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- **Slides 4:** Measuring Upward Mobility

Mobility centrally important in current debates:

- In the United States and Europe

Chetty et al (2017), Alesina et al (2018), Manduca et al (2020)

- Connection to growth, inequality, aspirations etc.

Krueger (2012), Genicot and Ray (2017, 2020), Narayan (2018)

- The concept refers to:

- the ease of transition between various social categories;
- income, wealth, location, political persuasions ...

What the Term Might Mean

■ **Non-Directional:**

- **Pure movement:** off-diagonals in transition matrix. Atkinson (1981), Bartholomew (1982), Conlisk (1974), Dardanoni (1993), Hart (1976), Prais (1955), Shorrocks (1978a,b) ...

■ **Directional:**

- **Movement up** \succ **movement down**; Chakravarty et al. (1985), Bénabou and Ok (2001), Chetty et al. (2014), Bhattacharya (2011), Fields and Ok (1996, 1999), Mitra and Ok (1998) ...
-

■ **Relative:**

- **Change relative to others**; Chakravarty et al. (1985), Bénabou and Ok (2001), Chetty et al. (2014), Fields (2007), Bhattacharya (2011)

■ **Absolute:**

- **Change per se: growth +/-**; Fields and Ok (1996, 1999), Mitra and Ok (1998), Chetty et al. (2017)

■ **+ all combinations of these ...**

A Large But Still Incomplete List

Name	Measure	Directional	Non-directional	Absolute	Relative
King (1983)	$M_K = 1 - \exp \left[-\frac{\lambda}{n} \sum \frac{ z_i - y_i }{\mu_y} \right]$		✓		✓
Shorrocks index (1978)	$M_S = \frac{n - \text{Tr}(P)}{n-1}$		✓		✓
Variability of the eigenvalues	$\sigma(\gamma_i)$		✓		✓
Bartholomew (1982)	$M_B = \frac{1}{n-1} \sum_i \sum_j \pi_i p_{ij} i - j $		✓		✓
IG Income Elasticity (IGE)	$\beta = \frac{\text{Cov}(S_{it}, S_{it-1})}{\text{Var}(S_{it-1})}$		✓	✓	
Correlation coefficient (CE)	$\rho_S = \frac{\text{Cov}(S_{it}, S_{it-1})}{\sqrt{\text{Var}(S_{it})} \sqrt{\text{Var}(S_{it-1})}}$		✓	✓	
Slope rank-rank	$\rho_{PR} = \text{Corr}(P_i, R_i)$		✓		✓
IG rank association (IRA)	$\beta = \frac{\text{Cov}(p_{it}^y, p_{it}^x)}{\text{Var}(p_{it}^x)}$		✓		✓
Mitra & Ok (1998)	$\text{MO}_\alpha(\mathbf{x}, \mathbf{y}) = \frac{1}{n} \gamma \left(\sum_i y_i - x_i ^\alpha \right)^{1/\alpha}$		✓	✓	
Gini symmetric index of mobility	$GS = \frac{\sum_i (y_i - x_i)(F_{y_i} - F_{x_i})}{\sum_i (y_i - 1)F_{y_i} + \sum_i (x_i - 1)F_{x_i}}$		✓	✓	
Great Gatsby curve	$\text{Corr}(\text{Gini}, \text{IGE})$		✓	✓	
Bhattacharya (2011)	$\nu = Pr(F_1(Y_1) - F_0(Y_0) > \tau s_1 \leq F_0(Y_0) \leq s_2, X = x)$	✓			✓
Absolute upward mobility (1)	$p_{25} = \mathbb{E}(Y X \leq 25)$	✓			✓
Absolute upward mobility (2)	$A = \Phi \left(\frac{\mu_o - \mu_p}{\sqrt{\sigma_o^2 + \sigma_p^2 + 2\rho\sigma_o\sigma_p}} \right)$	✓			✓
Chetty et al (2017)	$\text{AM}(\mathbf{x}, \mathbf{y}) = \frac{1}{n} \sum_i (1_{y_i \geq x_i})$	✓		✓	
Rising up-up	$P_{20to100} = \mathbb{E}[Y = 100 X = 20]$	✓			✓
Bottom half mobility	$\mu_0^{50} = \mathbb{E}(y x \in [0, 50])$	✓			✓
Fields & Ok (1999)	$\text{FO}(\mathbf{x}, \mathbf{y}) = \frac{1}{n} \sum_i (\ln(y_i) - \ln(x_i))$	✓		✓	
Card (2018)	$\mathbb{E}(y > 50 x \in [45, 70])$	✓		✓	
Pro-poor growth	$G = \sum_{k=1}^5 w_k g_k$	✓		✓	

Why Another Measure?

■ Conceptual reasons

- Foundations unclear
- When clear, they are problematic

■ In particular:

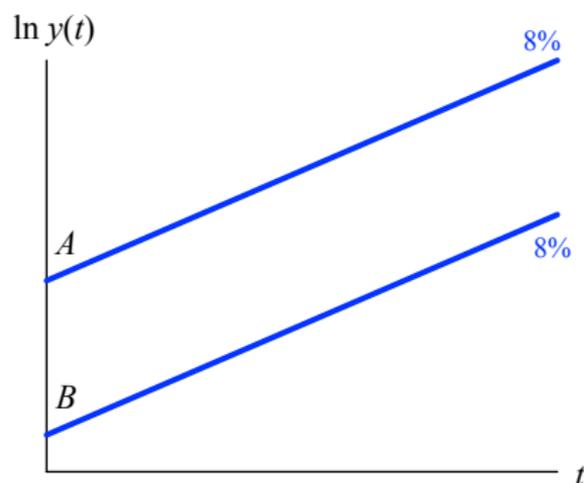
- “Mobility = mobility as pure movement + upward mobility”
- not interested in former component ([more later](#))

■ Data demands

- Existing measures rely heavily on panel data (more discussion later).
- This has held back empirical work, especially on developing countries.

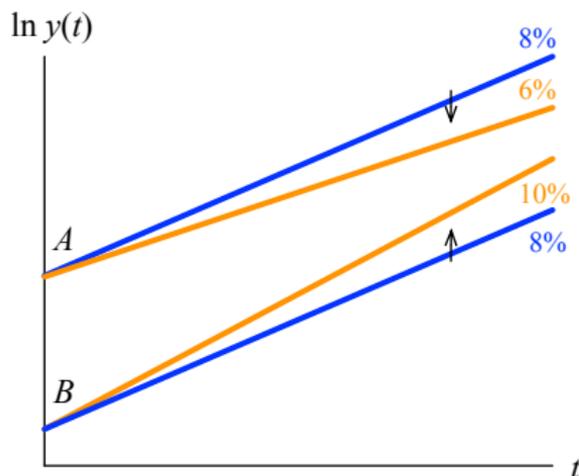
Upward Mobility

- We propose a measure of **upward mobility** that is:
 - **Directional**: rewards growth and punishes decline;
 - at least for absolute measures ([more on relative measures later](#)).
 - **Progressive**: higher if relatively poor enjoy faster growth.



