EC9AA Term 3: Lectures on Economic Inequality

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- Slides 2: Occupational Choice and Inequality

Markets and Personal Inequality

- Two views:
- Equalization: Inequality an ongoing battle between convergence and "luck."
  - Solow 1956, Brock-Mirman 1972, Becker-Tomes 1979, 1986, Loury 1981...
- Disequalization: Markets intrinsically create and maintain inequality.
  - Ray 1990, Banerjee-Newman 1993, Galor-Zeira 1993, Ljungqvist 1993, Freeman 1996, Mookherjee-Ray 2000, 2010...

Standard Accumulation Equations

Intertemporal allocation

$$\underset{\text{income}}{y_t} = \underset{\text{consumption}}{c_t} + \underset{\text{investment/bequest}}{k_t},$$

- **Production function**:

$$y_{t+1} = f(k_t) \text{ or } f(k_t, \alpha_t).$$

- Not surprising that this literature looks like growth theory.
  - Lots of "mini growth models", one per household.
  - But f can have various interpretations.

Interpreting f

- **Standard production function** as in growth theory

- **Competitive economy**: $f(k) = w + (1+r)k$.

- **Returns to skills or occupations**: for example,

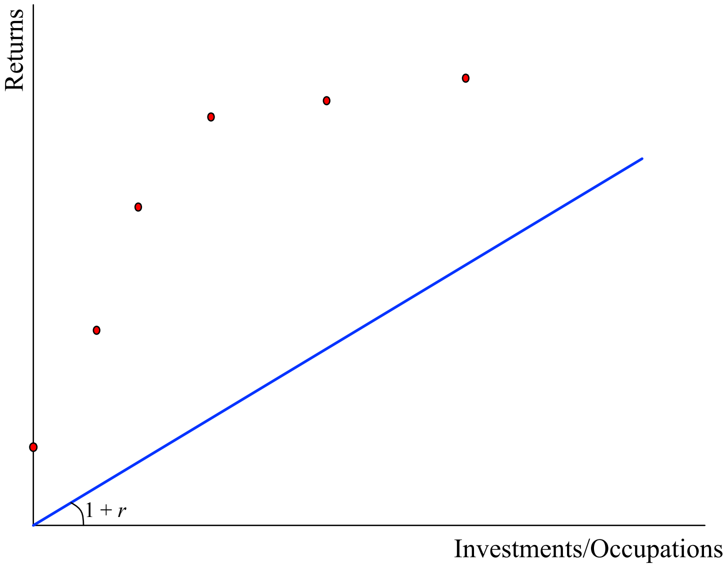
$$\begin{aligned} f(k) &= w_u \text{ for } k < \bar{x} \\ &= w_s \text{ for } k > \bar{x}. \end{aligned}$$

- May be exogenous to individual, but endogenous to the economy

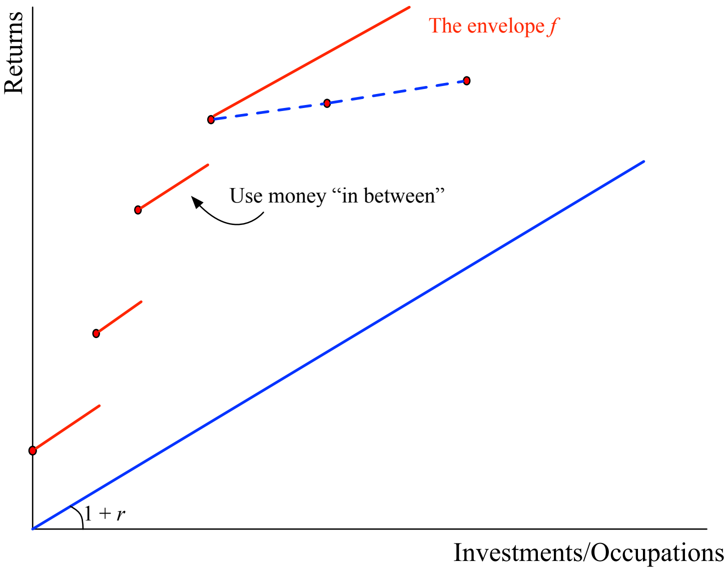
- So interpret f as **envelope of intergenerational investments**:

