EC9AA Term 3: Lectures on Economic Inequality

Debraj Ray, University of Warwick

- Slides 4: Measuring Upward Mobility

Introduction

Mobility centrally important in current debates:

- In the United States and Europe

  Chetty et al (2017), Alesina et al (2018), Manduca et al (2020)

- Connection to growth, inequality, aspirations etc.

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- The concept refers to:

- the ease of transition between various social categories;

- income, wealth, location, political persuasions ...

What the Term Might Mean

- **Non-Directional:**

  - Pure movement: off-diagonals in transition matrix. Atkinson (1981), Bartholomew (1982), Conlisk (1974), Dardanoni (1993), Hart (1976), Prais (1955), Shorrocks (1978a,b) …

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- **+ all combinations of these** …

A Large But Still Incomplete List

Name	Measure	Directional	Non-directional	Absolute	Relative
King (1983)	$M_\kappa = 1 - \exp\left[-\frac{\gamma}{n}\sum\frac{\lvert z_i - y_i\rvert}{\mu_y}\right]$		✓		✓
Shorrocks index (1978)	$M_S = \frac{n - \text{Tr}(P)}{n-1}$		✓		✓
Variability of the eigenvalues	$\sigma(\gamma_i)$		✓		✓
Bartholomew (1982)	$M_B = \frac{1}{n-1}\sum_i\sum_j \pi_i p_{ij} \mid i - j \mid$		✓		✓
IG Income Elasticity (IGE)	$\beta = \frac{\text{Cov}(S_{it}, S_{it-1})}{\text{Var}(S_{it-1})}$		✓	✓	
Correlation coefficient (CE)	$\rho_S = \frac{\text{Cov}(S_{it}, S_{it-1})}{\sqrt{\text{Var}(S_{it})}\sqrt{\text{Var}(S_{it-1})}}$		✓	✓	
Slope rank-rank	$\rho_{PR} = \text{Corr}(P_i, R_i)$		✓		✓
IG rank association (IRA)	$\beta = \frac{\text{Cov}(p_{it}^y, p_{it}^X)}{\text{Var}(p_{it}^X)}$		✓		✓
Mitra & Ok (1998)	$\text{MO}_\alpha(\mathbf{x}, \mathbf{y}) = \frac{1}{n}\gamma\left(\sum_i \lvert y_i - x_i\rvert^\alpha\right)^{1/\alpha}$		✓	✓	
Gini symmetric index of mobility	$GS = \frac{\sum_i(y_i - x_i)(F_{xi} - F_{yi})}{\sum_i(y_i - 1)F_{yi} + \sum_i(x_i - 1)F_{xi}}$		✓	✓	
Great Gatsby curve	Corr(Gini, IGE)		✓	✓	
Bhattacharya (2011)	$\nu = Pr(F_1(Y_1) - F_0(Y_0) > \tau \mid s_1 \leq F_0(Y_0) \leq s_2, X = x)$	✓			✓
Absolute upward mobility (1)	$p_{25} = \mathbb{E}(Y \mid X \leq 25)$	✓			✓
Absolute upward mobility (2)	$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 + \sigma_c^2 + 2\rho\sigma_p\sigma_c}}\right)$	✓			✓
Chetty et al (2017)	$\text{AM}(\mathbf{x}, \mathbf{y}) = \frac{1}{n}\sum_i(1_{y_i] \geq x_i})$	✓		✓	
Rising up-up	$P_{20to100} = \mathbb{E}[Y = 100 \mid X = 20]$	✓			✓
Bottom half mobility	$\mu_0^{50} = \mathbb{E}(y \mid x \in [0, 50])$	✓			✓
Fields & Ok (1999)	$\text{FO}(\mathbf{x}, \mathbf{y}) = \frac{1}{n}\sum_i(\ln(y_i) - \ln(x_i))$	✓		✓	
Card (2018)	$\mathbb{E}(y > 50 \mid x \in [45, 70])$	✓		✓	
Pro-poor growth	$G = \sum_{k=1}^5 w_k g_k$	✓		✓	

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  - not interested in former component (more later)
- **Data demands**
  - Existing measures rely heavily on panel data (more discussion later).
  - This has held back empirical work, especially on developing countries.

Upward Mobility

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  - Directional: rewards growth and punishes decline;
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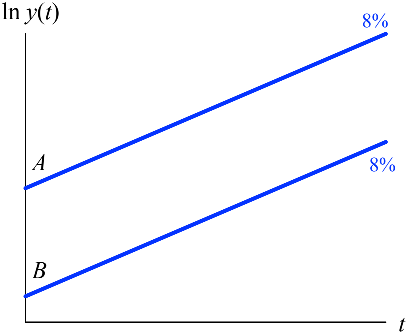

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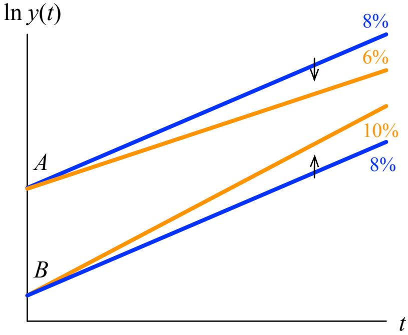

