EC9AA Term 3: Lectures on Economic Inequality

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- **Supplement to Slides 2**: A General Model of Occupational Choice

A General Model of Occupational Choice

- **Production with capital and occupations**.

  - Population distribution on occupations n (endogenous).

  - Physical capital k.

  - Production function $y = F(k, \boldsymbol{n})$, CRS and strictly quasiconcave.

- **Training cost function x on occupations**:

  - incurred up front.

  - parents pay directly, or bequeath and then children pay.

Prices

- **Perfect competition**.

  - Return on capital fixed at rate r (international k-mobility).

  - "Wage" vector $\mathbf{w} \equiv \{w(h)\}$ endogenously determined for each occupation h.

  - Together with r, $\mathbf{w}$ **supports profit-maximization**.

Supporting Profit Maximization

- $F(k, \boldsymbol{n})$ is associated with a **unit cost function** $c(\mathbf{w}, r)$.

- Find it by minimizing unit cost of production for any $(\mathbf{w}, r)$.

- If that unit cost $\neq$ output price:

- $(\mathbf{w}, r)$ **cannot support profit maximization at positive output**.

- Otherwise, it does.

- **Note**: For any $\mathbf{w}$, there is a unique **scaling** $\mu > 0$ such that $(\mu \mathbf{w}, r)$ supports profit maximization.

Households

- Continuum of households, each with one agent per generation.

- $y = \underset{\text{wealth}}{} \quad \underset{\text{consumption}}{z} \quad + \quad \underset{\text{fin. bequests}}{b} \quad + \quad \underset{\text{occ. choice}}{x(h)}$

- Child wealth $y' = (1+r)b + \mathbf{w}_{t+1}(h)$.

- Parent picks (b, h) to **maximize utility**.

- No debt! $b \geq 0$.

- Child grows up; back to the same cycle.

Preferences and Equilibrium

- **Preferences**: mix of income-based and nonpaternalistic

$$U(z) + \delta[\theta V(y') + (1 - \theta)P(y')]$$

- **Equilibrium**: wages $\mathbf{w}_t$, value functions V_t, occupational distributions n_t s.t.:

- Each family i chooses $\{h_t(i), b_t(i)\}$ optimally

- Occupational choices $\{h_t(i)\}$ aggregate to n_t;

- Firms willingly demand n_t at prices $(\mathbf{w}_t, r)$.

- **Note**: physical capital willingly supplied to meet any demand.

Steady State

■ A **steady state** is a stationary equilibrium with positive output and wages:

▪ $\mathbf{w}_t = \mathbf{w} \gg 0$, and

▪ $(k_t, \boldsymbol{n}_t) = (k, \boldsymbol{n})$ for all t, and $F(k, \boldsymbol{n}) > 0$.

- **The richness assumption [R]:**

  - The set of all training costs is a compact interval $[0, X]$.

  - If n is zero on any positive interval of training costs, then $y = 0$.
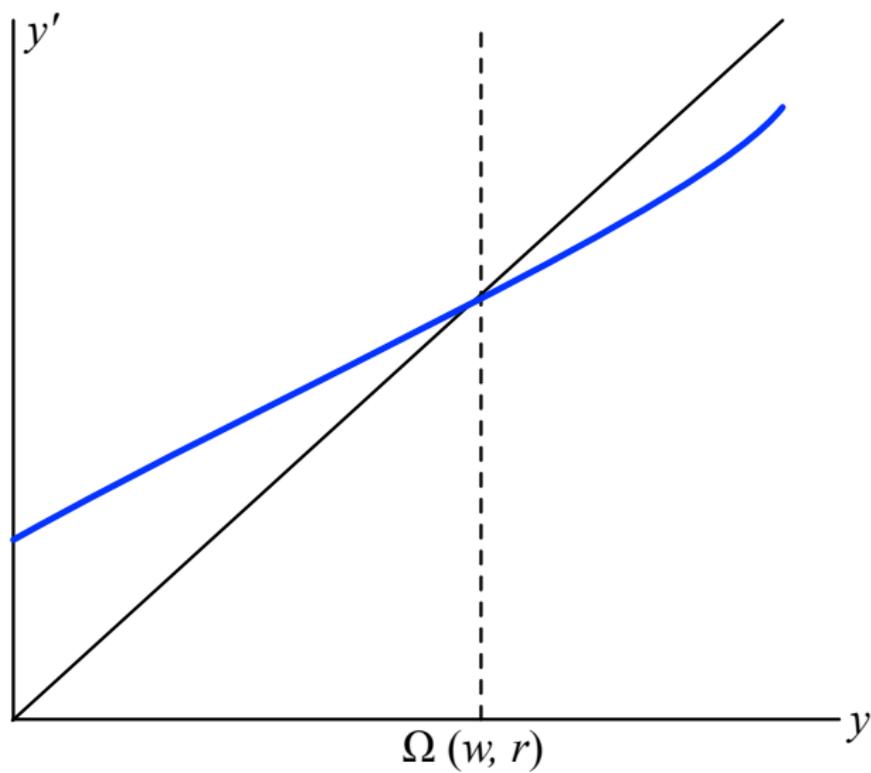
A Benchmark With No Occupational Choice