- Financial bequests

- Occupational choice



Returns
1 + r
Investments/Occupations


Returns
The envelope f
Use money "in between"
1 + r
Investments/Occupations

Parental Preferences and Limited Mobility

- **Parental utility** $U(c) + W(y^+)$, where $c =$ consumption and y^+ = child income, and:
  - U is increasing and strictly concave in c, and $W(y^+)$ is increasing in y^+.

$$W(y^+) \quad = \quad \delta[\theta V(y^+) \quad + \quad (1-\theta)P(y^+)]$$

Future utility — Bellman value — Exogenous value

  - "Reduced-form" **maximization problem**:

$$\max U(c) + \mathbb{E}_\alpha W(f(k, \alpha)).$$

Theorem 1

- *Let h describe all optimal choices of k for each y.*
- *Then if $y > y'$, $k \in h(y)$, and $k' \in h(y')$, it must be that $k \geq k'$.*

Proof of Theorem 1

- Pick $y > y'$, $k \in h(y)$, and $k' \in h(y')$. **Suppose $k' > k$.**
  - k beats k' under y, so:

$$U(y-k) + \mathbb{E}_\alpha W(f(k,\alpha)) \geq U(y-k') + \mathbb{E}_\alpha W(f(k',\alpha)).$$

  - k' beats k under y', so:

$$U(y'-k') + \mathbb{E}_\alpha W(f(k',\alpha)) \geq U(y'-k) + \mathbb{E}_\alpha W(f(k,\alpha)).$$

  - Adding, rearranging:

$$U(y-k) - U(y-k') \geq U(y'-k) - U(y'-k'),$$

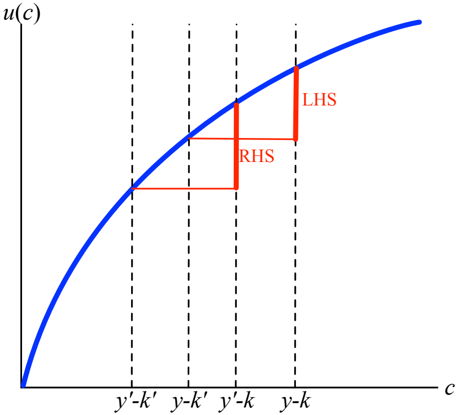
which **contradicts the strict concavity of U**. □

Illustration

For $y > y'$ and $k' > k$,

$$U(y-k) - U(y-k') \geq U(y'-k) - U(y'-k'),$$

contradicts this picture:

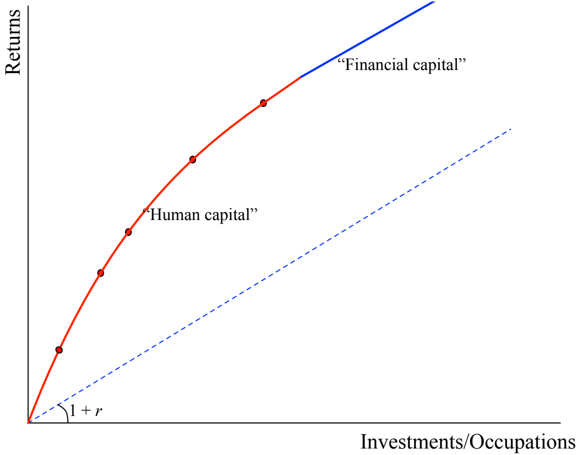


Remarks:

- h is “almost” a function.
- h can only jump up, not down.
- Same assertion is not true of optimal c.
- Note how curvature of U is important, that of W is unimportant.
- **Crucial for models in which f is endogenous** with uncontrolled curvature.

Standard Assumption

f **is *exogenous*, and *concave***:



- Generates convergence to unique steady state in the absence of uncertainty.

Convergence With Concavity: Intuition

- Look at **Bellman case** with no uncertainty:

$$V(y) \equiv \max_k \left[U(y-k) + \delta V(f(k))\right]. \tag{1}$$

- **First order condition** at y_t:

$$U'(c_t) = \delta V'(y_{t+1}) f'(k_t). \tag{2}$$

- But (1) + Envelope Theorem $\Rightarrow V'(y_{t+1}) = u'(c_{t+1})$, so:

$$U'(c_t) = \delta U'(c_{t+1}) f'(k_t). \tag{3}$$

- Theorem 1 + (3) imply **convergence to unique k^*, where $\delta f'(k^*) = 1$**.

And Without Concavity?