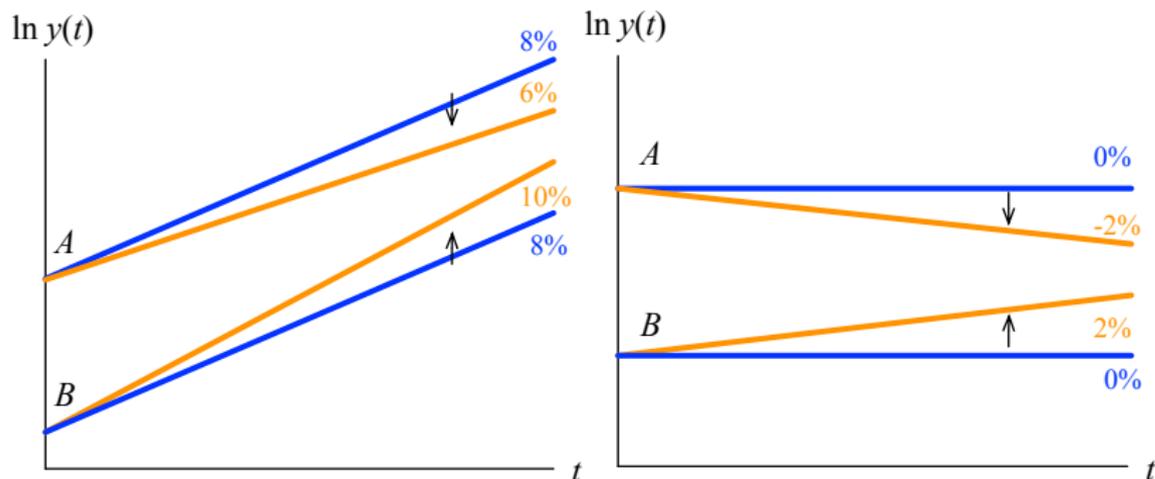
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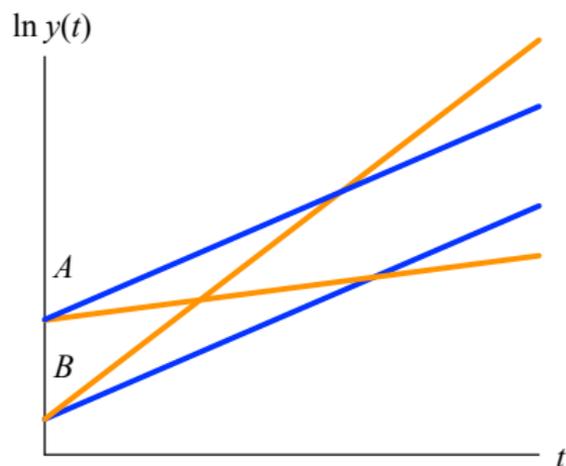
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Snapshots and Trajectories

- We divide our approach into two parts:
- An “instantaneous” measure or **upward mobility kernel** that is:
 - **intermediate step**
 - **directional** and **progressive**.
- A **mobility measure on trajectories** that is:
 - **what we're after**
 - based on the collection of instantaneous kernels.

Instantaneous Upward Mobility

- **Central variable:** y , “income.”
 - state variable for individual well-being.
 - e.g., “permanent income” or a proxy, such as consumption
- **Data:** For each person:
 - $y_i > 0$ baseline income
 - $g_i = \dot{y}_i/y_i$ instantaneous growth rate.
 - \mathbf{z} = the full collection $\{z_i\}_{i=1}^n$, where $z_i = (y_i, g_i)$.

Instantaneous Upward Mobility

- **Upward mobility kernel:** $M(\mathbf{z})$, where $\mathbf{z} = \{z_i\}_{i=1}^n$, and $z_i = (y_i, g_i)$.
 - Anonymous, continuous.
 - Zero-growth normalization:
 $g_i = 0$ all $i \mapsto M(\mathbf{z}) = 0$.
 - Consistency under population mergers.

Details

Core Axiom

■ Examples:

- $\mathbf{y} = (5000, 10000) + \mathbf{g} = (8\%, 8\%) \succ \mathbf{y} = (5000, 10000) + \mathbf{g} = (6\%, 10\%)$.
- $\mathbf{y} = (5000, 10000) + \mathbf{g} = (2\%, -2\%) \succ \mathbf{y} = (5000, 10000) + \mathbf{g} = (0\%, 0\%)$.
- No crossings in continuous time.

■ Growth Progressivity.

- For any \mathbf{z} , i and j with $y_i < y_j$, and $\epsilon > 0$, send g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$.
- Then $M(\mathbf{z}') > M(\mathbf{z})$.

■ Notes:

- Measure tolerates lower growth if poor can grow faster.
- Upward mobility \neq overall welfare.

Upward Mobility Kernel

Theorem 1

An upward mobility kernel is growth progressive if and only if it can be written as

$$M(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) g_i$$

for continuous permutation-invariant $\{\phi_i\}$, with $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$ when $y_i < y_j$.

Sharpening the Kernel

- **Income Neutrality.** $M(\mathbf{y}, \mathbf{g}) = M(\lambda\mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.
- **Growth Alignment.** $\mathbf{g} > \mathbf{g}' \Rightarrow M(\mathbf{y}, \mathbf{g}) > M(\mathbf{y}, \mathbf{g}')$ all \mathbf{y} .
- **Independent Pairwise Growth Tradeoffs:**

Is $M((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij})) \geq M((y_i, g'_i), (y_j, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$?

Answer insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$.

Theorem 2

Under additional three axioms and $n \geq 3$, M can be written as:

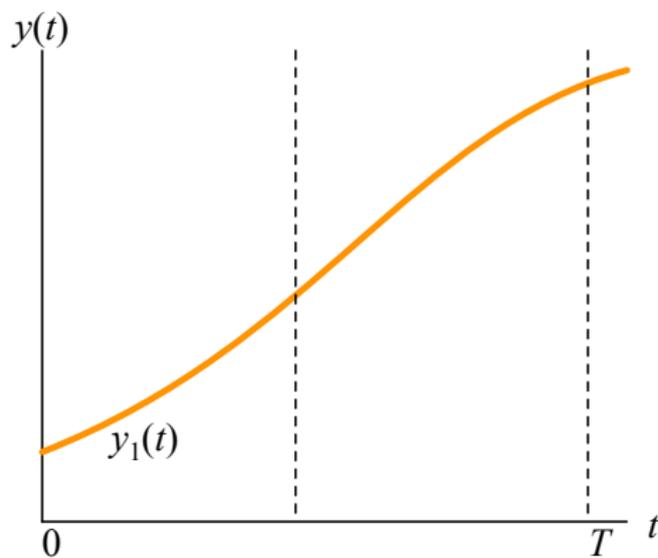
$$M_{\alpha}(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}}, \text{ for some } \alpha > 0.$$

- Proof employs a substantial extension of Gorman's separability theorem;

see Chatterjee (R) Ray (R) Sen (2021).