- **Financial bequests (at rate r) + just one occupation (wage w).**
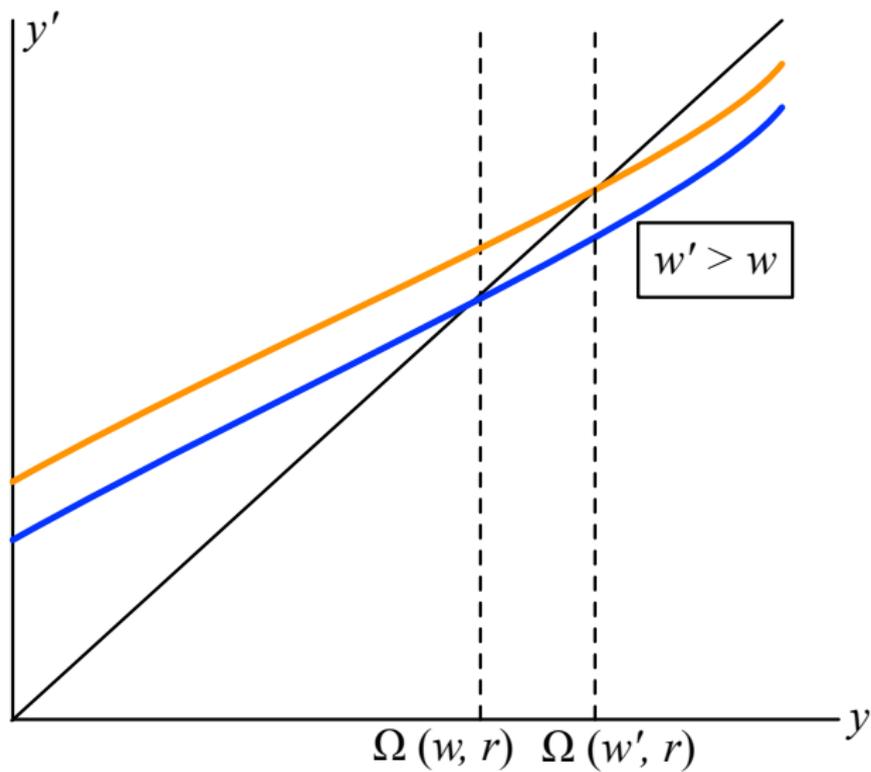
  - Parent with wealth y selects $b \geq 0$ to

$$\max U(c) + \delta[\theta V(y') + (1 - \theta)P(y')].$$

  - Child wealth $y' \equiv w + (1 + r)b$, increases in y.

- Converges to **limit wealth** $\Omega(w, r) < \infty$.

  - This needs $\theta < 1$.