- Without concavity: again, look at **Bellman case** with no uncertainty:

$$V(y) \equiv \max_k \left[U(y-k) + \delta V(f(k))\right]. \tag{4}$$

  - **First order condition** *still works* (necessary, after all):

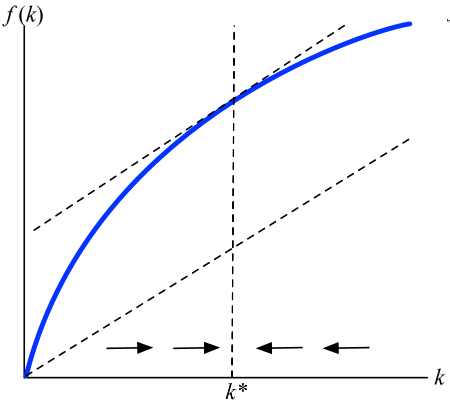
$$U'(c_t) = \delta V'(y_{t+1}) f'(k_t). \tag{5}$$

  - Envelope theorem *still works*, so:

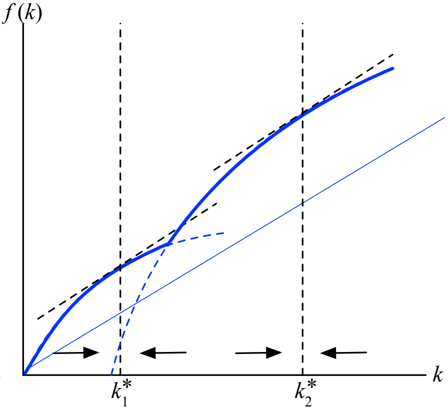
$$U'(c_t) = \delta U'(c_{t+1}) f'(k_t). \tag{6}$$

- So again convergence to k^*, where $\delta f'(k^*) = 1$, **but now k^* is not unique**.

Comparison



$f'(k^*) = 1$



$\delta f'(k_1^*) = \delta f'(k_2^*) = 1$

Stochastic Shocks

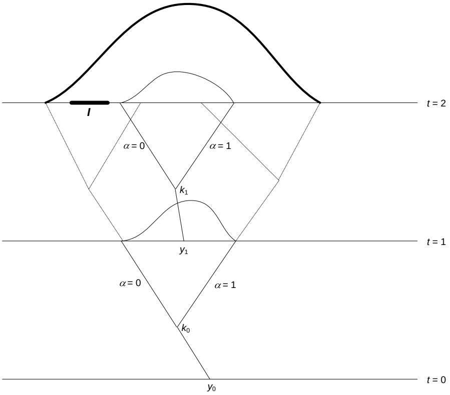
- What happens to these models with stochastic shocks?

- Something weird, at least conceptually.

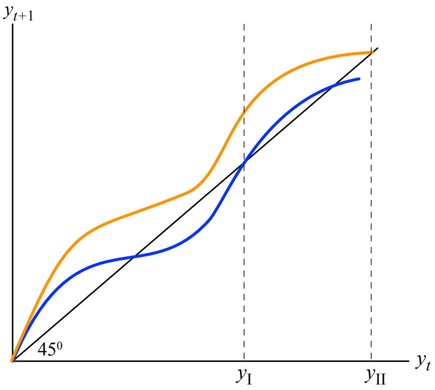
Theorem 2

Brock-Mirman 1976, Becker-Tomes 1979, Loury 1981, extended to drop concavity

- *Assume a mixing condition, such as $f(0,1) > 0$ (poor genius) and $f(k,0) < k$ for all $k > 0$ (rich fool).*
- *Then there exists a unique measure on incomes μ^* such that μ_t converges to μ^* as $t \to \infty$ from every μ_0.*



- **Core assumption**: a “mixing zone”:



Mixing

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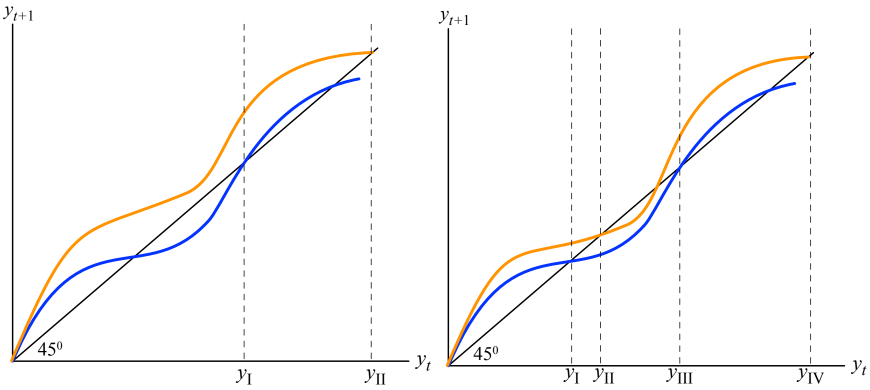


