tau,t]).$$

Upward Mobility Measure

Theorem 3

Progressivity and the extension axioms hold if and only if

$$M^{\alpha}(\vec{\mathbf{y}}[s,t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{i=1}^{n} y_i^{-\alpha}(t)}{\sum_{i=1}^{n} y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0.$$

Upward Mobility Measure

Theorem 3

Progressivity and the extension axioms hold if and only if

$$M^\alpha(\vec{\mathbf{y}}[s,t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \textit{ for some } \alpha > 0.$$

- **Remarks:**

- The measure is helpfully phrased in "growth units":

  when all growth rates are the same g, $M^\alpha(\vec{\mathbf{y}}[s,t]) = g$.

- Can also use income categories and population shares (see paper).

Absolute and Relative

- The formula

$$M^{\alpha}(\vec{\mathbf{y}}[s,t]) = \frac{1}{t-s}\ln\left[\frac{\sum_{i=1}^{n} y_i^{-\alpha}(t)}{\sum_{i=1}^{n} y_i^{-\alpha}(s)}\right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0$$

measures absolute upward mobility.

Absolute and Relative

- The formula

$$M^\alpha(\vec{\mathbf{y}}[s,t]) = \frac{1}{t-s}\ln\left[\frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)}\right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0$$

measures absolute upward mobility.

- **Relative upward mobility** nets out growth:

$$R^\alpha(\vec{\mathbf{y}}[s,t]) = M_\alpha(\vec{\mathbf{y}}[s,t]) - \frac{1}{t-s}\left[\ln(\bar{y}(t)) - \ln(\bar{y}(s))\right]$$
$$= \frac{1}{t-s}\ln\left[\frac{\sum_{i=1}^n e_i(t)^{-\alpha}}{\sum_{i=1}^n e_i(s)^{-\alpha}}\right]^{-\frac{1}{\alpha}}$$

where $\bar{y}$ is per-capita income and $e_i = y_i/\bar{y}$ is relative income

Upward Mobility as Pro-Poor Growth

- **Upward Mobility** $= \dfrac{1}{t-s} \ln \left[\dfrac{\sum_{j=1}^{n} y_j(t)^{-\alpha}}{\sum_{j=1}^{m} y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$
- **Growth** $= \dfrac{1}{t-s} \ln \left[\dfrac{\sum_{j=1}^{n} y_j(t)}{\sum_{j=1}^{m} y_j(s)} \right]$

Upward Mobility as Pro-Poor Growth

- **Upward Mobility** $= \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{n} y_j(t)^{-\alpha}}{\sum_{j=1}^{m} y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$
- **Growth** $= \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{n} y_j(t)}{\sum_{j=1}^{m} y_j(s)} \right] = M^{(-1)}(\mathbf{y}[s,t])$

Upward Mobility as Pro-Poor Growth

- **Upward Mobility** $= \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{n} y_j(t)^{-\alpha}}{\sum_{j=1}^{m} y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$
- **Growth** $= \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{n} y_j(t)}{\sum_{j=1}^{m} y_j(s)} \right] = M^{(-1)}(\mathbf{y}[s,t])$

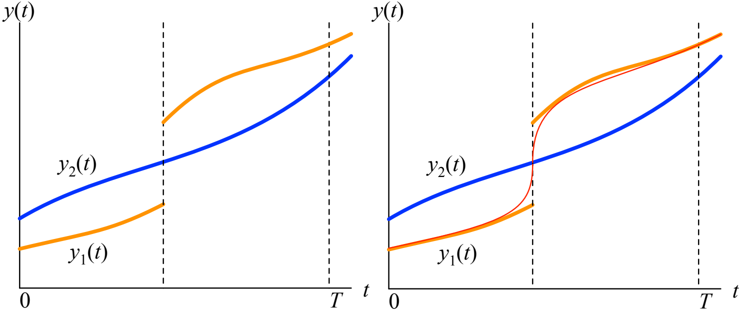
- Isn't even on our "boundary" as $\alpha \to 0$.
- Nevertheless, when all growth rates are the same, $M^{\alpha} =$ growth rate.