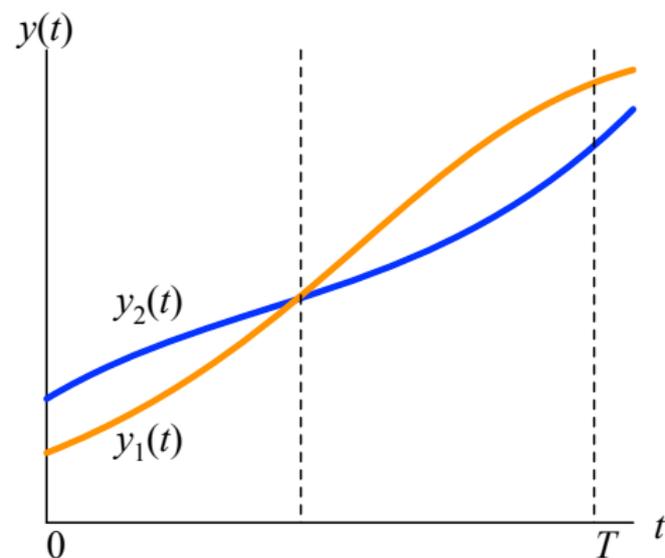
Income Trajectories

Towards a measure on trajectories:



Income Trajectories

Towards a measure on trajectories:



- $\mathbf{y}[s, t] = \{y_i(\tau)_s^t\}_{i=1}^n$
- **Upward mobility measure:** $\mu(\mathbf{y}[s, t])$.

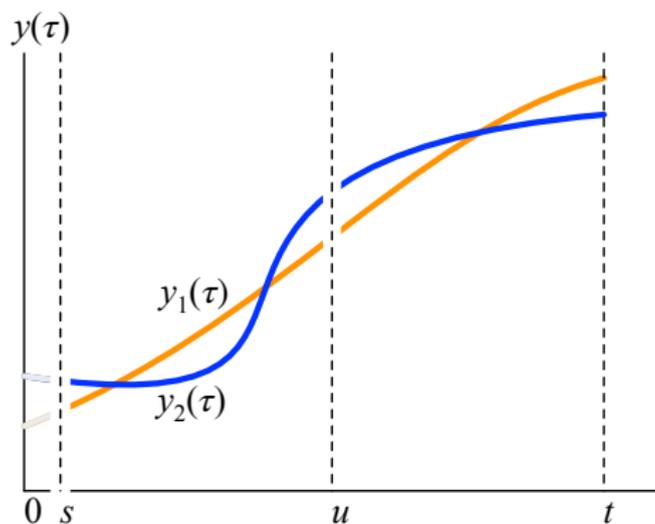
Reducibility

- Assume $\mathbf{y}[s, t]$ continuously differentiable. Then:
 - Well-defined $\mathbf{z}(\tau) = (\mathbf{y}(\tau), \mathbf{g}(\tau))$ for each $\tau \in [s, t]$.
 - Well-defined $M(\mathbf{z}(\tau))$ for each $\tau \in [s, t]$.
- μ is **reducible** if it's expressible as a function of all these M 's:

$$\mu(\mathbf{y}[s, t]) = \Psi(\{M(\mathbf{z}(\tau))\}_s^t)$$

- with $\mu(\mathbf{y}[s, t]) = m$ whenever $M(\mathbf{z}(\tau)) = m$ for all $\tau \in [s, t]$ (**normalization**)

Additivity



- μ is **additive** if for all $s < u < t$,
- $(t - s)\mu(\mathbf{y}[s, t]) = (u - s)\mu(\mathbf{y}[s, u]) + (t - u)\mu(\mathbf{y}[u, t])$.

Theorem 3

Kernel axioms, reducibility, and additivity hold if and only if

$$\mu_{\alpha}(\mathbf{y}[s, t]) = \frac{1}{t - s} \ln \left[\frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0.$$

- **Remark:** Can also use income categories and population shares (see paper).
- In what follows, we look at different aspects of this measure.

Upward Mobility as Change in Welfare

- **Mobility measure:**

$$\mu_{\alpha}(\mathbf{y}[s, t]) = \frac{1}{t - s} \ln \left[\frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \quad \text{for some } \alpha > 0.$$

- **Atkinson welfare function, or Atkinson equivalent income:**

$$a_{\alpha}(\mathbf{y}) = \left(\frac{1}{n} \sum_{j=1}^n y_j^{-\alpha} \right)^{-\frac{1}{\alpha}},$$

for $\alpha > 0$ (elasticity restricted).

- $\mu_{\alpha}(\mathbf{y}[s, t]) =$ **average growth of Atkinson equiv income** on $[s, t]$.

- Not a measure of equality per se.

Upward Mobility as Pro-Poor Growth

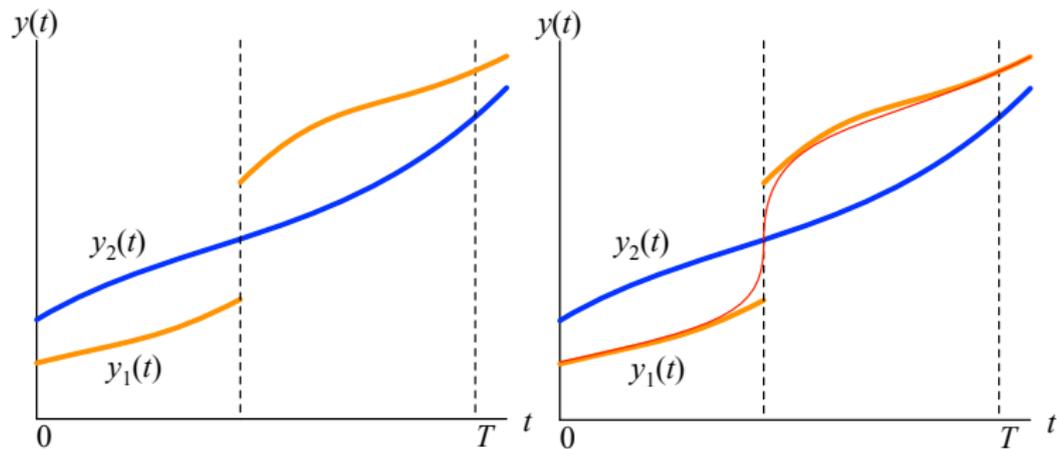
$$\begin{aligned} \text{Upward Mobility} &= \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^n y_j(t)^{-\alpha}}{\sum_{j=1}^m y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}} \\ \text{Growth} &= \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^n y_j(t)}{\sum_{j=1}^m y_j(s)} \right] \end{aligned}$$

Upward Mobility as Pro-Poor Growth

- **Upward Mobility** = $\frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^n y_j(t)^{-\alpha}}{\sum_{j=1}^m y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$
- **Growth** = $\frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^n y_j(t)}{\sum_{j=1}^m y_j(s)} \right] = \mu_{-1}(\mathbf{y}[s, t])$
- Isn't even on our "boundary" as $\alpha \rightarrow 0$.
- Nevertheless, when all growth rates are the same, $\mu_{\alpha} = \text{growth rate}$.