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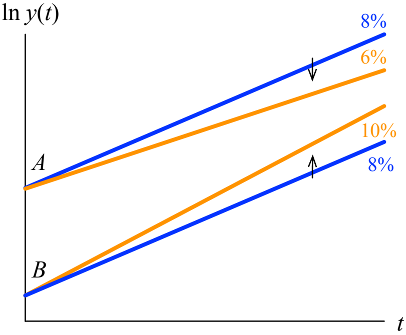

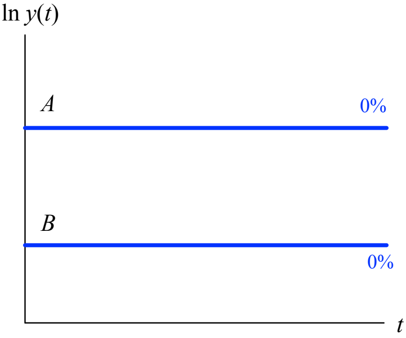

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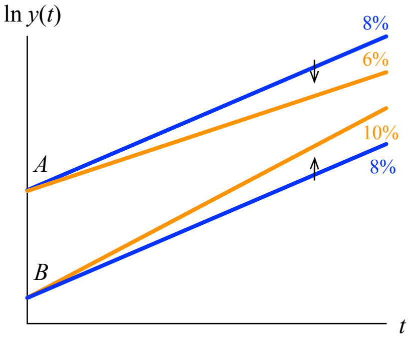

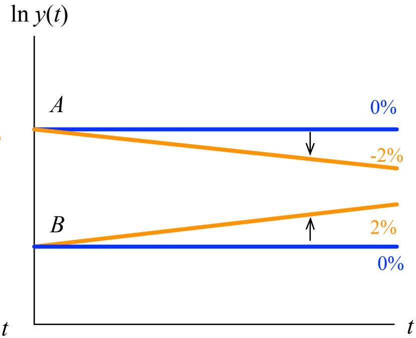

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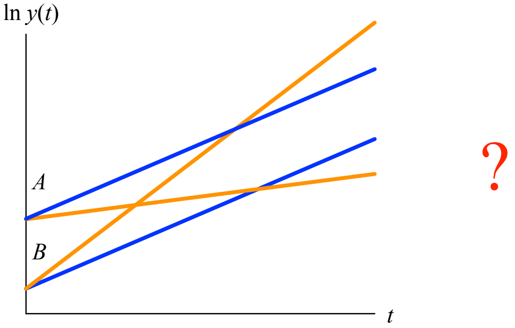

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Snapshots and Trajectories

- We divide our approach into two parts:
  - An "instantaneous" measure or **upward mobility kernel** (intermediate step) that is:
    - directional and progressive.
  - A **mobility measure on trajectories** (what we're after) that is:
    - based on the collection of instantaneous kernels.

Instantaneous Upward Mobility

- **Central variable**: y, "income."
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  - state variable for individual well-being.
  - e.g., "permanent income" or a proxy, such as consumption
- **Data**: For each person:
  - $y_i > 0$ baseline income
  - $g_i = \dot{y}_i / y_i$ instantaneous growth rate.
  - $\mathbf{z} =$ the full collection $\{z_i\}_{i=1}^n$, where $z_i = (y_i, g_i)$.

Instantaneous Upward Mobility

- **Upward mobility kernel**: $M(\mathbf{z})$, where $\mathbf{z} = \{z_i\}_{i=1}^{n}$, and $z_i = (y_i, g_i)$.
  - Anonymous, continuous.

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  - Anonymous, continuous.
  - Zero-growth normalization:

    $g_i = 0$ all $i \mapsto M(\mathbf{z}) = 0$.
  - Consistency under population mergers.

Details

Core Axiom

- **Examples:**

- $\mathbf{y} = (5000, 10000)$ + $\mathbf{g} = (8\%, 8\%) \succ \mathbf{y} = (5000, 10000)$ + $\mathbf{g} = (6\%, 10\%)$.
- $\mathbf{y} = (5000, 10000)$ + $\mathbf{g} = (2\%, -2\%) \succ \mathbf{y} = (5000, 10000)$ + $\mathbf{g} = (0\%, 0\%)$.
- No crossings in continuous time.

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  - No crossings in continuous time.
- **Growth Progressivity.**
  - For any $\mathbf{z}$, i and j with $y_i < y_j$, and $\epsilon > 0$, send g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$.
  - Then $M(\mathbf{z}') > M(\mathbf{z})$.

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- **Notes:**
  - Measure tolerates lower growth if poor can grow faster.
  - Upward mobility $\neq$ overall welfare.

Upward Mobility Kernel

Theorem 1

An upward mobility kernel is growth progressive if and only if it can be written as

$$M(\mathbf{z}) = \sum_{i=1}^{n} \phi_i(\mathbf{y}) g_i$$

for continuous permutation-invariant $\{\phi_i\}$*, with* $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$ *when* $y_i < y_j$*.*

Proof Outline

Sharpening the Kernel

- **Income Neutrality.** $M(\mathbf{y}, \mathbf{g}) = M(\lambda\mathbf{y}, \mathbf{g})$ for all $\lambda > 0$.

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- **Independent Pairwise Growth Tradeoffs:**

  Is $M((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij})) \geq M((y_i, g_i'), (y_j, g_j'), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$?

  Answer insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$.