Discontinuous Trajectories

- If there are jumps, then mobility kernels aren’t defined at some points.
  - Examples: inheritance, job change, promotions …

Discontinuous Trajectories

- If there are jumps, then mobility kernels aren't defined at some points.
  - Examples: inheritance, job change, promotions …



  - Approximate by smooth functions and use continuity: **same answer.**

Panel Independence

- $M^\alpha(\vec{\mathbf{y}}[s,t])$ and $R^\alpha(\vec{\mathbf{y}}[s,t])$ are **panel independent**.

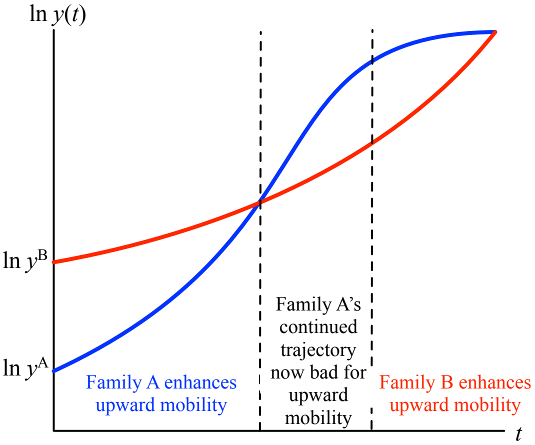
Panel Independence

- $M^\alpha(\vec{\mathbf{y}}[s,t])$ and $R^\alpha(\vec{\mathbf{y}}[s,t])$ are **panel independent**.
  - To assess a family's changing fortunes, *that* family must be tracked.
  - But to assess upward mobility overall, it is *society* that must be tracked.

Panel Independence

- $M^{\alpha}(\vec{\mathbf{y}}[s,t])$ and $R^{\alpha}(\vec{\mathbf{y}}[s,t])$ are **panel independent**.
  - To assess a family's changing fortunes, *that* family must be tracked.
  - But to assess upward mobility overall, it is *society* that must be tracked.
  - A family receives time-varying weights depending on its relative location.
  - Dynastic effects or pure movements "cancel out."

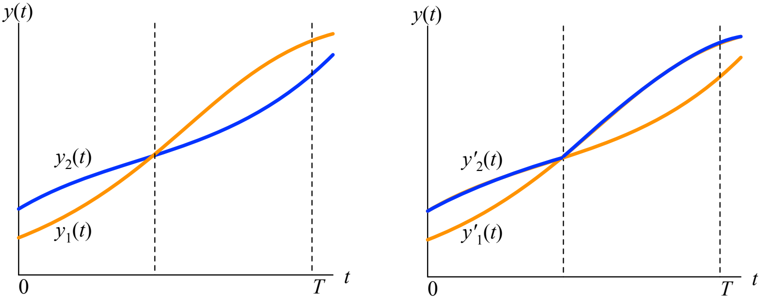
Panel Independence



- Same lineage contributes differently.
- **Caveat:** Variable y must be a "good" sufficient statistic for welfare.

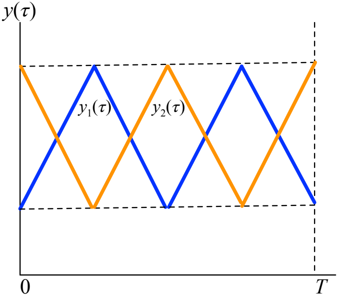
Panel Independence

- But what about mobility as pure movement “back and forth”?



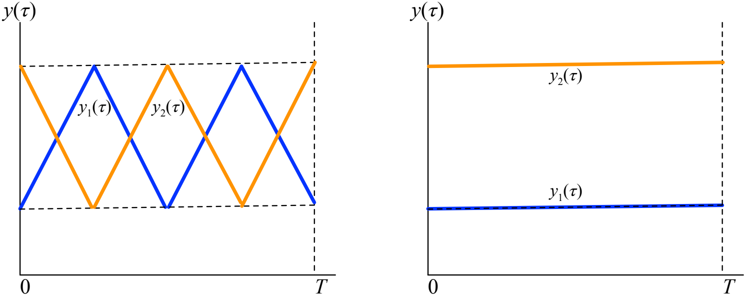
Panel Independence

- But what about mobility as pure movement “back and forth”?



Panel Independence

- But what about mobility as pure movement “back and forth”?



- Different exchange mobility or pure movement. ✓
- Different inequalities. ✓

- But **upward** mobility in both panels is zero.