Mixing

No Mixing

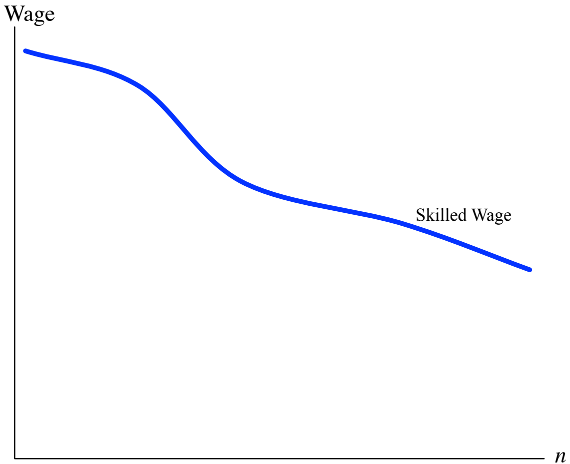
Three major drawbacks of this model:

- **I**. The reliance on stochastic shocks.
  - Participation in national lottery $\Rightarrow$ mixing.
  - Ergodicity could be a misleading concept.
- **II**. Disjoint supports.
  - No mixing condition $\Rightarrow$ multiple steady states:
  - But must have disjoint supports, which is weird.
- **III**. The reliance on efficiency units.
  - No way to endogenize the returns to different occupations.
  - Whether f concave at the household level **should depend on markets**.

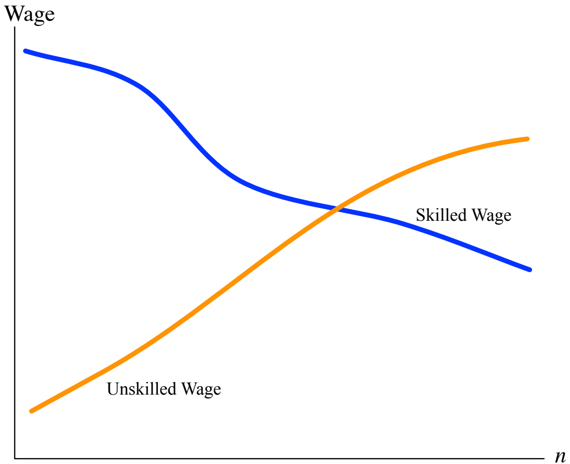
Inequality and Markets

- Return to the **interpretation of f as occupational choice**.
  - Dropping efficiency units creates movements in relative prices:
  - f isn't "just technology" anymore.
- **An Extended Example** with just two occupations
  - Two occupations, skilled S and unskilled U. Training cost X.
  - Population allocation $(n, 1-n)$.
  - Output: $F(n, 1-n)$, where $F =$ is a macro production function on skilled and unskilled labor:
- Skilled wage: $w_s(n) \equiv F_1(n, 1-n)$
- Unskilled wage: $w_u(n) \equiv F_2(n, 1-n)$

Skilled and Unskilled Wages



Skilled and Unskilled Wages



Households

- **Continuum of households, each with one agent per generation.**
  - Starting wealth y; $y = c + k$, where $k \in \{0, X\}$.
  - Child wealth $y' = w$, where $w = w_s$ or w_u.
- **Parent maxes $U(c) + \delta V(y')$ (Bellman equation)**
  - No debt!
  - Child grows up; back to the same cycle.

Equilibrium

- A sequence $\{n^t, w_s^t, w_u^t\}$ such that
  - $w_s^t = w_s(n^t)$ and $w_u^t = w_u(n^t)$ for every t.
  - n^0 given and the other n^ts agree with utility maximization.
- **Steady states:**
  - A constant fraction n are skilled
  - Wages are constant at $w_s = F_1(n, 1-n)$ and $w_u = F_2(n, 1-n)$
  - All parents keep replicating their skill status in their children.
  - Replication of skills follows from Theorem 1.