Sharpening the Kernel

Theorem 2

Under additional three axioms and $n \geq 3$, M can be written as:

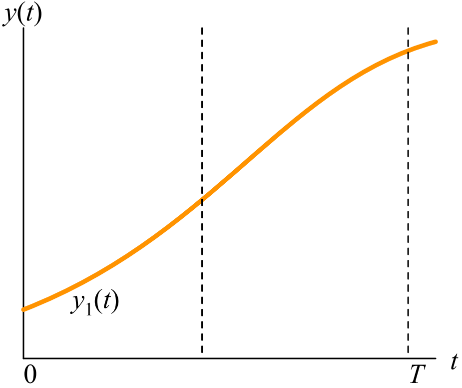
$$M_\alpha(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}}, \text{ for some } \alpha > 0.$$

- Proof employs a substantial extension of Gorman's separability theorem;

  see Chatterjee ⓡ Ray ⓡ Sen (2021).

Income Trajectories

Towards a measure on trajectories:



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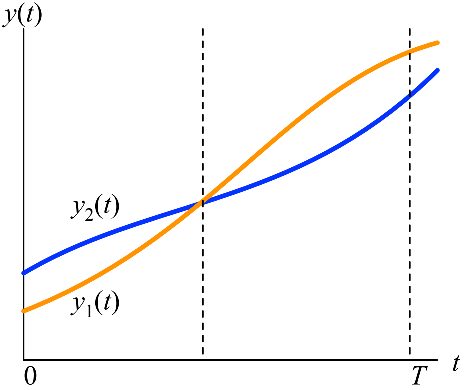

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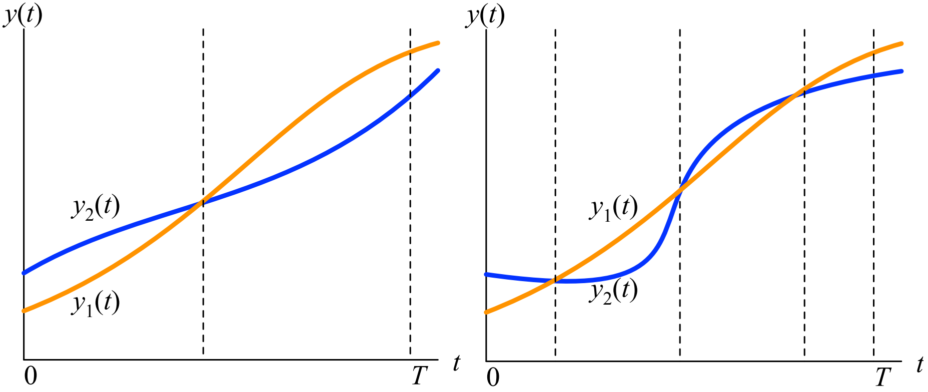

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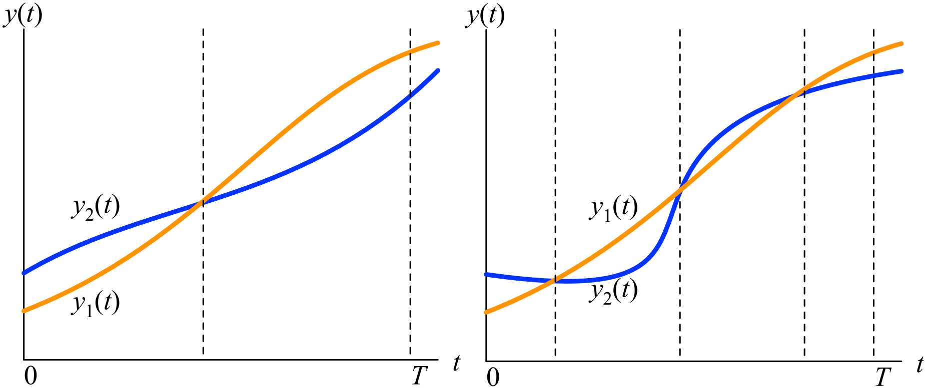

- $\mathbf{y}[s,t] = \{y_i(\tau)_s^t\}_{i=1}^n$
- **Upward mobility measure**: $\mu(\mathbf{y}[s,t])$.

Reducibility

- Assume $\mathbf{y}[s,t]$ continuously differentiable. Then:
  - Well-defined $\mathbf{z}(\tau) = (\mathbf{y}(\tau), \mathbf{g}(\tau))$ for each $\tau \in [s,t]$.
  - Well-defined $M(\mathbf{z}(\tau))$ for each $\tau \in [s,t]$.

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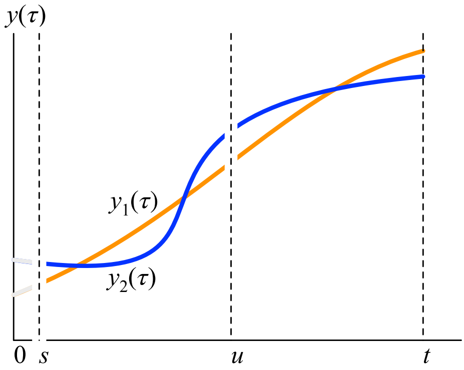
- μ is **reducible** if it's expressible as a function of all these M's:

$$\mu(\mathbf{y}[s,t]) = \Psi(\{M(\mathbf{z}(\tau))\}_s^t)$$

  - with $\mu(\mathbf{y}[s,t]) = m$ whenever $M(\mathbf{z}(\tau)) = m$ for all $\tau \in [s,t]$ (normalization)

Additivity



- μ is **additive** if for all $s < u < t$,
  - $(t-s)\mu(\mathbf{y}[s,t]) = (u-s)\mu(\mathbf{y}\,[s,u]) + (t-u)\mu(\mathbf{y}\,[u,t])$.

