

EC9AA Term 3: Lectures on Economic Inequality

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- **Supplement to Slides 2:** A General Model of Occupational Choice

A General Model of Occupational Choice

■ Production with capital and occupations.

- Population distribution on occupations n (endogenous).
- Physical capital k .
- Production function $y = F(k, n)$, CRS and strictly quasiconcave.

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■ Training cost function x on occupations:

- incurred up front.
- parents pay directly, or bequeath and then children pay.

■ Perfect competition.

- Return on capital fixed at rate r (international k -mobility).
- “Wage” vector $\mathbf{w} \equiv \{w(h)\}$ endogenously determined for each occupation h .
- Together with r , $\mathbf{w}$ **supports profit-maximization**.

Supporting Profit Maximization

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- If that unit cost $\neq$ output price:
 - $(\mathbf{w}, r)$ **cannot support profit maximization at positive output.**
 - Otherwise, it does.
- **Note:** For any $\mathbf{w}$, there is a unique **scaling** $\mu > 0$ such that $(\mu\mathbf{w}, r)$ supports profit maximization.

Households

- Continuum of households, each with one agent per generation.

- $y = z + b + x(h)$
wealth consumption fin. bequests occ. choice

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- Child wealth $y' = (1 + r)b + \mathbf{w}_{t+1}(h)$.
- Parent picks (b, h) to **maximize utility**.
- No debt! $b \geq 0$.
- Child grows up; back to the same cycle.

Preferences and Equilibrium

- **Preferences:** mix of income-based and nonpaternalistic

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- **Equilibrium:** wages $\mathbf{w}_t$, value functions V_t , occupational distributions $\mathbf{n}_t$ s.t.:
 - Each family i chooses $\{h_t(i), b_t(i)\}$ optimally
 - Occupational choices $\{h_t(i)\}$ aggregate to $\mathbf{n}_t$;
 - Firms willingly demand $\mathbf{n}_t$ at prices $(\mathbf{w}_t, r)$.
 - **Note:** physical capital willingly supplied to meet any demand.

Steady State

- A **steady state** is a **stationary equilibrium** with positive output and wages:
 - $w_t = w \gg 0$, and
 - $(k_t, n_t) = (k, n)$ for all t , and $F(k, n) > 0$.

■ The richness assumption [R]:

- The set of all training costs is a compact interval $[0, X]$.
- If n is zero on any positive interval of training costs, then $y = 0$.

A Benchmark With No Occupational Choice

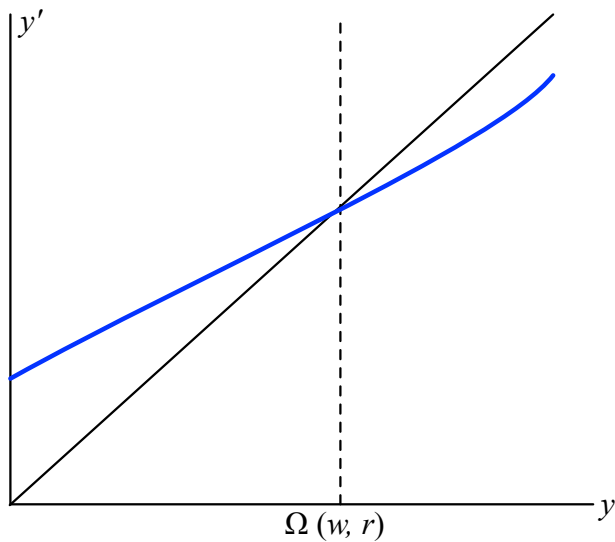
- **Financial bequests (at rate r) + just one occupation (wage w).**