Discontinuous Trajectories

- If there are jumps, then mobility kernels aren't defined at some points.
- Examples: inheritance, job change, promotions ...



- Approximate by smooth functions and use continuity: **same answer.**

Relative Upward Mobility

- **Relative upward mobility** nets out growth:

$$\begin{aligned}\rho_{\alpha}(\mathbf{y}[s, t]) &= \mu_{\alpha}(\mathbf{y}[s, t]) - \frac{1}{t-s} [\ln(\bar{y}(t)) - \ln(\bar{y}(s))] \\ &= \frac{1}{t-s} \ln \left[\frac{\sum_{i=1}^n e_i(t)^{-\alpha}}{\sum_{i=1}^n e_i(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}\end{aligned}$$

- where $e_i = y_i/\bar{y}$ is **excess growth factor** relative to per-capita income \bar{y} .
- ρ_{α} admissible under Theorem 1; can be further axiomatized.

Upward Mobility and Panel Independence

- We now arrive at a central point of the paper:

- **Upward Mobility** = $\frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^n y_j(t)^{-\alpha}}{\sum_{j=1}^m y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$ is **panel independent**.

Upward Mobility and Panel Independence

1. Oh come on. Mobility is a construct for *dynasties* or *lineages*.

- **Answer:** To assess a family's changing fortunes, *that* family must be tracked.
- But to assess upward mobility overall, it is *society* that must be tracked.
- A family receives *time-varying weights* depending on its relative location.
- The impact on overall mobility feeds through the impact on mobility kernels.
- Such nimble weight switches are central to our argument.

Upward Mobility and Panel Independence

1. Oh come on. Mobility is a construct for *dynasties* or *lineages*.

■ Specifically, study how the axioms work:

■ **Growth Progressivity** \Rightarrow linearity of the kernel in growth rates.

■ **Reducibility** \Rightarrow

$$\mu(\mathbf{y}[s, t]) = \Psi \left(\left\{ \sum_{i=1}^n \phi_i(\mathbf{y}(\tau)) g_i(\tau) \right\}_s^t \right) = \Psi \left(\left\{ \sum_{i=1}^n \frac{\phi_i(\mathbf{y}(\tau))}{y_i(\tau)} \dot{y}_i(\tau) \right\}_s^t \right).$$

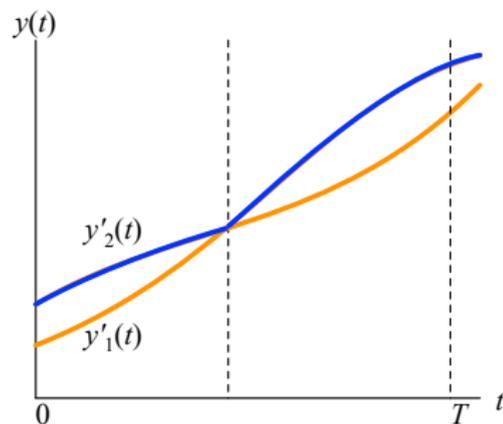
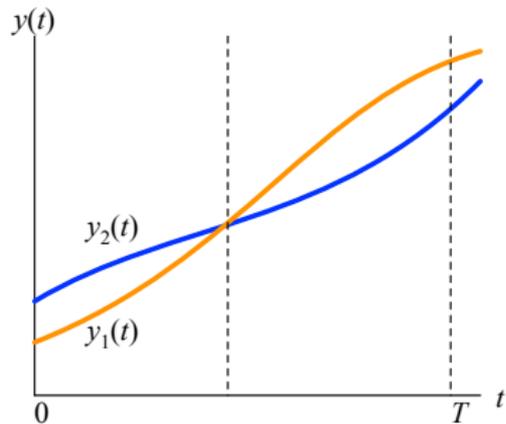
■ **Additivity** \Rightarrow

$$\mu(\mathbf{y}[s, t]) = \int_s^t \sum_{i=1}^n \frac{\phi_i(\mathbf{y}(\tau))}{y_i(\tau)} \dot{y}_i(\tau) d\tau.$$

■ $\frac{\phi_i(\mathbf{y})}{y_i} = \frac{y_i^{-\alpha-1}}{\sum_j y_j^{-\alpha}}$, which integrates out to Atkinson welfare. jumps?

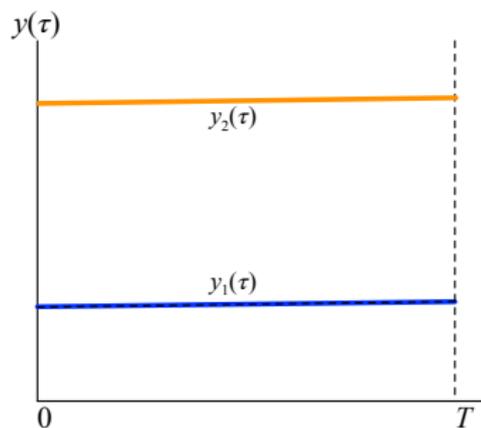
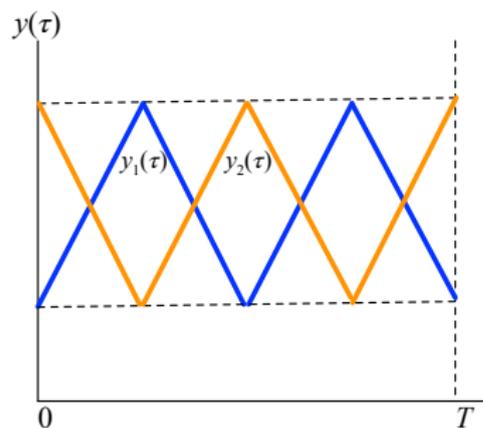
Upward Mobility and Panel Independence

2. But what about mobility as pure movement “back and forth”?



Upward Mobility and Panel Independence

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- Different exchange mobility or pure movement. ✓
- Different inequalities. ✓
- But **upward** mobility in both panels is zero.