Upward Mobility

Theorem 3

Kernel axioms, reducibility, and additivity hold if and only if

$$\mu_\alpha(\mathbf{y}[s,t]) = \frac{1}{t-s} \ln \left[\frac{\sum_{i=1}^{n} y_i^{-\alpha}(t)}{\sum_{i=1}^{n} y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \textit{ for some } \alpha > 0.$$

- **Remark:** Can also use income categories and population shares (see paper).

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- In what follows, we look at different aspects of this measure.

Upward Mobility as Change in Welfare

- **Mobility measure**:

$$\mu_\alpha(\mathbf{y}[s,t]) = \frac{1}{t-s}\ln\left[\frac{\sum_{i=1}^{n} y_i^{-\alpha}(t)}{\sum_{i=1}^{n} y_i^{-\alpha}(s)}\right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0.$$

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- Atkinson welfare function, or Atkinson equivalent income:

$$a_\alpha(\mathbf{y}) = \left(\frac{1}{n} \sum_{j=1}^n y_j^{-\alpha} \right)^{-\frac{1}{\alpha}},$$

for $\alpha > 0$ (elasticity restricted).

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- $\mu_\alpha(\mathbf{y}[s,t])$ = **average growth of Atkinson equiv income** on $[s,t]$.

- Not a measure of equality per se.

Upward Mobility as Pro-Poor Growth

- **Upward Mobility** $= \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{n} y_j(t)^{-\alpha}}{\sum_{j=1}^{m} y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$
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- Isn't even on our "boundary" as $\alpha \to 0$.

- Nevertheless, when all growth rates are the same, $\mu_\alpha =$ growth rate.

Discontinuous Trajectories

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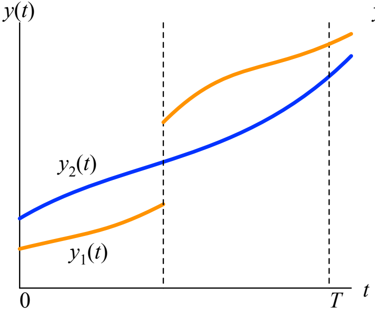

  - Approximate by smooth functions and use continuity: **same answer.**

Relative Upward Mobility

- **Relative upward mobility** nets out growth:

$$\rho_\alpha(\mathbf{y}[s,t]) = \mu_\alpha(\mathbf{y}[s,t]) - \frac{1}{t-s}\big[\ln(\bar{y}(t)) - \ln(\bar{y}(s))\big]$$
$$= \frac{1}{t-s}\ln\left[\frac{\sum_{i=1}^n e_i(t)^{-\alpha}}{\sum_{i=1}^n e_i(s)^{-\alpha}}\right]^{-\frac{1}{\alpha}}$$

  - where $e_i = y_i/\bar{y}$ is excess growth factor relative to per-capita income $\bar{y}$.

- ρ_α admissible under Theorem 1; can be further axiomatized.

Upward Mobility and Panel Independence

- We now arrive at a central point of the paper:

- **Upward Mobility** $= \dfrac{1}{t-s} \ln \left[\dfrac{\sum_{j=1}^{n} y_j(t)^{-\alpha}}{\sum_{j=1}^{m} y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$ is **panel independent.**

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- **Answer:** To assess a family's changing fortunes, *that* family must be tracked.
  - But to assess upward mobility overall, it is *society* that must be tracked.

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- **Answer:** To assess a family's changing fortunes, *that* family must be tracked.
  - But to assess upward mobility overall, it is *society* that must be tracked.
- A family receives time-varying weights depending on its relative location.
  - The impact on overall mobility feeds through the impact on mobility kernels.
  - Such nimble weight switches are central to our argument.

Upward Mobility and Panel Independence

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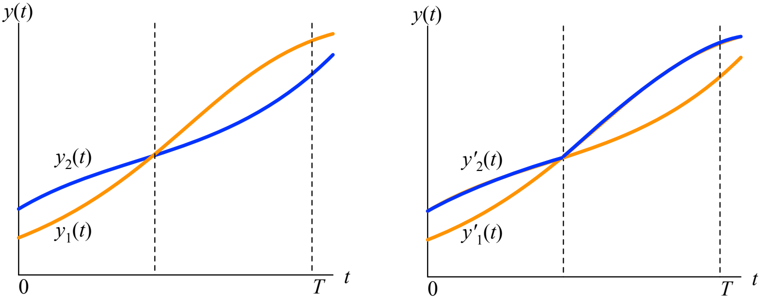
  - $\dfrac{\phi_i(\mathbf{y})}{y_i} = \dfrac{y_i^{-\alpha-1}}{\sum_j y_j^{-\alpha}}$, which integrates out to Atkinson welfare. Jumps?

Upward Mobility and Panel Independence

2. But what about mobility as pure movement "back and forth"?



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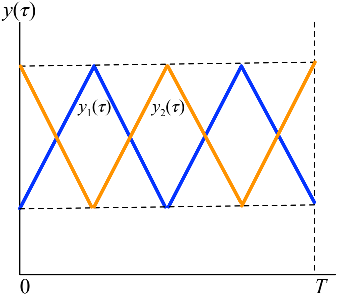

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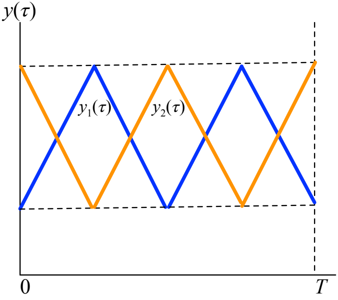

- Different exchange mobility or pure movement. ✓
- Different inequalities. ✓
- But **upward** mobility in both panels is zero.