- Parent with wealth y selects $b \geq 0$ to

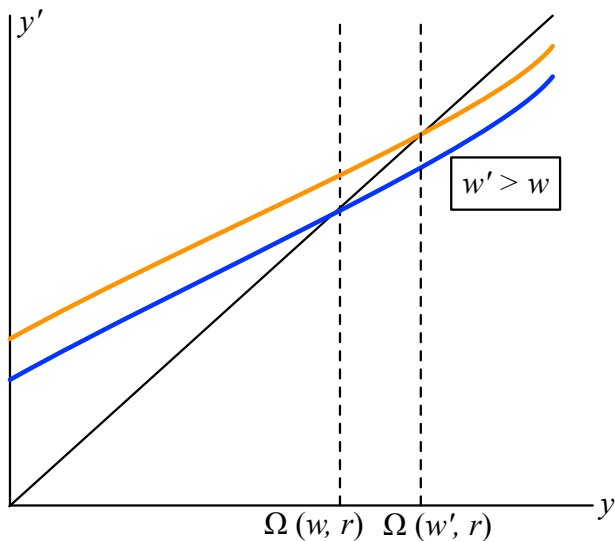
$$\max U(c) + \delta[\theta V(y') + (1 - \theta)P(y')].$$

- **Child wealth** $y' \equiv w + (1 + r)b$, increases in y .
- Converges to **limit wealth** $\Omega(w, r) < \infty$.
- This needs $\theta < 1$.
- Could depend on initial y (as in non-concave Ramsey model); we exclude that.

Limit Wealth in Benchmark Model



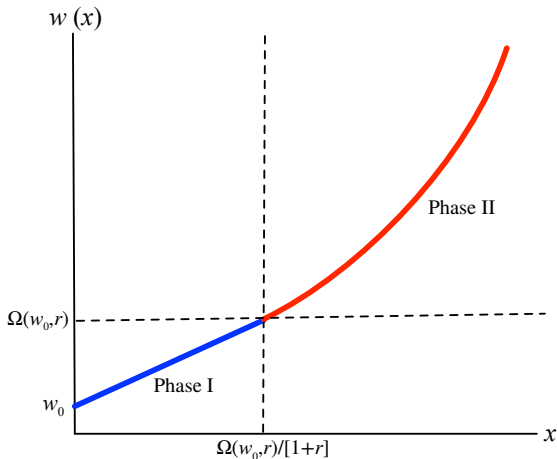
Limit Wealth in Benchmark Model



Back to Occupational Choice

Theorem 1

- Every steady state w is fully described by a **two-phase property**:



- **In Phase I w is linear in x :** there is $w_0 > 0$ such that

$$w(x) = w_0 + (1 + r)x \text{ for all } x \leq \frac{\Omega(w_0, r)}{1 + r}$$

- All families in Phase I have the **same overall wealth** $\Omega(w_0, r)$.

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- **In Phase II, w follows the differential equation**

$$w'(x) = \frac{U'(w(x) - x)}{\delta[\theta U'(w(x) - x) + (1 - \theta)P'(w(x))]}$$

- with endpoint to patch with I: $w(x) = w_0 + (1 + r)x$ as $x \downarrow \frac{\Omega(w_0, r)}{1 + r}$.
- Families located in Phase II have **different wealths and lifetime consumptions**.

■ Closer look at Phase II

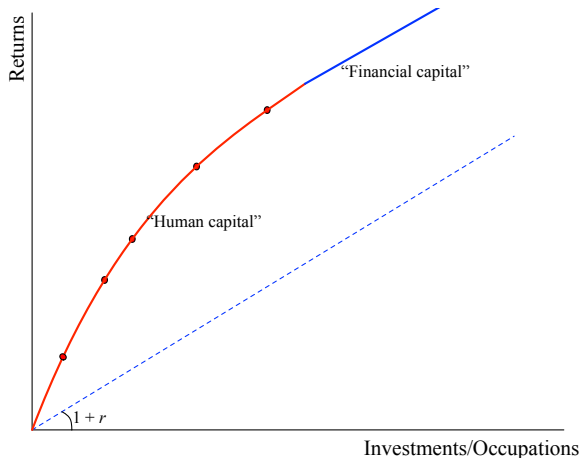
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■ Shape comes from **Euler equation**:

- depends fundamentally on preferences
- **technology only serves to pin down baseline** w_0 (remember remark on scaling)