Upward Mobility and Panel Independence

2. But what about mobility as pure movement “back and forth”?

- Upward mobility **subtracts** downward movements from upward movement
- Exchange mobility **adds** them.

$$M_{\alpha}(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) g_i = M_{\alpha}^{+}(\mathbf{z}) - M_{\alpha}^{-}(\mathbf{z})$$

- where $M_{\alpha}^{+}(\mathbf{z}) = \sum_{i=1}^n \phi_i^{+}(\mathbf{y}) \max\{g_i, 0\}$ and $M_{\alpha}^{-}(\mathbf{z}) = \sum_{i=1}^n \phi_i^{-}(\mathbf{y}) \max\{-g_i, 0\}$.

$$E_{\alpha}(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) |g_i| = M_{\alpha}^{+}(\mathbf{z}) + M_{\alpha}^{-}(\mathbf{z})$$

- Our preferred approach to exchange mobility.
- Such a measure would not be panel-independent.

Upward Mobility and Panel Independence

3. Income isn't a sufficient statistic for lifetime welfare.

- **Answer:** That's entirely possible.
- But imperfection of measurement is not an excuse for changing the measure.
- It is a reason to use the best data we have.
- Try **consumption** or **wealth** as proxies for a state variable; Deaton and Zaidi (2002)
- Or some other measure of permanent income (time-averaged?).
- Similar recommendations apply to poverty or inequality measurement.

Upward Mobility and Panel Independence

4. But even then, individuals may belong to different **social** groups. How do we take that into account?

- **Answer:** K social groups. Each person i belongs to one $k(i) \in K$.
- Data for kernel: (\mathbf{z}, \mathbf{w}) , with $z_i = (y_i, g_i)$, w_k the mean income of group k .

Social Growth Progressivity. For any \mathbf{z} , i and j with $(y_i, w_{k(i)}) \leq (y_j, w_{k(j)})$, form \mathbf{z}' by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $M(\mathbf{z}') > M(\mathbf{z})$.

Social Income Neutrality. $M(\lambda \mathbf{y}, \mathbf{g}, \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$ & $M(\mathbf{y}, \mathbf{g}, \lambda \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$.

Social Binary Growth Tradeoffs. For any i, j , any $(y_i, y_j, w_{k(i)}, w_{k(j)})$, comparing $((y_i, w_{k(i)}, g_i), (y_j, w_{k(j)}, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ and $((y_i, w_{k(i)}, g'_i), (y_j, w_{k(j)}, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ is insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)})$.

Upward Mobility and Panel Independence

4, contd.

Theorem 4

The above axioms hold if and only if for $n \geq 3$ and groupings K ,

$$\mu_{\alpha,\beta}(\mathbf{y}[s, t], K) = \frac{1}{t-s} \left\{ \ln \left[\frac{\sum_{i=1}^n y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^n y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}} \right]^{-1/\alpha} - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau \right\},$$

for some $(\alpha, \beta) \gg 0$, where $a_k(\tau)$ is Atkinson equivalent group income.

- First term on RHS is panel-independent.
- Second term depends on trajectories, but **only at the group level**.
- Can approximate group Atkinson by standard inequality measures (see paper).

Upward Mobility and Panel Independence

5. Anyway, we typically have panel data, don't we?

■ **Answer:** No.

■ For the United States, [Chetty et al \(2017\)](#) estimate:

■ % population share: children \succ parents (US birth cohorts, 1940–84).

■ Transitions estimated from a [unique panel of tax records](#)

■ \oplus marginal income distributions from CPS and Census.

■ Generally very hard to get hold of.

■ Though similar studies exist for other countries; e.g., [Acciari et al \(2021\)](#).

Upward Mobility: Other Measures

Skip?

- The **Chetty et al** (2017) measure (also Berman 2021, Acciari et al 2021):

$$\mu^c(\mathbf{y}[0, 1]) = \sum_{i=1}^n I(y_i(0), y_i(1)).$$

- where $I(y_i(0), y_i(1))$ is indicator for $y_i(0) < y_i(1)$.
- Population share for whom future \succ present.

- The **Fields-Ok** (1999) measure:

$$\mu^{\text{FO}}(\mathbf{z}) = \frac{1}{n} \sum_{i=1}^n [\ln(y_i(0)) - \ln(y_i(1))] = \frac{1}{n} \sum_{i=1}^n \left[\int_0^1 g_i(\tau) d\tau \right].$$

- Both must fail growth progressivity.

Upward Mobility: Other Measures

■ Example for μ^c :

- Two persons at incomes \$10,000 and \$20,000.
- Growth rates 1% for both. Then $\mu^c = 1$.
- Transfer 2 points of growth from rich to poor. Then $\mu^c = 1/2$.
- But growth progressivity asks that mobility must rise.

Upward Mobility: Other Measures

- **Rank-weighted measures:**

- Such measures fail our axioms in a seemingly technical way:
- They are not continuous — and this isn't just a technicality.

- **Tiny changes** in incomes can generate **discrete jumps** in mobility.

- And worse: large changes in *relative* income could go unnoticed.

- **Our measure is indeed correlated with rank-based measures.**

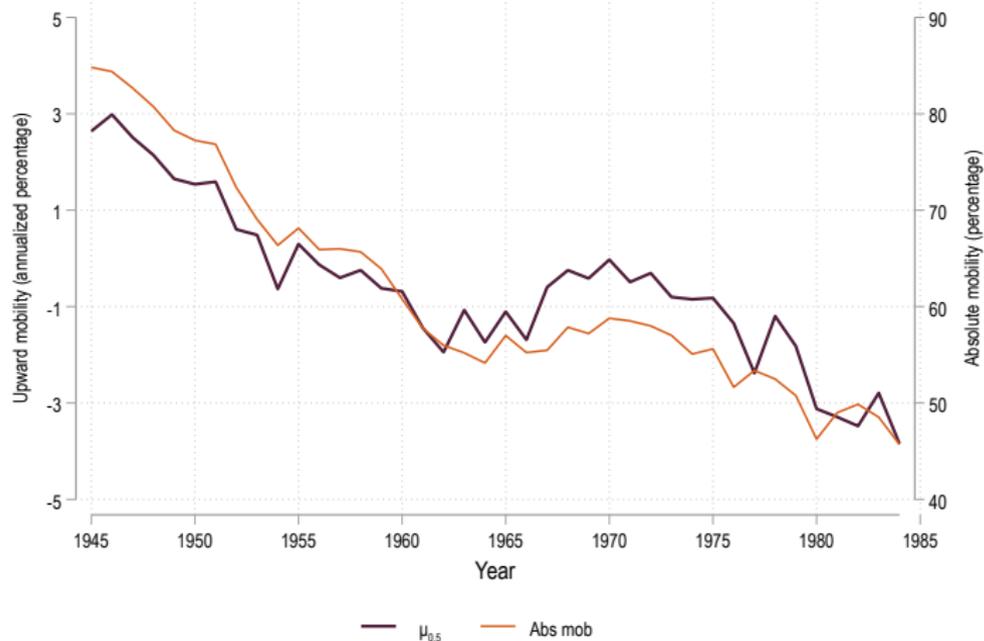
- But is sensitive throughout, without being unduly affected by a rank switch.