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- $$E_\alpha(\mathbf{z}) = \sum_{i=1}^{n} \phi_i(\mathbf{y}) |g_i| = M_\alpha^+(\mathbf{z}) + M_\alpha^-(\mathbf{z})$$
- Our preferred approach to exchange mobility.
- Such a measure would not be panel-independent.

Upward Mobility and Panel Independence

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- Similar recommendations apply to poverty or inequality measurement.

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Social Growth Progressivity. For any $\mathbf{z}$, i and j with $(y_i, w_{k(i)}) \leqslant (y_j, w_{k(j)})$, form $\mathbf{z}'$ by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then $M(\mathbf{z}') > M(\mathbf{z})$.

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Social Binary Growth Tradeoffs. For any i, j, any $(y_i, y_j, w_{k(i)}, w_{k(j)})$, comparing $((y_i, w_{k(i)}, g_i), (y_j, w_{k(j)}, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$ and $((y_i, w_{k(i)}, g_i'), (y_j, w_{k(j)}, g_j'), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)})))$ is insensitive to $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i),k(j)}))$.

4, contd.

Theorem 4

The above axioms hold if and only if for $n \geq 3$ and groupings K,

$$\mu_{\alpha,\beta}(\mathbf{y}[s,t],K) = \frac{1}{t-s}\left\{\ln\left[\frac{\sum_{i=1}^{n} y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^{n} y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}}\right]^{-1/\alpha} - \frac{\beta}{\alpha}\int_s^t \frac{\sum_{k\in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k\in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau\right\},$$

for some $(\alpha,\beta) \gg 0$, where $a_k(\tau)$ is Atkinson equivalent group income.

- First term on RHS is panel-independent.
- Second term depends on trajectories, but **only at the group level**.
- Can approximate group Atkinson by standard inequality measures (see paper).

Upward Mobility and Panel Independence

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- For the United States, Chetty et al (2017) estimate:
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- Transitions estimated from a unique panel of tax records
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  - Transitions estimated from a unique panel of tax records
  - $\oplus$ marginal income distributions from CPS and Census.
- Generally very hard to get hold of.
  - Though similar studies exist for other countries; e.g., Acciari et al (2021).

Upward Mobility: Other Measures

Skip?

- The **Chetty et al** (2017) measure (also Berman 2021, Acciari et al 2021):

$$\mu^{\mathsf{c}}(\mathbf{y}[0,1]) = \sum_{i=1}^{n} I(y_i(0), y_i(1)).$$

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- The **Fields-Ok** (1999) measure:

$$\mu^{\mathsf{FO}}(\mathbf{z}) = \frac{1}{n}\sum_{i=1}^{n}\left[\ln(y_i(0)) - \ln(y_i(1))\right] = \frac{1}{n}\sum_{i=1}^{n}\left[\int_0^1 g_i(\tau)d\tau\right].$$

- Both must fail growth progressivity.

Upward Mobility: Other Measures

- **Example for μ^c:**

- Two persons at incomes $10,000 and $20,000.

- Growth rates 1% for both. Then $\mu^c = 1$.

- Transfer 2 points of growth from rich to poor. Then $\mu^c = 1/2$.

- But growth progressivity asks that mobility must rise.

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  - They are not continuous — and this isn't just a technicality.
- **Tiny changes** in incomes can generate **discrete jumps** in mobility.
  - And worse: large changes in *relative* income could go unnoticed.
- **Our measure is indeed correlated with rank-based measures**.
  - But is sensitive throughout, without being unduly affected by a rank switch.

Upward Mobility in the Data

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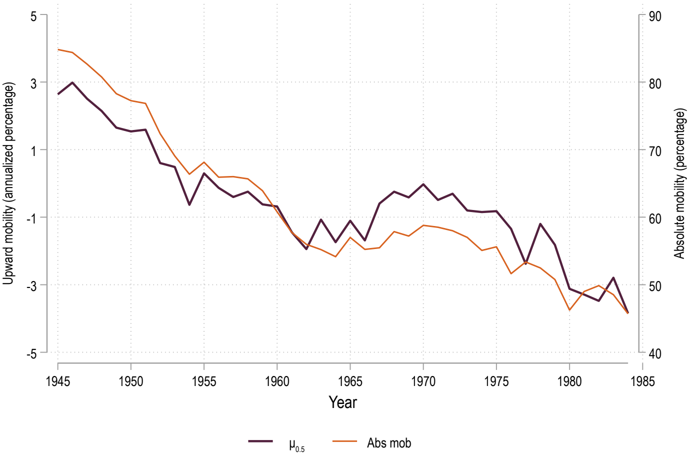
- *In practice*, the dependence on exact copulas seems limited; Berman (2021)

"Estimating the absolute mobility in the United States with different copulas, some of which are very different from the one characterizing the United States, results in a similar evolution in time."