  - Could depend on initial y (as in non-concave Ramsey model); we exclude that.
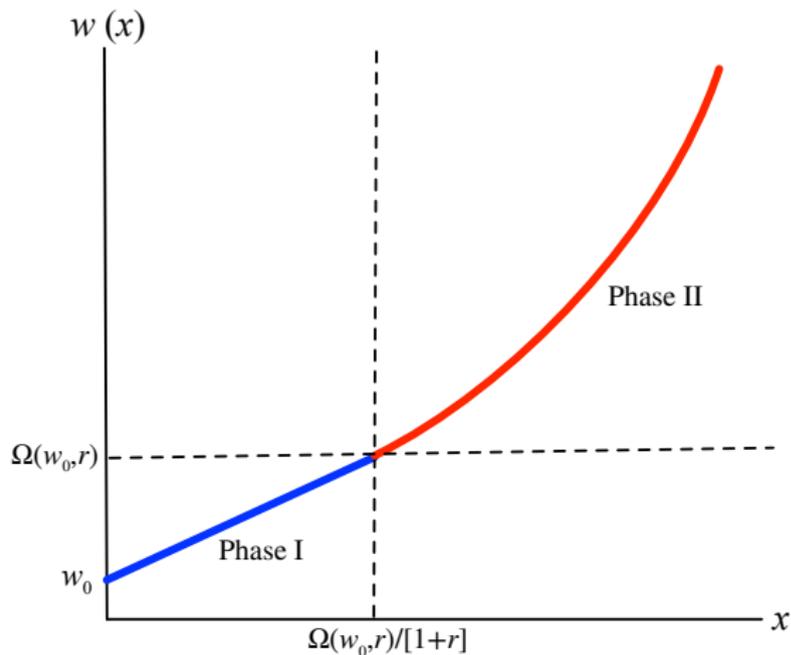
Limit Wealth in Benchmark Model

Back to Occupational Choice

Theorem 1

■ *Every steady state* $\mathbf{w}$ *is fully described by a **two-phase property***:

- **In Phase I w is linear in x:** there is $w_0 > 0$ such that

$$w(x) = w_0 + (1+r)x \text{ for all } x \le \frac{\Omega(w_0, r)}{1+r}$$

- All families in Phase I have the **same overall wealth** $\Omega(w_0, r)$.

- **In Phase II, w follows the differential equation**

$$w'(x) = \frac{U'\left(w(x) - x\right)}{\delta[\theta U'\left(w(x) - x\right) + (1-\theta)P'(w(x))]}$$

- with endpoint to patch with I: $w(x) = w_0 + (1+r)x$ as $x \downarrow \frac{\Omega(w_0,r)}{1+r}$.

- Families located in Phase II have **different wealths and lifetime consumptions**.
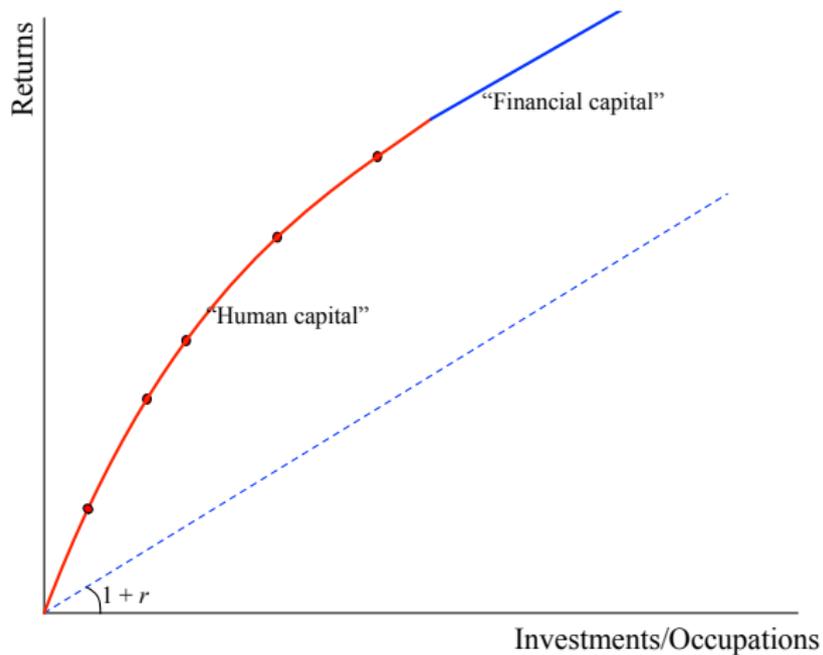
- **Closer look at Phase II**

$$w'(x) = \frac{U'\left(w(x) - x\right)}{\delta[\theta U'\left(w(x) - x\right) + (1 - \theta)P'(w(x))]}$$