Panel Independence

- But what about mobility as pure movement "back and forth"?

- Upward mobility **subtracts** downward movements from upward movement
- Exchange mobility **adds** them.

Panel Independence

- But what about mobility as pure movement “back and forth”?

- Upward mobility **subtracts** downward movements from upward movement
- Exchange mobility **adds** them.
- $$M_\alpha(\mathbf{z}) = \sum_{i=1}^{n} \phi_i(\mathbf{y}) g_i = M_\alpha^+(\mathbf{z}) - M_\alpha^-(\mathbf{z})$$
- where $M_\alpha^+(\mathbf{z}) = \sum_{i=1}^{n} \phi_i^+(\mathbf{y}) \max\{g_i, 0\}$ and $M_\alpha^-(\mathbf{z}) = \sum_{i=1}^{n} \phi_i^-(\mathbf{y}) \max\{-g_i, 0\}$.

Panel Independence

- But what about mobility as pure movement "back and forth"?

- Upward mobility **subtracts** downward movements from upward movement
- Exchange mobility **adds** them.
- $$M_\alpha(\mathbf{z}) = \sum_{i=1}^{n} \phi_i(\mathbf{y}) g_i = M_\alpha^+(\mathbf{z}) - M_\alpha^-(\mathbf{z})$$
- where $M_\alpha^+(\mathbf{z}) = \sum_{i=1}^{n} \phi_i^+(\mathbf{y}) \max\{g_i, 0\}$ and $M_\alpha^-(\mathbf{z}) = \sum_{i=1}^{n} \phi_i^-(\mathbf{y}) \max\{-g_i, 0\}$.
- $$E_\alpha(\mathbf{z}) = \sum_{i=1}^{n} \phi_i(\mathbf{y}) |g_i| = M_\alpha^+(\mathbf{z}) + M_\alpha^-(\mathbf{z})$$
- Our preferred approach to exchange mobility.
- Such a measure would not be panel-independent.

Panel Independence

- But income may not be a sufficient statistic for lifetime welfare.

Panel Independence

- But income may not be a sufficient statistic for lifetime welfare.

- **Answer:** That's entirely possible.
  - But imperfection of measurement is not an excuse for changing the measure.
  - It is a reason to use the best data we have.

Panel Independence

- But income may not be a sufficient statistic for lifetime welfare.

- **Answer:** That's entirely possible.
  - But imperfection of measurement is not an excuse for changing the measure.
  - It is a reason to use the best data we have.
- Try consumption or wealth as proxies for a state variable; Deaton and Zaidi (2002)
  - Or some other measure of permanent income (time-averaged?).

Panel Independence

- But income may not be a sufficient statistic for lifetime welfare.

- **Answer:** That's entirely possible.
  - But imperfection of measurement is not an excuse for changing the measure.
  - It is a reason to use the best data we have.
- Try consumption or wealth as proxies for a state variable; Deaton and Zaidi (2002)
  - Or some other measure of permanent income (time-averaged?).
- Similar recommendations apply to poverty or inequality measurement.

Panel Independence

- But even so, individuals may belong to different **social** groups. How do we take that into account?

Panel Independence

- But even so, individuals may belong to different **social** groups. How do we take that into account?

- **Answer:** K social groups. Each person i belongs to one $k(i) \in K$.
  - Data for kernel: $(\mathbf{z}, \mathbf{w})$, with $z_i = (y_i, g_i)$, w_k the mean income of group k.

Panel Independence

- But even so, individuals may belong to different **social** groups. How do we take that into account?

- **Answer:** K social groups. Each person i belongs to one $k(i) \in K$.
  - Data for kernel: $(\mathbf{z}, \mathbf{w})$, with $z_i = (y_i, g_i)$, w_k the mean income of group k.

Social Growth Progressivity. For any $\mathbf{z}$, i and j with $(y_i, w_{k(i)}) < (y_j, w_{k(j)})$, form $\mathbf{z}'$ by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $M(\mathbf{z}') > M(\mathbf{z})$.

Panel Independence

- But even so, individuals may belong to different **social** groups. How do we take that into account?

- **Answer:** K social groups. Each person i belongs to one $k(i) \in K$.
- Data for kernel: $(\mathbf{z}, \mathbf{w})$, with $z_i = (y_i, g_i)$, w_k the mean income of group k.