Steady States in Occupational Choice

- **Conditions for n to be a steady state:**

[Skilled parent] $$V(w_s) = \frac{u(w_s - X)}{1-\delta} \geq u(w_s) + \frac{\delta}{1-\delta}u(w_u)$$

[Unskilled parent] $$V(w_u) = \frac{u(w_u)}{1-\delta} \geq u(w_u - X) + \frac{\delta}{1-\delta}u(w_s - X)$$

Steady States in Occupational Choice

[Skilled parent] $$\frac{u(w_s - X)}{1-\delta} \geq u(w_s) + \frac{\delta}{1-\delta} u(w_u)$$

[Unskilled parent] $$\frac{u(w_u)}{1-\delta} \geq u(w_u - X) + \frac{\delta}{1-\delta} u(w_s - X)$$

Theorem 3

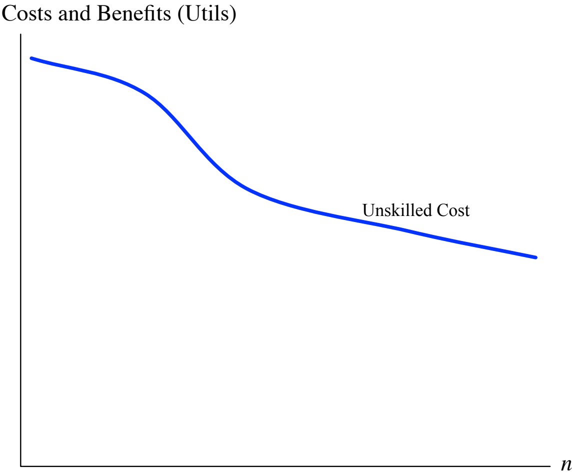
Every n with $w_s = F_1(n, 1-n)$ and $w_u = F_2(n, 1-n)$ such that

$$\underbrace{u(w_u) - u(w_u - X)}_{\text{Unskilled Cost}} \geq \underbrace{\frac{\delta}{1-\delta}\left[u(w_s - X) - u(w_u)\right]}_{\text{Future Benefit}} \geq \underbrace{u(w_s) - u(w_s - X)}_{\text{Skilled Cost}}$$

must be a steady state.

Steady States in Occupational Choice

$$\underbrace{u(w_u) - u(w_u - X)}_{\text{Unskilled Cost}} \geq \underbrace{\frac{\delta}{1-\delta}\left[u(w_s - X) - u(w_u)\right]}_{\text{Future Benefit}} \geq \underbrace{u(w_s) - u(w_s - X)}_{\text{Skilled Cost}}$$



Steady States in Occupational Choice

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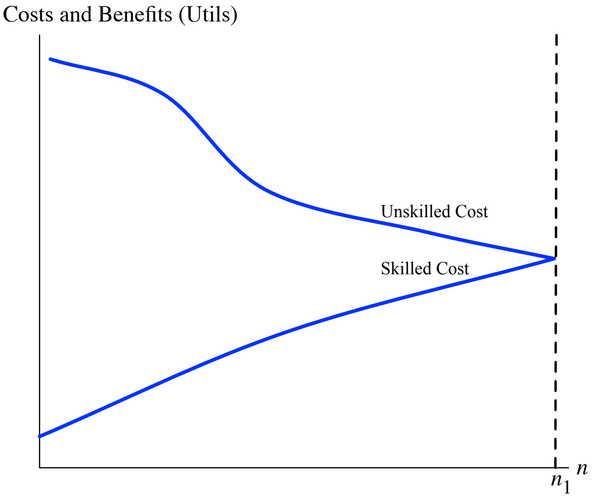

Steady States in Occupational Choice

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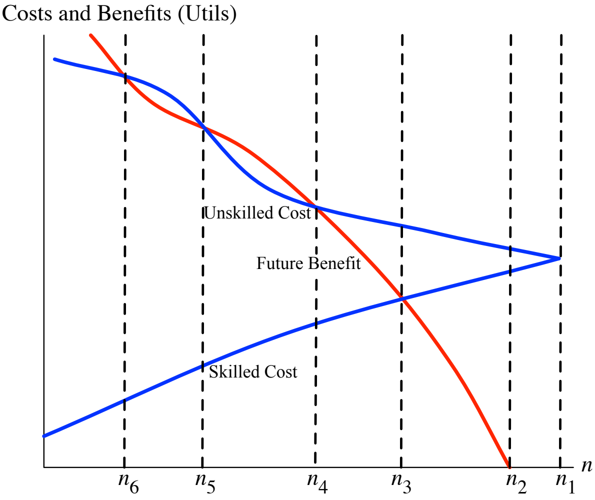


Features of the Two-Occupation Model

- Two-occupation model useful for number of insights:

1. **Steady states exist**:

   - The first one (from right to left) is at n_3.