- Shape comes from **Euler equation**:

- depends fundamentally on preferences

- **technology only serves to pin down baseline** w_0 (remember remark on scaling)
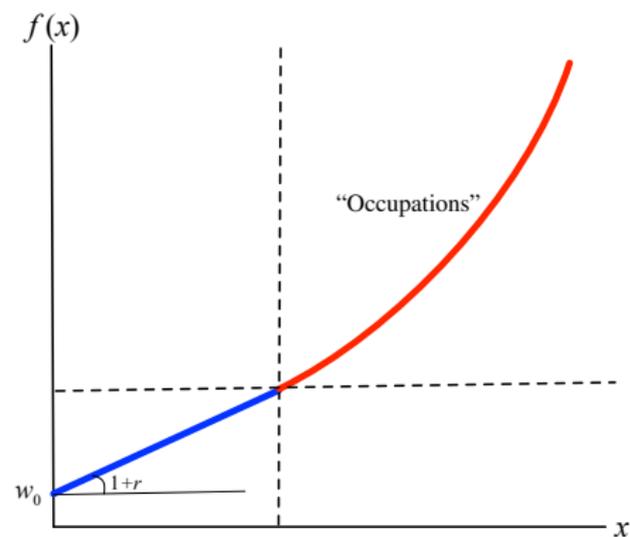
■ **Recall standard model**. By assumption:
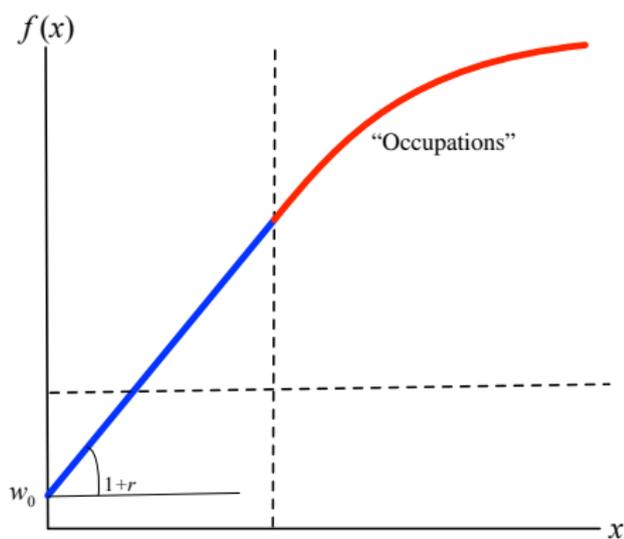
A Testable Implication

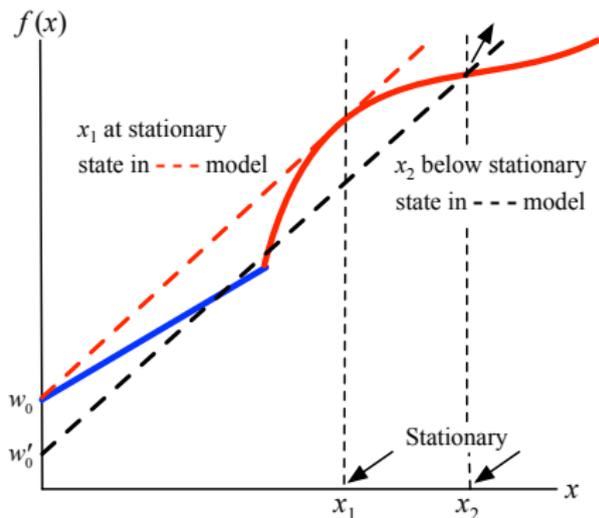- **Compare to Phases I and II**: *This?*   *Or this?*

A Testable Implication

Theorem 2

The average return $\frac{w(x)-w_0}{x}$ to occupational investment is flat in Phase I and strictly increasing in Phase II.