Upward Mobility in the Data

- **Chetty et al (2017) estimate $M^I(\mathbf{z})$** for US birth cohorts, 1940–84.
 - They estimate a copula from a unique panel of tax records.
- *In practice, the dependence on exact copulas seems limited*; Berman (2021)

“Estimating the absolute mobility in the United States with different copulas, some of which are very different from the one characterizing the United States, results in a similar evolution in time.”

μ_α Compared to Chetty et al (2017) for the United States



- Robust to different α .
- Robust to using other publicly available databases (e.g., WID).

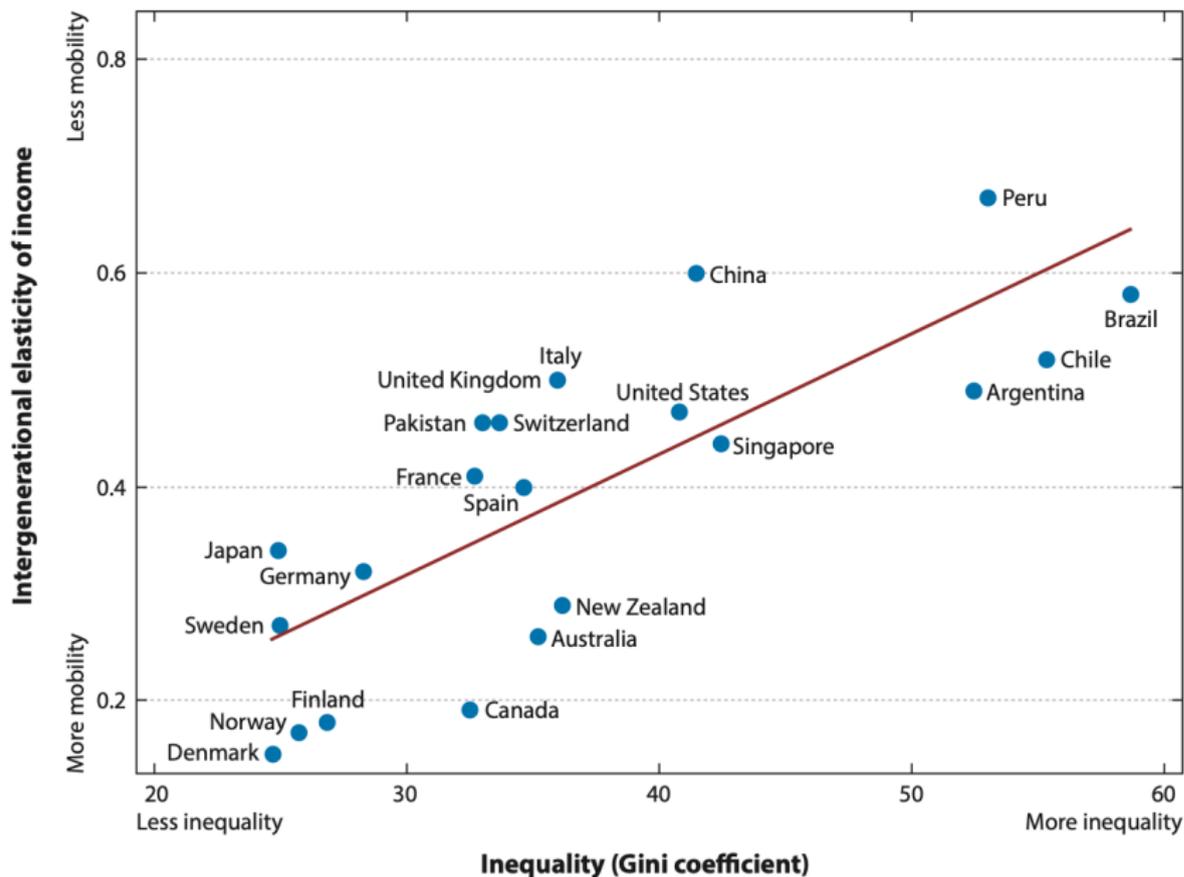
Upward Mobility in Brazil, India and France

- **Ten-year upward mobility in Brazil, India and France:**
 - Data from the World Inequality Database (repeated cross-sections).
 - Measure $\mu_{0.5}(\mathbf{y}[t, t + 10])$ and $\rho_{0.5}(\mathbf{y}[t, t + 10])$.
 - Robust with respect to choice of α (see paper).

Upward Mobility in Brazil, India and France



The Great Gatsby Curve



What is the Intergenerational Elasticity of Income?

- **IGE** is the estimate of β in

$$\log y(t+1) = \alpha + \beta \log y(t) + u(t)$$

where $(t, t+1)$ tracks a “parent-child pair”.

$$\text{IGE} = \hat{\beta} = \frac{\text{Cov}(\log y_i(t), \log y_i(t+1))}{\sigma(\log y_i(t))^2}. \quad (1)$$

- Pure correlation between $\log y_i(t)$ and $\log y_i(t+1)$ given by

$$\rho = \frac{\text{Cov}(\log y_i(t), \log y_i(t+1))}{\sigma(\log y_i(t))\sigma(\log y_i(t+1))}, \quad (2)$$

where σ = standard deviation.

- Combine (1) and (2) to get ...

IGE Decomposition

$$\text{IGE} = \rho \frac{\sigma(\log y_i(t+1))}{\sigma(\log y_i(t))}.$$

- ρ is **exchange immobility**: scale, growth-rate, distribution invariant.
- $\frac{\sigma(\log y_i(t+1))}{\sigma(\log y_i(t))}$ is **ratio of standard deviations** of log income across time.

An often-used (though flawed) measure of inequality.

- Changes in inequality affect IGE separately from ρ .
- Our measures can be interpreted as **getting rid of ρ entirely and replacing second component.**

The Great Gatsby Curve, Respecified

- Our **re-specification**:

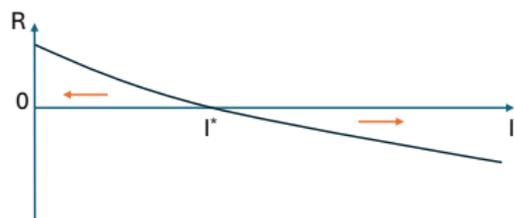
$$\mathbf{Mobility}_{jst} = \Psi(\mathbf{Inequality}_{js}) + \phi y_{js} + f_j + h_t + \epsilon_{jst}.$$

- **Mobility** = upward mobility (μ_α or ρ_α) over $[s, t]$; e.g., $t = s + 10, s + 30 \dots$
- Ψ is a flexible (nonparametric or pre-specified) functional form.
- **Inequality** $_{js}$: Gini — or Atkinson, given link with upward mobility.
- y_{js} is **income per-capita** for country j at date s .