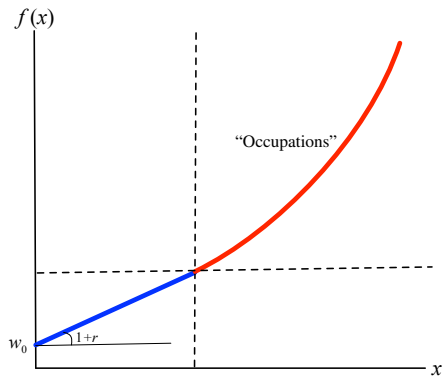
A Testable Implication

■ **Recall standard model.** By assumption:



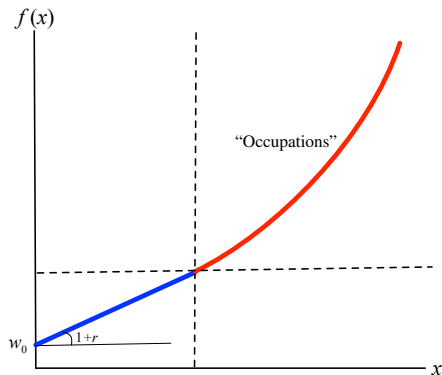
A Testable Implication

- Compare to Phases I and II: *This?*

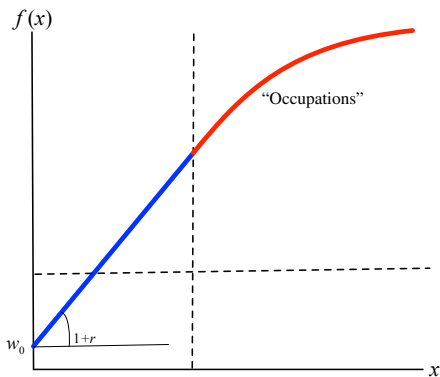


A Testable Implication

■ Compare to Phases I and II: *This?*



Or this?



A Testable Implication

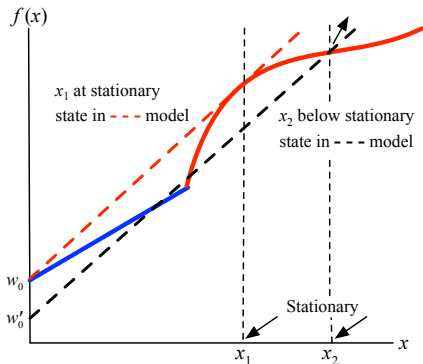
Theorem 2

The average return $\frac{w(x)-w_0}{x}$ to occupational investment is flat in Phase I and strictly increasing in Phase II.

A Testable Implication

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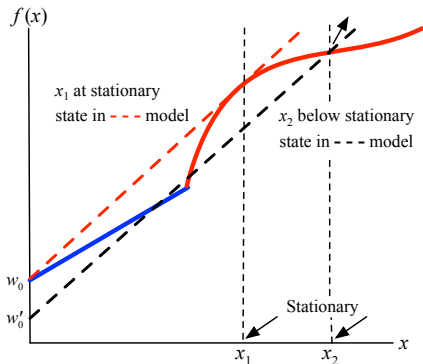
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The average return $\frac{w(x)-w_0}{x}$ to occupational investment is flat in Phase I and strictly increasing in Phase II.



- Contradiction to unique limit wealth in benchmark, increasing in w .

Unique Steady State with Rich Occupational Structure

- We end with a **fundamental difference from two-occupation case**:

Theorem 3

There is at most one steady state.