Social Growth Progressivity. For any $\mathbf{z}$, i and j with $(y_i, w_{k(i)}) < (y_j, w_{k(j)})$, form $\mathbf{z}'$ by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $M(\mathbf{z}') > M(\mathbf{z})$.

Social Income Neutrality. $M(\lambda \mathbf{y}, \mathbf{g}, \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$ & $M(\mathbf{y}, \mathbf{g}, \lambda \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$.

Panel Independence

- But even so, individuals may belong to different **social** groups. How do we take that into account?

- **Answer:** K social groups. Each person i belongs to one $k(i) \in K$.
  - Data for kernel: $(\mathbf{z}, \mathbf{w})$, with $z_i = (y_i, g_i)$, w_k the mean income of group k.

Social Growth Progressivity. For any $\mathbf{z}$, i and j with $(y_i, w_{k(i)}) < (y_j, w_{k(j)})$, form $\mathbf{z}'$ by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $M(\mathbf{z}') > M(\mathbf{z})$.

Social Income Neutrality. $M(\lambda\mathbf{y}, \mathbf{g}, \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$ & $M(\mathbf{y}, \mathbf{g}, \lambda\mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$.

Social Binary Growth Tradeoffs. For any i, j, any $(y_i, y_j, w_{k(i)}, w_{k(j)})$, comparing $((y_i, w_{k(i)}, g_i), (y_j, w_{k(j)}, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ and $((y_i, w_{k(i)}, g_i'), (y_j, w_{k(j)}, g_j'), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)})))$ is insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$.

Panel Independence

Social groups, contd.

Theorem 4

The above axioms hold if and only if for $n \geq 3$ and groupings K,

$$\mu_{\alpha,\beta}(\mathbf{y}[s,t],K) = \frac{1}{t-s}\left\{\ln\left[\frac{\sum_{i=1}^{n} y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^{n} y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}}\right]^{-1/\alpha} - \frac{\beta}{\alpha}\int_s^t \frac{\sum_{k\in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k\in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau\right\},$$

for some $(\alpha,\beta) \gg 0$, where $a_k(\tau)$ is Atkinson equivalent group income.

- First term on RHS is panel-independent.
- Second term depends on trajectories, but **only at the group level**.
- Can approximate group Atkinson by standard inequality measures (see paper).

Panel Independence

- Anyway, we typically have panel data, don’t we?

Panel Independence

- Anyway, we typically have panel data, don't we?

- **Answer:** No.

- For the United States, Chetty et al (2017) estimate:
- % population share: children $\succ$ parents (US birth cohorts, 1940–84).
- Transitions estimated from a unique panel of tax records
- $\oplus$ marginal income distributions from CPS and Census.

Panel Independence

- Anyway, we typically have panel data, don't we?

- **Answer:** No.
    - For the United States, Chetty et al (2017) estimate:
    - % population share: children $\succ$ parents (US birth cohorts, 1940–84).
    - Transitions estimated from a unique panel of tax records
    - $\oplus$ marginal income distributions from CPS and Census.
- Generally very hard to get hold of.
    - Though similar studies exist for other countries; e.g., Acciari et al (2021).

Upward Mobility: Other Measures

Skip?

- The **Chetty et al** (2017) measure (also Berman 2021, Acciari et al 2021):

$$m^{c}(\mathbf{y}[0,1]) = \frac{1}{n}\sum_{i=1}^{n} I(y_i(0), y_i(1)).$$

  - where $I(y_i(0), y_i(1))$ is indicator for $y_i(0) < y_i(1)$.
  - Population share for whom future $\succ$ present.

Upward Mobility: Other Measures

Skip?

- The **Chetty et al** (2017) measure (also Berman 2021, Acciari et al 2021):

$$m^{\mathsf{c}}(\mathbf{y}[0,1]) = \frac{1}{n}\sum_{i=1}^{n} I(y_i(0), y_i(1)).$$

  - where $I(y_i(0), y_i(1))$ is indicator for $y_i(0) < y_i(1)$.
  - Population share for whom future $\succ$ present.