μ_α Compared to Chetty et al (2017) for the United States



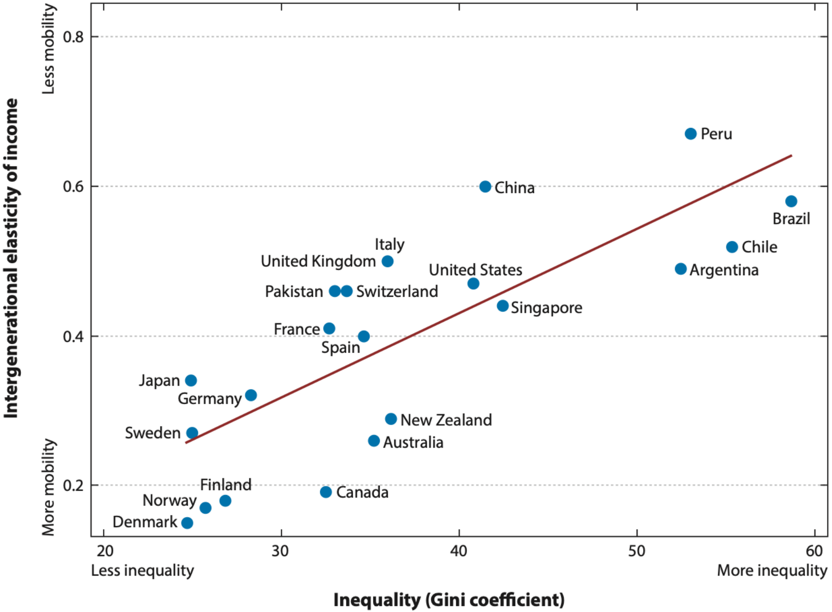
- Robust to different α.
- Robust to using other publicly available databases (e.g., WID).

Upward Mobility in Brazil, India and France

- **Ten-year upward mobility in Brazil, India and France:**
  - Data from the World Inequality Database (repeated cross-sections).
  - Measure $\mu_{0.5}(\mathbf{y}[t, t+10])$ and $\rho_{0.5}(\mathbf{y}[t, t+10])$.
  - Robust with respect to choice of α (see paper).

Upward Mobility in Brazil, India and France

The Great Gatsby Curve



What is the Intergenerational Elasticity of Income?

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- Combine (1) and (2) to get …

IGE Decomposition

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- Our measures can be interpreted as getting rid of ρ entirely and replacing second component.

The Great Gatsby Curve, Respecified

- Our **re-specification**:

$$\textbf{Mobility}_{jst} = \Psi\big(\text{Inequality}_{js}\big) + \phi y_{js} + f_j + h_t + \epsilon_{jst}.$$

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  - y_{js} is income per-capita for country j at date s.

The Great Gatsby Curve and Inequality Traps

$$\text{Mobility}_{jst} = \Psi\big(\text{Inequality}_{js}\big) + \phi y_{js} + f_j + h_t + \epsilon_{jst}$$

is a dynamic equation for the evolution of economic inequality in society.

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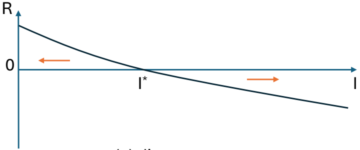
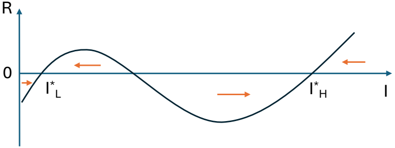
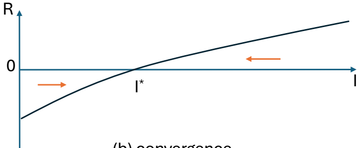


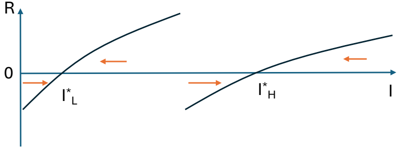
(a) divergence



(c) multiple equilibria



(b) convergence



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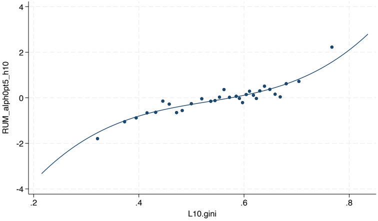
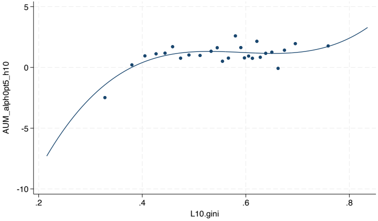
- Two datasets:
  - **World Inequality Database** (WID): 133 countries, 1980-2021
  - **Poverty and Inequality Platform** (PIP): harmonized global survey data.

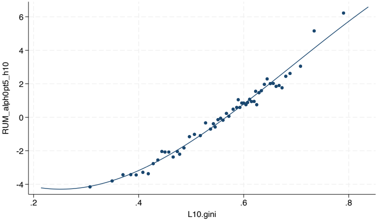
Graphical Analysis: Gini (same with Atkinson)



Rel. mobility vs Gini, $\alpha = 0.5$



Abs. mobility vs Gini, $\alpha = 0.5$



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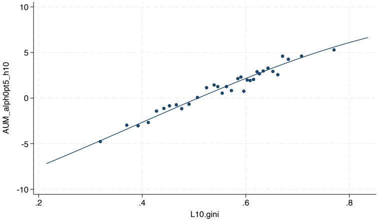