Unique Steady State with Rich Occupational Structure

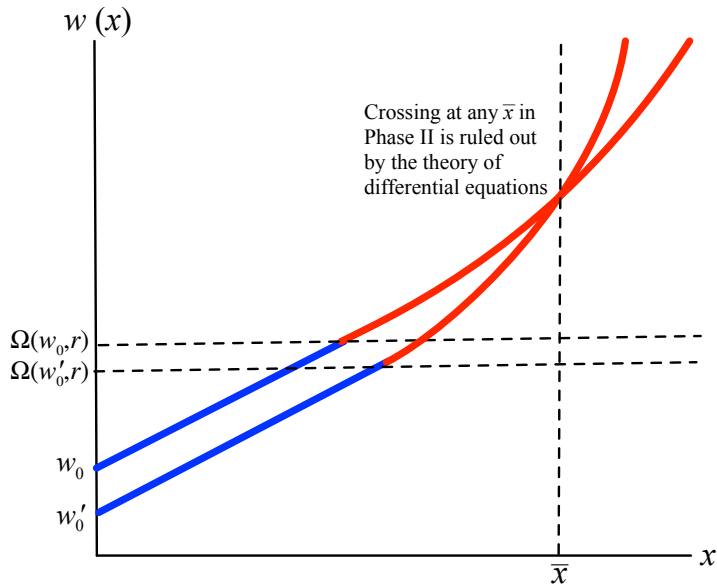
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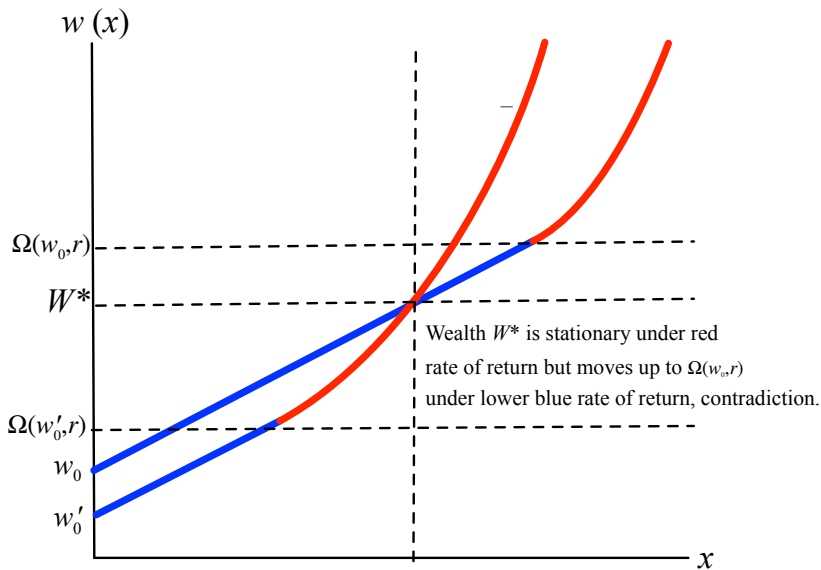
There is at most one steady state.

- **Proof idea:**
- No two members of the two-phase family (indexed only by w_0) can cross.
- Then **only one w_0 can support profit maximization** with positive output.
(For all wages must co-move with intercept wage w_0 .)

■ No-crossing argument, part I



■ No-crossing argument, part II



Three Remarks

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- In Phase I, there is **perfect equality** of overall wealth.
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- Families at different occupations in Phase II **cannot have the same wealth**.
- Thus, “most” inequality in this model comes from nonalienable capital.

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■ I. Alienable and Inalienable Capital

- In Phase I, there is **perfect equality** of overall wealth.
- (All families in Phase I must have wealth equal to $\Omega(w, r)$.)
- Families at different occupations in Phase II **cannot have the same wealth**.
- Thus, “most” inequality in this model comes from nonalienable capital.
- This focus will change when we consider automation in the next set of models and the decline in functional labor share.

Three Remarks

■ II. When is Phase II nonempty?

- When there is a **large occupation span relative to bequest motive**:
 - Discounting.
 - Poverty, via TFP differences.
 - Growth in TFP, lowers effective bequest motive
 - World return on capital.
 - Globalization: new occupations.

Three Remarks

■ III. Two Notions of History-Dependence

- At the macro-level, history-dependence depends on occupational richness.
- A lot of history-dependence at the individual level.
- The distribution **as a whole** is pinned down, but not who occupies which slot.