- Contradiction to unique limit wealth in benchmark, increasing in w.
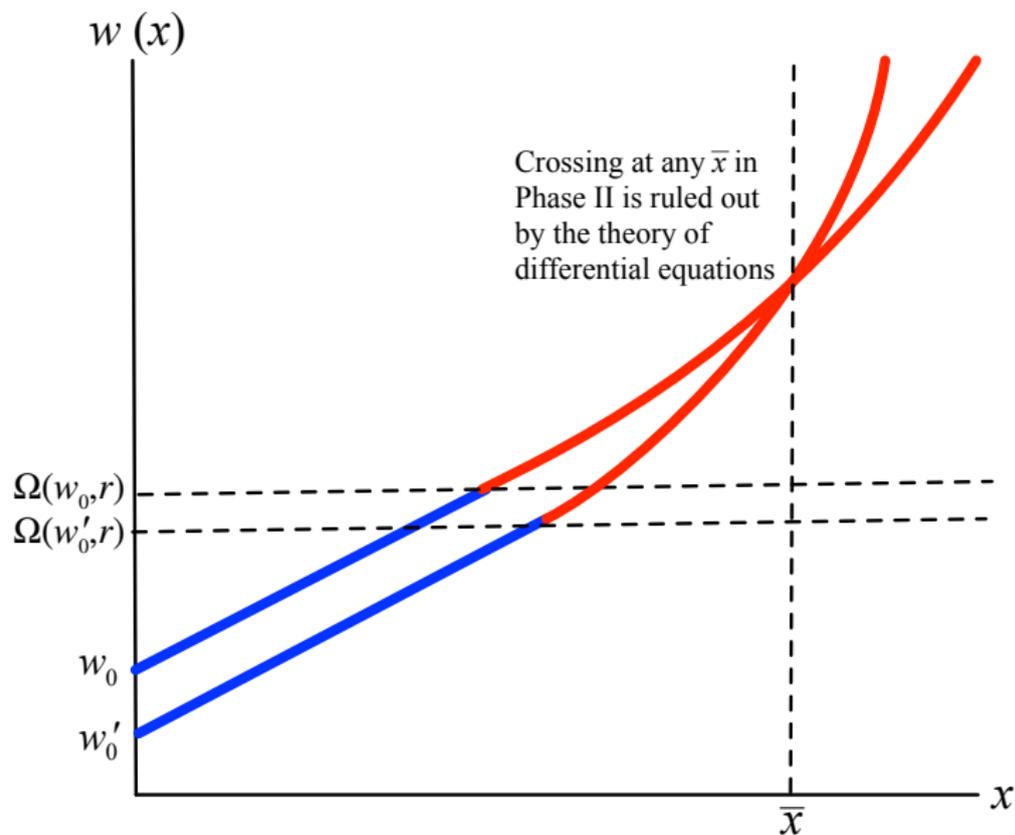
Unique Steady State with Rich Occupational Structure