2. **Multiple steady states** must exist.

   - See diagram for multiple instances of red line sandwiched between blue line3.

3. **No convergence**; persistent inequality in *utilities*.

   - Symmetry-breaking argument.

Features of the Two-Occupation Model

4. **Dynamics and history-dependence**.

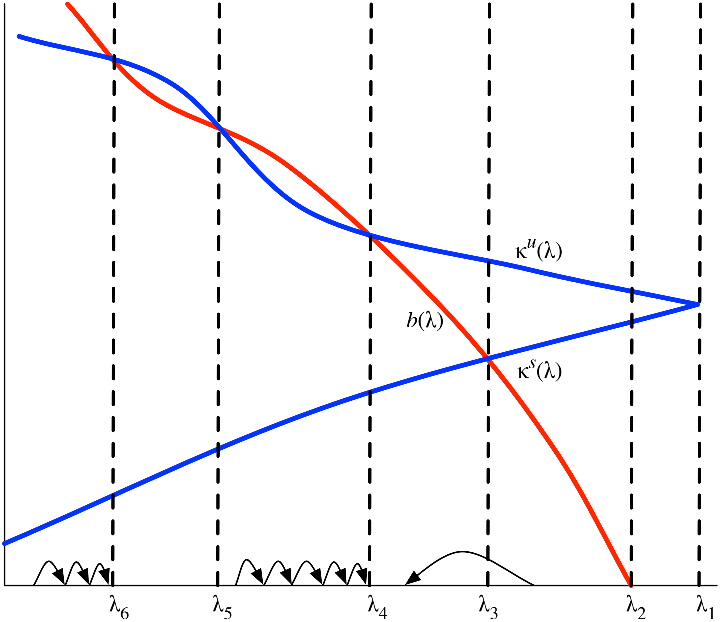
Theorem 4

(i) From any initial n that is a steady state, the system remains there: $n_t = n$ for all t.

(ii) From any initial n that is not a steady state, but with some steady state $n' > n$, n_t converges monotonically up to the ***smallest*** *steady state exceeding n.*

(iii) From any initial n that is larger than any steady state, n_t converges down in ***one period*** *to some steady state.*

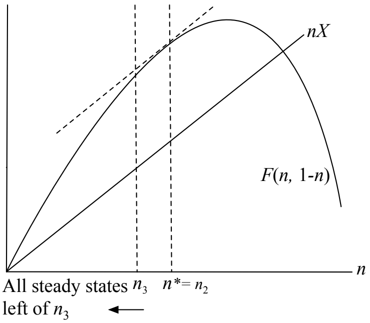
Dynamics



Features of the Two-Occupation Model

5. **Every steady state is inefficient**.

- Efficient allocation maximizes $F(n, 1-n) - nX$:



$F_1(n^*, 1-n^*) - F_2(n^*, 1-n^*) = X$, $\Rightarrow w_s^* - X = w_u^* \Rightarrow n^* = n_2$. But every steady state is to the left of n_3 (see steady state diagram).

Features of the Two-Occupation Model

6. **Can easily embed other models here, such as entrepreneurship**.

- Reinterpret s as entrepreneur, u as worker.
- X is setup cost for industrialization.
- $F(s,u) = sf\left(\frac{u}{s}\right)$

- Then:

- $F_2(s,u) = f'\left(\frac{u}{s}\right) = w$, and
- $F_1(s,u) = f\left(\frac{u}{s}\right) - \frac{u}{s}f'\left(\frac{u}{s}\right) = f\left(\frac{u}{s}\right) - \frac{u}{s}w$ = profit.