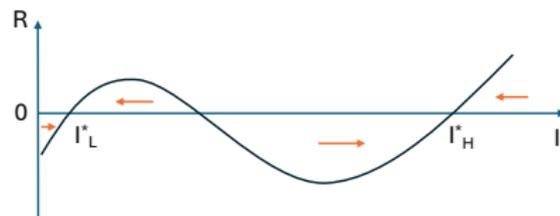
The Great Gatsby Curve and Inequality Traps

$$\text{Mobility}_{jst} = \Psi(\text{Inequality}_{js}) + \phi y_{js} + f_j + h_t + \epsilon_{jst}$$

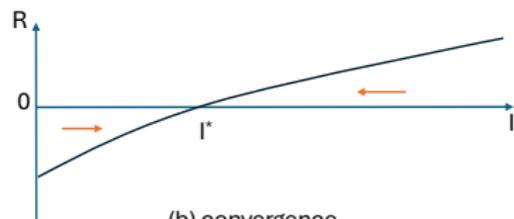
is a dynamic equation for the evolution of economic inequality in society.



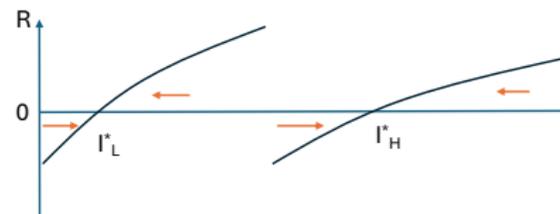
(a) divergence



(c) multiple equilibria



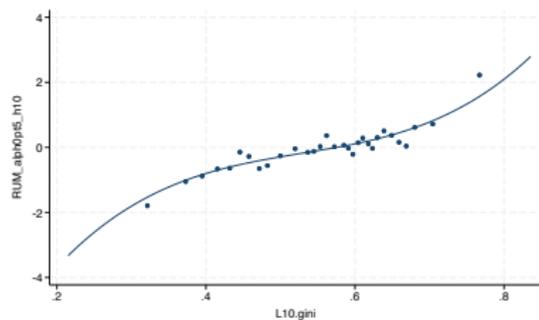
(b) convergence



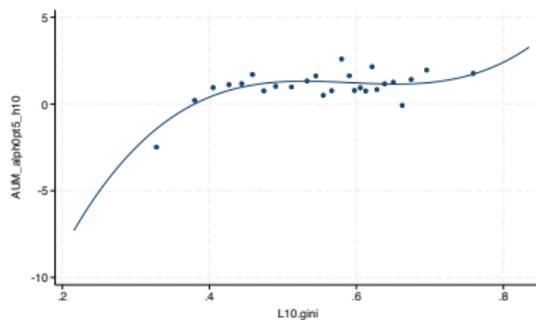
(d) multiple equilibria

- Deciles (or percentiles) of the income distribution
- lack individual-level data for large sample of country-year observations.
- Two datasets:
 - **World Inequality Database** (WID): 133 countries, 1980-2021
 - **Poverty and Inequality Platform** (PIP): harmonized global survey data.

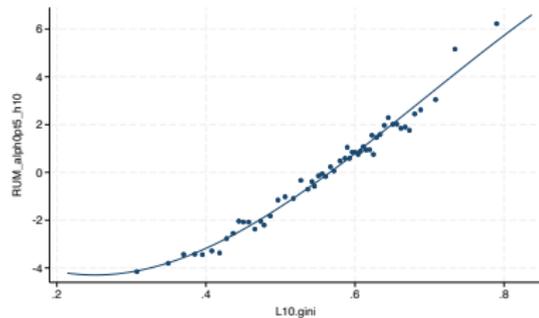
Graphical Analysis: Gini (same with Atkinson)



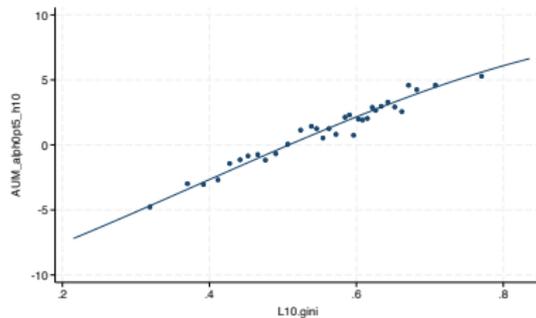
Rel. mobility vs Gini, $\alpha = 0.5$



Abs. mobility vs Gini, $\alpha = 0.5$



Rel. mobility vs Gini, FE $\alpha = 0.5$



Abs. mobility vs Gini, FE $\alpha = 0.5$

$$\text{Mob}^\alpha(\mathbf{y}[h])_{ct} = \beta I_{ct-h} + \sum_{j=1}^p \gamma_j \text{Mob}^\alpha(\mathbf{y}[h])_{ct-j} + \eta_c + \delta_t + \epsilon_{ct},$$

- $h = t - s$ denotes the **horizon** of the mobility measure ($h = 10$ here).
- j : **lagged mobility** ranging from 0 to 4 (elimination of serial autocorrelation discussed in paper)
- **Robust standard errors** clustered at the country level

Linear Model: Gini (same with Atkinson)

	DEPENDENT VARIABLE: Relative Upward Mobility										
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
G_{t-10}	19.491*** (0.000)	7.099*** (0.000)	7.654*** (0.000)	7.520*** (0.000)	7.372*** (0.000)	7.400*** (0.000)	6.907*** (0.000)	7.792*** (0.000)	7.807*** (0.000)	8.250*** (0.001)	9.692*** (0.004)
$R(y[10])_{ct-1}$		0.780*** (0.000)	0.752*** (0.000)	0.777*** (0.000)	0.756*** (0.000)	0.755*** (0.000)	0.828*** (0.000)	0.755*** (0.000)	0.764*** (0.000)	0.718*** (0.000)	0.688*** (0.000)
$R(y[10])_{ct-2}$			0.015 (0.840)	0.051 (0.148)	0.071 (0.164)	0.071 (0.164)		0.061 (0.570)	0.077 (0.268)	0.090 (0.323)	0.088 (0.321)
$R(y[10])_{ct-3}$				-0.068 (0.115)	0.047 (0.519)	0.047 (0.517)			-0.036 (0.469)	0.050 (0.508)	0.053 (0.478)
$R(y[10])_{ct-4}$					-0.140** (0.047)	-0.140** (0.047)				-0.122* (0.094)	-0.123* (0.090)
LOG INCOME P.C. _{t-1}						-0.025 (0.748)					-0.490 (0.454)
AR(2) autoc. test (p-val)							0.783	0.981	0.836	0.467	0.460
R ²	0.270	0.760	0.768	0.773	0.777	0.777					
Obs	4253	4120	3987	3854	3721	3721	3987	3854	3721	3588	3588