- The **Fields-Ok** (1999) measure:

$$m^{\mathsf{FO}}(\mathbf{z}) = \frac{1}{n}\sum_{i=1}^{n}[\ln(y_i(0)) - \ln(y_i(1))] = \frac{1}{n}\sum_{i=1}^{n}\left[\int_0^1 g_i(\tau)d\tau\right].$$

- Both must fail growth progressivity.

Upward Mobility: Other Measures

- **Example for m^{c}:**

- Two persons at incomes \$10,000 and \$20,000.
- Growth rates 1% for both. Then $m^{\mathrm{c}} = 1$.
- Transfer 2 points of growth from rich to poor. Then $m^{\mathrm{c}} = 1/2$.
- But growth progressivity asks that mobility must rise.

Upward Mobility: Other Measures

- **Rank-weighted measures:**

  - Such measures fail our axioms in a seemingly technical way:

  - They are not continuous — and this isn’t just a technicality.

Upward Mobility: Other Measures

- **Rank-weighted measures:**
  - Such measures fail our axioms in a seemingly technical way:
  - They are not continuous — and this isn't just a technicality.
- **Tiny changes** in incomes can generate **discrete jumps** in mobility.
  - And worse: large changes in *relative* income could go unnoticed.

Upward Mobility: Other Measures

- **Rank-weighted measures:**
  - Such measures fail our axioms in a seemingly technical way:
  - They are not continuous — and this isn't just a technicality.
- **Tiny changes** in incomes can generate **discrete jumps** in mobility.
  - And worse: large changes in *relative* income could go unnoticed.
- **Our measure is indeed correlated with rank-based measures**.
  - But is sensitive throughout, without being unduly affected by a rank switch.

Upward Mobility in the Data

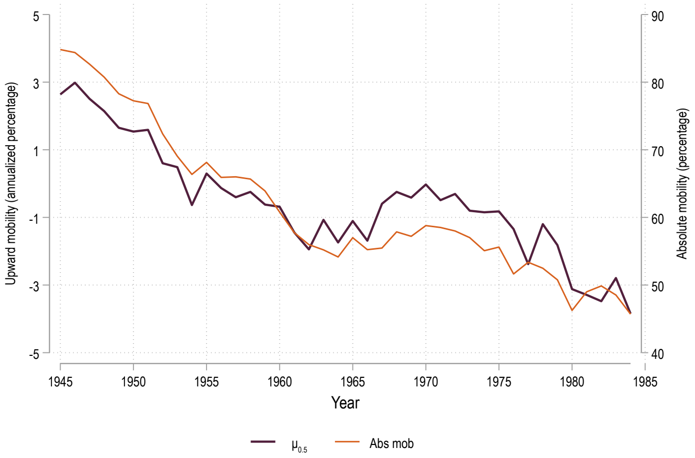
- **Chetty et al (2017) estimate** m^c for US birth cohorts, 1940–84.
  - They estimate a copula from a unique panel of tax records.

Upward Mobility in the Data

- **Chetty et al (2017) estimate** m^c for US birth cohorts, 1940–84.
  - They estimate a copula from a unique panel of tax records.
- *In practice*, the dependence on exact copulas seems limited; Berman (2021)

"Estimating the absolute mobility in the United States with different copulas, some of which are very different from the one characterizing the United States, results in a similar evolution in time."

μ_α Compared to Chetty et al (2017) for the United States



- Robust to different α.
- Robust to using other publicly available databases (e.g., WID).

Upward Mobility in Brazil, India and France

- **Ten-year upward mobility in Brazil, India and France:**
  - Data from the World Inequality Database (repeated cross-sections).
  - Measure $M^{0.5}(\mathbf{y}[t, t+10])$ and $R^{0.5}(\mathbf{y}[t, t+10])$.
  - Robust with respect to choice of α (see paper).