- We end with a **fundamental difference from two-occupation case**:
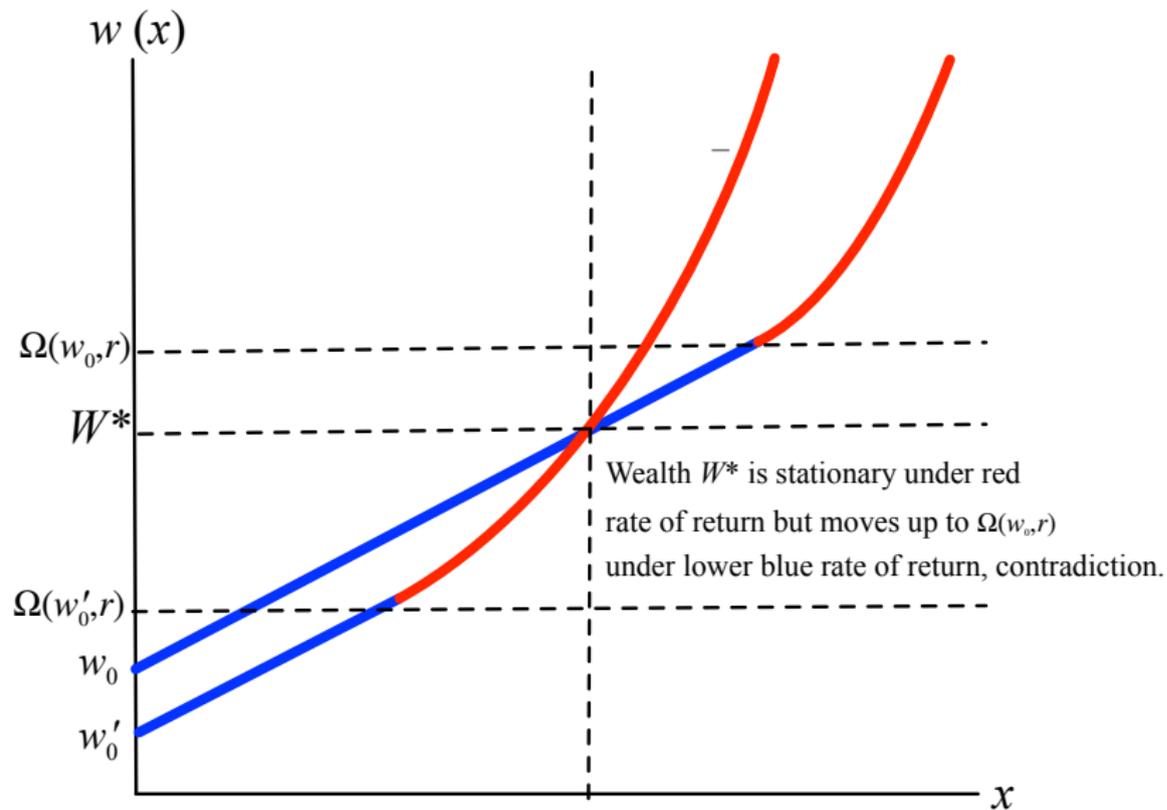
Theorem 3

There is at most one steady state.

- Proof idea:

- No two members of the two-phase family (indexed only by w_0) can cross.

- Then **only one w_0 can support profit maximization** with positive output.

  (For all wages must co-move with intercept wage w_0.)

No-crossing argument, part I

$w(x)$

Crossing at any $\overline{x}$ in Phase II is ruled out by the theory of differential equations

$\Omega(w_0, r)$

$\Omega(w_0', r)$

w_0

w_0'

$\overline{x}$

x

No-crossing argument, part II

$w(x)$

$\Omega(w_0, r)$

W^*

$\Omega(w_0', r)$

w_0

w_0'

x

Wealth W^* is stationary under red
rate of return but moves up to $\Omega(w_0, r)$
under lower blue rate of return, contradiction.

Three Remarks

- **I. Alienable and Inalienable Capital**

- In Phase I, there is **perfect equality** of overall wealth.

- (All families in Phase I must have wealth equal to $\Omega(w, r)$.)

- Families at different occupations in Phase II **cannot have the same wealth**.

- Thus, "most" inequality in this model comes from nonalienable capital.

- This focus will change when we consider automation in the next set of models and the decline in functional labor share.

Three Remarks

- **II. When is Phase II nonempty?**

- When there is a **large occupation span relative to bequest motive**:

  - Discounting.

  - Poverty, via TFP differences.

  - Growth in TFP, lowers effective bequest motive

  - World return on capital.

  - Globalization: new occupations.

Three Remarks

- **III. Two Notions of History-Dependence**

- At the macro-level, history-dependence depends on occupational richness.

- A lot of history-dependence at the individual level.

- The distribution as a whole is pinned down, but not who occupies which slot.