Linear Model: Gini (same with Atkinson)

	DEPENDENT VARIABLE: Absolute Upward Mobility										
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
G_{t-10}	23.149*** (0.000)	6.013*** (0.000)	5.790*** (0.000)	5.387*** (0.000)	4.996*** (0.000)	5.019*** (0.000)	5.391*** (0.001)	3.896** (0.010)	3.617** (0.034)	8.477*** (0.001)	5.732** (0.040)
$M(y[10])_{ct-1}$		0.884*** (0.000)	1.031*** (0.000)	1.013*** (0.000)	0.967*** (0.000)	0.976*** (0.000)	0.960*** (0.000)	1.185*** (0.000)	1.144*** (0.000)	0.843*** (0.000)	1.034*** (0.000)
$M(y[10])_{ct-2}$			-0.168*** (0.001)	-0.028 (0.419)	-0.002 (0.951)	-0.001 (0.968)		-0.250*** (0.007)	-0.132 (0.155)	0.006 (0.952)	-0.031 (0.690)
$M(y[10])_{ct-3}$				-0.135*** (0.000)	0.009 (0.818)	0.008 (0.840)			-0.083*** (0.006)	-0.013 (0.860)	0.010 (0.803)
$M(y[10])_{ct-4}$					-0.144*** (0.000)	-0.141*** (0.000)				-0.133 (0.137)	-0.105** (0.012)
LOG INCOME P.C. _{t-1}							-0.345 (0.241)				-1.847 (0.160)
R ²	0.236	0.842	0.851	0.859	0.861	0.861					
Obs	4253	4120	3987	3854	3721	3721	3987	3854	3721	3588	3588

Remarks

- Positive and significant relationship between mobility and inequality
- One sd \uparrow in Gini (0.1) \Rightarrow relative upward mobility \uparrow 0.74 (\approx 65% of sd)
- Weaker but still positive association with absolute upward mobility:
 - negative relationship between inequality and growth

Measurement Error and Mean Reversion: Gini

Average incomes over 3 years:

	DEPENDENT VARIABLE: Relative Upward Mobility											
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	
G_{t-10}	19.563*** (0.000)	8.552*** (0.000)	8.595*** (0.000)	8.784*** (0.000)	8.233*** (0.000)	7.989*** (0.000)	8.163*** (0.000)	9.417*** (0.000)	9.965*** (0.000)	9.209*** (0.004)	12.894*** (0.000)	
$R(y[10])_{ct-1}$		0.674*** (0.000)	0.481*** (0.000)	0.479*** (0.000)	0.469*** (0.000)	0.469*** (0.000)	0.748*** (0.000)	0.551*** (0.000)	0.585*** (0.000)	0.573*** (0.000)	0.358*** (0.000)	
$R(y[10])_{ct-2}$			0.256*** (0.000)	0.273*** (0.000)	0.334*** (0.000)	0.332*** (0.000)		0.187*** (0.000)	0.188*** (0.000)	0.251*** (0.000)	0.316*** (0.000)	
$R(y[10])_{ct-3}$				-0.027 (0.697)	0.071 (0.238)	0.069 (0.241)			-0.049 (0.462)	0.041 (0.563)	0.080 (0.119)	
$R(y[10])_{ct-4}$					-0.188*** (0.000)	-0.185*** (0.000)				-0.169*** (0.000)	-0.173*** (0.000)	
INCOME P.C. (LOG) $_{t-10}$							0.696** (0.029)				0.499 (0.569)	
AR(2) autoc. test (p-val)								0.000	0.289	0.444	0.076	0.113
R ²	0.224	0.589	0.619	0.620	0.635	0.644						
Obs	4555	4403	4251	4099	3947	3947	4251	4099	3947	3795	3795	

Measuring Upward Mobility: A Summary

- A **bewildering variety** of mobility indices:
 - directional/non-directional; absolute/relative.
- We axiomatize a **class of upward mobility measures**
 - At the core is the **growth progressivity axiom**.
 - Analogue of the Lorenz criterion for inequality measurement
- Our **trajectory-based measure** is pinned down by two conditions
 - **reducibility** and **additivity**.
 - It is **panel-independent**
- If convincing, this **significantly expands the scope of empirical inquiry**

Appendix 1: Population Consistency

Given: $\mathbf{z} = (y_1, g_1, \dots, y_k, g_k, \dots, y_n, g_n)$

$$\mathbf{z}' = (y_1, g_1, \dots, y_k, g_k - \epsilon, \dots, y_n, g_n) \quad |$$

$$\mathbf{z}'' = (y_1, g_1, \dots, y_k, g_k + \epsilon, \dots, y_n, g_n)$$

and \mathbf{z}' and \mathbf{z}'' have average mobility distinct from \mathbf{z} : $\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$,

Then: $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$.

Appendix 2: Proof of Theorem 1

- **Step 1. For every k , $m(g_k) \equiv M(g_k | \mathbf{y}, \mathbf{g}_{-k})$ is affine in g_k , or equivalently:**

$$m(g_k) = \frac{1}{2} [m(g_k - \epsilon) + m(g_k + \epsilon)] \text{ for every } \epsilon > 0.$$

- Suppose false for some g_k and ϵ .
- Define $\mathbf{z} = (\mathbf{y}, \mathbf{g}_{-k}, g_k)$, $\mathbf{z}' = (\mathbf{y}, \mathbf{g}_{-k}, g_k - \epsilon)$, and $\mathbf{z}'' = (\mathbf{y}, \mathbf{g}_{-k}, g_k + \epsilon)$.
- Then $M(\mathbf{z}') + M(\mathbf{z}'') \neq M(\mathbf{z}) + M(\mathbf{z})$.
- By Local Merge, $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$.
- Say $M(\mathbf{z}' \oplus \mathbf{z}'') > M(\mathbf{z} \oplus \mathbf{z})$.

Appendix: Proof of Theorem 1

- **Step 2. (Gallier 1999)** $M(z)$ multiaffine so can be written as:

$$M(\mathbf{z}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right].$$

for a collection $\{\phi_S\}$ defined for every $\emptyset \neq S \subset \{1, \dots, n\}$.

- **Step 3. All nontrivial product terms above *must have zero coefficients*.**

Suppose $\{ij\} \subset S$ for some S with $\phi_S(\mathbf{y}) \neq 0$. We will only move g_i and g_j but with $g_i + g_j = G$, so hold all else fixed and write

$$\begin{aligned} M(\mathbf{y}, \mathbf{g}) &= \alpha g_i(G - g_i) + \beta g_i + \gamma(G - g_i) + \delta. \\ \Rightarrow \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_i} - \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_j} &= \alpha G - 2\alpha g_i + \beta - \gamma. \end{aligned}$$

Choose G and g_i to violate Growth Progressivity. [back](#)