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Linear Model

$$\mathsf{Mob}^{\alpha}(\mathbf{y}[h])_{ct} = \beta I_{ct-h} + \sum_{j=1}^{p} \gamma_j \mathsf{Mob}^{\alpha}(\mathbf{y}[h])_{ct-j} + \eta_c + \delta_t + \epsilon_{ct},$$

- $h = t - s$ denotes the horizon of the mobility measure ($h = 10$ here).
- j: lagged mobility ranging from 0 to 4 (elimination of serial autocorrelation discussed in paper)
- Robust standard errors clustered at the country level

Linear Model: Gini (same with Atkinson)

	DEPENDENT VARIABLE: **Relative Upward Mobility**										
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
$\mathbf{G}_{t-10}$	**19.491*** **	**7.099*** **	**7.654*** **	**7.520*** **	**7.372*** **	**7.400*** **	**6.907*** **	**7.792*** **	**7.807*** **	**8.250*** **	**9.692*** **
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.004)
$R(\mathbf{y}[10])_{ct-1}$		0.780***	0.752***	0.777***	0.756***	0.755***	0.828***	0.755***	0.764***	0.718***	0.688***
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$R(\mathbf{y}[10])_{ct-2}$			0.015	0.051	0.071	0.071		0.061	0.077	0.090	0.088
			(0.840)	(0.148)	(0.164)	(0.164)		(0.570)	(0.268)	(0.323)	(0.321)
$R(\mathbf{y}[10])_{ct-3}$				-0.068	0.047	0.047			-0.036	0.050	0.053
				(0.115)	(0.519)	(0.517)			(0.469)	(0.508)	(0.478)
$R(\mathbf{y}[10])_{ct-4}$					-0.140**	-0.140**				-0.122*	-0.123*
					(0.047)	(0.047)				(0.094)	(0.090)
LOG INCOME P.C.$_{t-1}$						-0.025					-0.490
						(0.748)					(0.454)
AR(2) autoc. test (p-val)							0.783	0.981	0.836	0.467	0.460
R^2	0.270	0.760	0.768	0.773	0.777	0.777					
Obs	4253	4120	3987	3854	3721	3721	3987	3854	3721	3588	3588

Linear Model: Gini (same with Atkinson)

	Dependent Variable: **Absolute Upward Mobility**										
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
$\mathbf{G}_{t-10}$	**23.149*** **	**6.013*** **	**5.790*** **	**5.387*** **	**4.996*** **	**5.019*** **	**5.391*** **	**3.896** **	**3.617** **	**8.477*** **	**5.732** **
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.010)	(0.034)	(0.001)	(0.040)
$M(\mathbf{y}[10])_{ct-1}$		0.884***	1.031***	1.013***	0.967***	0.976***	0.960***	1.185***	1.144***	0.843***	1.034***
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$M(\mathbf{y}[10])_{ct-2}$			-0.168***	-0.028	-0.002	-0.001		-0.250***	-0.132	0.006	-0.031
			(0.001)	(0.419)	(0.951)	(0.968)		(0.007)	(0.155)	(0.952)	(0.690)
$M(\mathbf{y}[10])_{ct-3}$				-0.135***	0.009	0.008			-0.083***	-0.013	0.010
				(0.000)	(0.818)	(0.840)			(0.006)	(0.860)	(0.803)
$M(\mathbf{y}[10])_{ct-4}$					-0.144***	-0.141***				-0.133	-0.105**
					(0.000)	(0.000)				(0.137)	(0.012)
Log Income p.c.$_{t-1}$						-0.345					-1.847
						(0.241)					(0.160)
R^2	0.236	0.842	0.851	0.859	0.861	0.861					
Obs	4253	4120	3987	3854	3721	3721	3987	3854	3721	3588	3588

Remarks

- Positive and significant relationship between mobility and inequality
  - One sd $\uparrow$ in Gini (0.1) $\Rightarrow$ relative upward mobility $\uparrow 0.74$ ($\approx$ 65% of sd)

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- Positive and significant relationship between mobility and inequality
  - One sd $\uparrow$ in Gini $(0.1) \Rightarrow$ relative upward mobility $\uparrow 0.74$ ($\approx$ 65% of sd)
- Weaker but still positive association with absolute upward mobility:
  - negative relationship between inequality and growth

Measurement Error and Mean Reversion: Gini

- Average incomes over 3 years:

	Dependent Variable: **Relative Upward Mobility**										
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
G$_{t-10}$	**19.563*** **	**8.552*** **	**8.595*** **	**8.784*** **	**8.233*** **	**7.989*** **	**8.163*** **	**9.417*** **	**9.965*** **	**9.209*** **	**12.894*** **
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)
$R(\mathbf{y}[10])_{ct-1}$		0.674***	0.481***	0.479***	0.469***	0.469***	0.748***	0.551***	0.585***	0.573***	0.358***
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$R(\mathbf{y}[10])_{ct-2}$			0.256***	0.273***	0.334***	0.332***		0.187***	0.188***	0.251***	0.316***
			(0.000)	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)
$R(\mathbf{y}[10])_{ct-3}$				-0.027	0.071	0.069			-0.049	0.041	0.080
				(0.697)	(0.238)	(0.241)			(0.462)	(0.563)	(0.119)
$R(\mathbf{y}[10])_{ct-4}$					-0.188***	-0.185***				-0.169***	-0.173***
					(0.000)	(0.000)				(0.000)	(0.000)
Income p.c. (log)$_{t-10}$						0.696**					0.499
						(0.029)					(0.569)
AR(2) autoc. test (p-val)							0.000	0.289	0.444	0.076	0.113
R^2	0.224	0.589	0.619	0.620	0.635	0.644					
Obs	4555	4403	4251	4099	3947	3947	4251	4099	3947	3795	3795

Measuring Upward Mobility: A Summary

- A bewildering variety of mobility indices:
  - directional/non-directional; absolute/relative.