Upward Mobility in Brazil, India and France

APPENDIX

Appendix 1: Population Consistency

Given: $$\mathbf{z} = (y_1, g_1, \ldots, y_k, g_k, \ldots, y_n, g_n)$$

$$\mathbf{z}' = (y_1, g_1, \ldots, y_k, g_k - \epsilon, \ldots, y_n, g_n)$$

$$\mathbf{z}'' = (y_1, g_1, \ldots, y_k, g_k + \epsilon, \ldots, y_n, g_n)$$

and $\mathbf{z}'$ and $\mathbf{z}''$ have average mobility distinct from $\mathbf{z}$: $\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$,

Appendix 1: Population Consistency

Given: $$\mathbf{z} = (y_1, g_1, \ldots, y_k, g_k, \ldots, y_n, g_n)$$

$$\mathbf{z}' = (y_1, g_1, \ldots, y_k, g_k - \epsilon, \ldots, y_n, g_n)$$

$$\mathbf{z}'' = (y_1, g_1, \ldots, y_k, g_k + \epsilon, \ldots, y_n, g_n)$$

and $\mathbf{z}'$ and $\mathbf{z}''$ have average mobility distinct from $\mathbf{z}$: $\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$,

Then: $$M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z}).$$

back

Appendix 2: Proof of Theorem 1

- **Step 1. For every k, $m(g_k) \equiv M(g_k|\mathbf{y}, \mathbf{g}_{-k})$ is affine in g_k, or equivalently**:

$$m(g_k) = \frac{1}{2}\left[m(g_k - \epsilon) + m(g_k + \epsilon)\right] \text{ for every } \epsilon > 0.$$

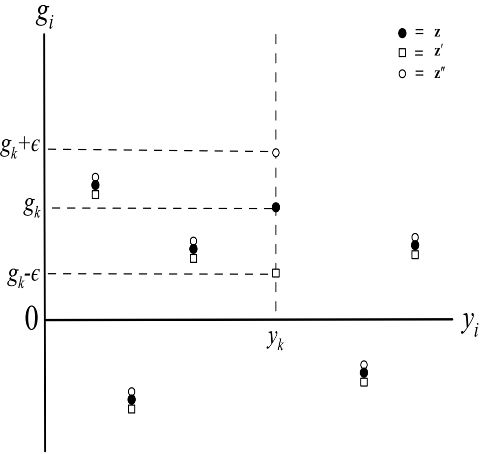
Appendix 2: Proof of Theorem 1

- **Step 1. For every k, $m(g_k) \equiv M(g_k|\mathbf{y}, \mathbf{g}_{-k})$ is affine in g_k, or equivalently**:

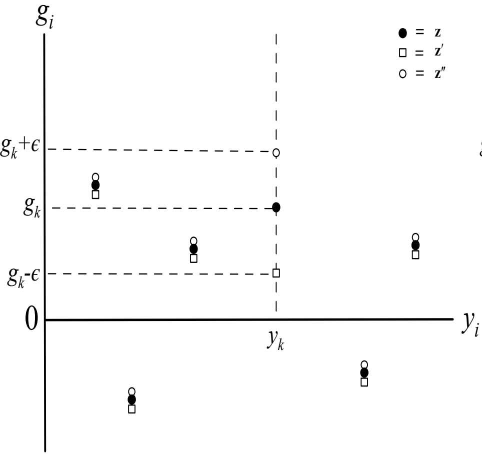
$$m(g_k) = \frac{1}{2}\big[m(g_k - \epsilon) + m(g_k + \epsilon)\big] \text{ for every } \epsilon > 0.$$

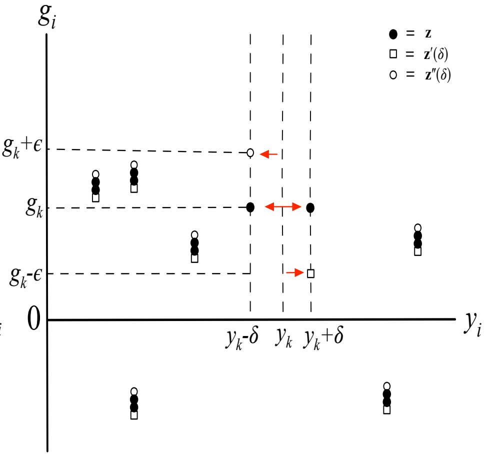
- Suppose false for some g_k and ϵ.
  - Define $\mathbf{z} = (\mathbf{y}, \mathbf{g}_{-k}, g_k)$, $\mathbf{z}' = (\mathbf{y}, \mathbf{g}_{-k}, g_k - \epsilon)$, and $\mathbf{z}'' = (\mathbf{y}, \mathbf{g}_{-k}, g_k + \epsilon)$.
  - Then $M(\mathbf{z}') + M(\mathbf{z}'') \neq M(\mathbf{z}) + M(\mathbf{z})$.
  - By Population Consistency, $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$.
  - Say $M(\mathbf{z}' \oplus \mathbf{z}'') < M(\mathbf{z} \oplus \mathbf{z})$.