Features of the Two-Occupation Model

7. **Policy questions, such as conditionality in educational subsidies**

- Recall social's planner's n^* had higher net output than any steady state:
  - So there could be a role for **educational subsidies**.
  - Assume all subsidies funded by taxing w_s at rate τ.
- **Unconditional**: give equally to currently unskilled parents:

$$T_t = \frac{n_t \tau}{1 - n_t} w_s(n_t).$$

- **Conditional**: give to *all* parents conditional on educating children.

$$Z_t = \frac{n_t \tau}{n_{t+1}} w_s(n_t).$$

(can contemplate other obvious variants with similar results)

Features of the Two-Occupation Model

Theorem 5

- *With unconditional subsidies, every left-edge steady state declines, lowering the proportion of skilled labor and increasing pre-tax inequality, which undoes some or all of the initial subsidy.*
- *With conditional subsidies, every left-edge steady state goes up, increasing the proportion of skilled labor. In steady state, no direct transfer occurs from skilled to unskilled, yet unskilled incomes go up and skilled incomes fall.*
- *Conditional subsidies therefore generate superior macroeconomic performance (per capita skill ratio, output and consumption).*

Other Applications

- **Trade theory** in which autarkic inequality determines comparative advantage.
- **Country-level specialization** when national infrastructure is goods-specific.
- **Fertility patterns** in models of occupational choice.

A General Model with Occupational Choice

- **Why study a general model?**

- Rich occupational structure

  “Curvature” of household production function is fully endogenous.

- New insights

  Are there multiple steady states as in the two-occupation model?

- Financial bequests

  Won’t do it here, but easy to incorporate.

Steady States With Multiple Occupations

- Occupations $1, \ldots, n$, setup costs $x_1 < \cdots < x_n$.
- **Steady state conditions:** For all i and j,

$$u(w_i - x_i) + \delta[\theta V(w_i) + (1-\theta)P(w_i)] \geq u(w_i - x_j) + \delta[\theta V(w_j) + (1-\theta)P(w_j)]$$

- Theorem 1 rules out crossings, permits focus on local one-step deviations.
- Take limits as occupations become a continuum ...

$$\begin{aligned} u'(w(x) - x) &= \delta[\theta V'(w(x)) + (1-\theta)P'(w(x))]w'(x) \\ &= \delta[\theta u'(w(x) - x) + (1-\theta)P'(w(x))]w'(x). \end{aligned}$$

- Obtain a **differential equation for the wage function**:

$$w'(x) = \frac{u'(w(x) - x)}{\delta[\theta u'(w(x) - x) + (1-\theta)P'(w(x))]}$$

The Wage Function in Steady State

- A **differential equation for the wage function**:

$$w'(x) = \frac{u'(w(x)-x)}{\delta[\theta u'(w(x)-x) + (1-\theta)P'(w(x))]}$$

  - Fully defined by preferences, except for $w(0)$, the unskilled wage.
  - That is pinned down by technology.
- Endogenous inequality, **but no multiplicity of steady states**.
  - Macro- versus micro-history-dependence.

How are Wages Pinned Down?

$$w'(x) = \frac{u'(w(x)-x)}{\delta[\theta u'(w(x)-x) + (1-\theta)P'(w(x))]}.$$

- Try out the pure value function model with $\theta = 1$:
  - Then $w'(x) = (1/\delta)x$, or $w(x) = w(0) + (1/\delta)x$.
- The production function determines **quantities of labor** in different skills.

  E.g., if:

$$y = A\left[\int_x a(x)\lambda(x)^\rho\right]^{1/\rho},$$

  - then first-order condition for profit-maximization is

$$w(0) + \frac{1}{\delta}x = A^\rho y^{1-\rho} a(x)\lambda(x)^{\rho-1}.$$

  - which pins down $w(0)$, y and every $\lambda(x)$.

Is the Wage Function Concave?

$$w'(x) = \frac{u'(w(x) - x)}{\delta[\theta u'(w(x) - x) + (1 - \theta)P'(w(x)]}.$$

- Is this function (= household "production function") concave?
  - In the Bellman case, yes, because it is affine.
  - Otherwise, no.

Luck versus Markets: Philosophy of Inequality

Equalization: Inequality an ongoing battle between convergence and "luck"

Disequalization: Markets intrinsically create and maintain inequality

- We've explored here the second approach, which:

(i) relies on symmetry-breaking to generate inequality in non-alienable activities.

(ii) is fundamentally interactive across agents (inequality is not the ergodic distribution of some isolated stochastic process).

(iii) generates new predictions for the curvature of the rate of return (and does not assume that curvature via efficiency units and an aggregate production function)

(iv) exhibits history-dependence at the level of individual dynasties, but less so at the macro level

- It remains to be seen if this is the right view of the world.