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  - It is panel-independent
- If convincing, this **significantly expands the scope of empirical inquiry**

Appendix 1: Population Consistency

Given: $\mathbf{z} = (y_1, g_1, \ldots, y_k, g_k, \ldots, y_n, g_n)$

$$\mathbf{z}' = (y_1, g_1, \ldots, y_k, g_k - \epsilon, \ldots, y_n, g_n)$$

$$\mathbf{z}'' = (y_1, g_1, \ldots, y_k, g_k + \epsilon, \ldots, y_n, g_n)$$

and $\mathbf{z}'$ and $\mathbf{z}''$ have average mobility distinct from $\mathbf{z}$: $\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$,

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Then: $$M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z}).$$

back

Appendix 2: Proof of Theorem 1

- **Step 1. For every k, $m(g_k) \equiv M(g_k|\mathbf{y}, \mathbf{g}_{-k})$ is affine in g_k, or equivalently**:

$$m(g_k) = \frac{1}{2}\left[m(g_k - \epsilon) + m(g_k + \epsilon)\right] \text{ for every } \epsilon > 0.$$

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- Suppose false for some g_k and ϵ.
  - Define $\mathbf{z} = (\mathbf{y}, \mathbf{g}_{-k}, g_k)$, $\mathbf{z}' = (\mathbf{y}, \mathbf{g}_{-k}, g_k - \epsilon)$, and $\mathbf{z}'' = (\mathbf{y}, \mathbf{g}_{-k}, g_k + \epsilon)$.
  - Then $M(\mathbf{z}') + M(\mathbf{z}'') \neq M(\mathbf{z}) + M(\mathbf{z})$.
  - By Local Merge, $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$.
  - Say $M(\mathbf{z}' \oplus \mathbf{z}'') > M(\mathbf{z} \oplus \mathbf{z})$.

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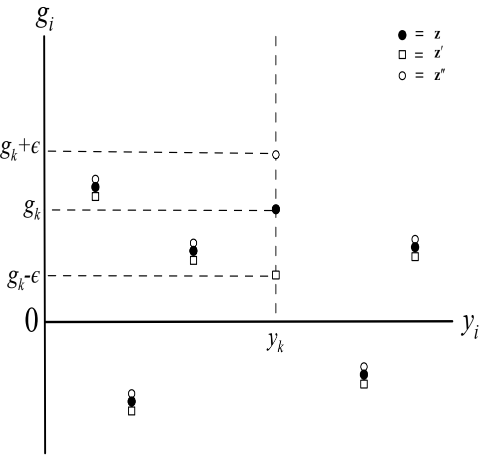

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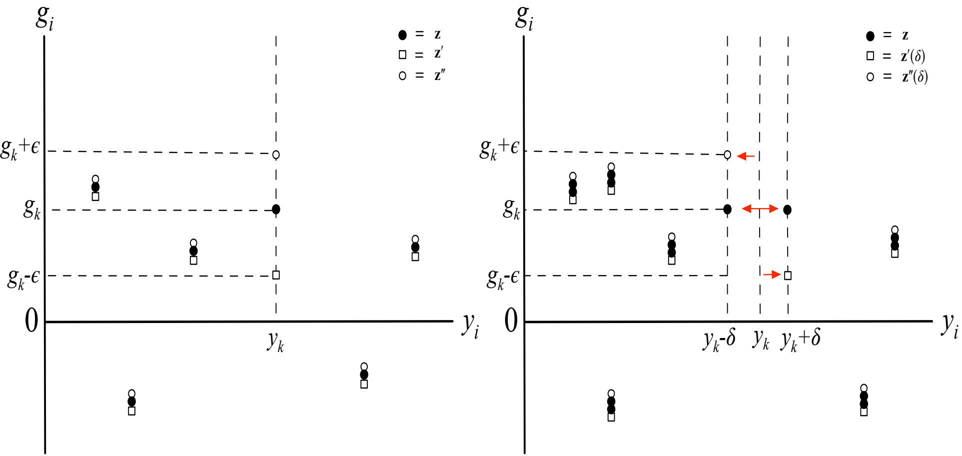

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- **Step 2. (Gallier 1999) $M(z)$ multiaffine so can be written as**:

$$M(\mathbf{z}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right].$$

for a collection $\{\phi_S\}$ defined for every $\emptyset \neq S \subset \{1, \ldots, n\}$.

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- **Step 3. All nontrivial product terms above *must have zero coefficients*.**

Suppose $\{ij\} \subset S$ for some S with $\phi_S(\mathbf{y}) \neq 0$. We will only move g_i and g_j but with $g_i + g_j = G$, so hold all else fixed and write

$$M(\mathbf{y}, \mathbf{g}) = \alpha g_i (G - g_i) + \beta g_i + \gamma (G - g_i) + \delta.$$

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$$\Rightarrow \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_i} - \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_j} = \alpha G - 2\alpha g_i + \beta - \gamma.$$

Choose G and g_i to violate Growth Progressivity. back