Appendix 2: Proof of Theorem 1



Appendix 2: Proof of Theorem 1





- $M(\mathbf{z}' \oplus \mathbf{z}'') < M(\mathbf{z} \oplus \mathbf{z})$ contradicts continuity and growth progressivity.
- And so does $M(\mathbf{z}' \oplus \mathbf{z}'') > M(\mathbf{z} \oplus \mathbf{z})$, by a similar argument.

Appendix 2: Proof of Theorem 1

- **Step 2. (Gallier 1999) $m(\mathbf{y},\mathbf{g})$ multiaffine so can be written as**:

$$m(\mathbf{y},\mathbf{g}) = \sum_{S} \phi_S(\mathbf{y}) \left[\prod_{j\in S} g_j\right].$$

for a collection $\{\phi_S\}$ defined for every $\emptyset \neq S \subset \{1,\ldots,n\}$.

Appendix 2: Proof of Theorem 1

- **Step 2. (Gallier 1999) $m(\mathbf{y}, \mathbf{g})$ multiaffine so can be written as**:

$$m(\mathbf{y}, \mathbf{g}) = \sum_{S} \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right].$$

for a collection $\{\phi_S\}$ defined for every $\emptyset \neq S \subset \{1, \ldots, n\}$.

- **Step 3. All nontrivial product terms above *must have zero coefficients***.

Suppose $\{ij\} \subset S$ for some S with $\phi_S(\mathbf{y}) \neq 0$. By continuity, $y_i \neq y_j$; say $y_i > y_j$.

We will only move g_i and g_j but with $g_i + g_j = G$, so hold all else fixed and write

$$m(\mathbf{y}, \mathbf{g}) = \alpha g_i (G - g_i) + \beta g_i + \gamma (G - g_i) + \text{ all other terms.}$$

Appendix 2: Proof of Theorem 1

- **Step 2. (Gallier 1999) $m(\mathbf{y}, \mathbf{g})$ multiaffine so can be written as**:

$$m(\mathbf{y}, \mathbf{g}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right].$$

for a collection $\{\phi_S\}$ defined for every $\emptyset \neq S \subset \{1, \ldots, n\}$.

- **Step 3. All nontrivial product terms above *must have zero coefficients*.**

Suppose $\{ij\} \subset S$ for some S with $\phi_S(\mathbf{y}) \neq 0$. By continuity, $y_i \neq y_j$; say $y_i > y_j$.

We will only move g_i and g_j but with $g_i + g_j = G$, so hold all else fixed and write

$$m(\mathbf{y}, \mathbf{g}) = \alpha g_i (G - g_i) + \beta g_i + \gamma (G - g_i) + \text{ all other terms.}$$

$$\Rightarrow \frac{\partial m(\mathbf{y}, \mathbf{g})}{\partial g_i} - \frac{\partial m(\mathbf{y}, \mathbf{g})}{\partial g_j} = \alpha G - 2\alpha g_i + \beta - \gamma.$$

Choose G and g_i to make this < 0, violates Growth Progressivity. back

Details for Additional Axioms

1. **Income Scaling.** $m(\mathbf{y}, \mathbf{g}) = m(\lambda\mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.

2. **Population Scaling.** Given $\mathbf{z} \equiv (y_1, g_1, \ldots, y_k, g_k, \ldots, y_n, g_n)$, form

   $\mathbf{z}' \equiv (y_1, g_1, \ldots, y_k, g_k - \epsilon, \ldots, y_n, g_n)$, $\mathbf{z}'' = (y_1, g_1, \ldots, y_k, g_k + \epsilon, \ldots, y_n, g_n)$.

   Then $[\frac{1}{2}[m(\mathbf{z}') + m(\mathbf{z}'')] \neq m(\mathbf{z})] \Rightarrow [m(\mathbf{z}' \oplus \mathbf{z}'') \neq m(\mathbf{z} \oplus \mathbf{z})]$.

3. **IIA:** Is $m((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij})) \geq m((y_i, g_i'), (y_j, g_j'), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$?

   Answer insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$. back