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- **Supplement to Slides 2:** A General Model of Occupational Choice

## A General Model of Occupational Choice

### ■ Production with capital and occupations.

- Population distribution on occupations  $n$  (endogenous).
- Physical capital  $k$ .
- Production function  $y = F(k, n)$ , CRS and strictly quasiconcave.

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### ■ Training cost function $x$ on occupations:

- incurred up front.
- parents pay directly, or bequeath and then children pay.

- **Perfect competition.**

- Return on capital fixed at rate  $r$  (international  $k$ -mobility).
- “Wage” vector  $\mathbf{w} \equiv \{w(h)\}$  endogenously determined for each occupation  $h$ .
- Together with  $r$ ,  $\mathbf{w}$  **supports profit-maximization.**

## Supporting Profit Maximization

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- If that unit cost  $\neq$  output price:
  - $(\mathbf{w}, r)$  **cannot support profit maximization at positive output.**
  - Otherwise, it does.
- **Note:** For any  $\mathbf{w}$ , there is a unique **scaling**  $\mu > 0$  such that  $(\mu\mathbf{w}, r)$  supports profit maximization.

# Households

- Continuum of households, each with one agent per generation.

- $y = \quad z \quad + \quad b \quad + \quad x(h)$   
wealth    consumption    fin. bequests    occ. choice

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- Child wealth  $y' = (1 + r)b + \mathbf{w}_{t+1}(h)$ .
- Parent picks  $(b, h)$  to **maximize utility**.
- No debt!  $b \geq 0$ .
- Child grows up; back to the same cycle.

## Preferences and Equilibrium

- **Preferences:** mix of income-based and nonpaternalistic

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- **Equilibrium:** wages  $\mathbf{w}_t$ , value functions  $V_t$ , occupational distributions  $\mathbf{n}_t$  s.t.:
  - Each family  $i$  chooses  $\{h_t(i), b_t(i)\}$  optimally
  - Occupational choices  $\{h_t(i)\}$  aggregate to  $\mathbf{n}_t$ ;
  - Firms willingly demand  $\mathbf{n}_t$  at prices  $(\mathbf{w}_t, r)$ .
  - **Note:** physical capital willingly supplied to meet any demand.

# Steady State

- A **steady state** is a **stationary equilibrium** with positive output and wages:
  - $w_t = w \gg 0$ , and
  - $(k_t, n_t) = (k, n)$  for all  $t$ , and  $F(k, n) > 0$ .

## Rich Occupational Structure

- **The richness assumption [R]:**
  - The set of all training costs is a compact interval  $[0, X]$ .
  - If  $n$  is zero on any positive interval of training costs, then  $y = 0$ .

## A Benchmark With No Occupational Choice

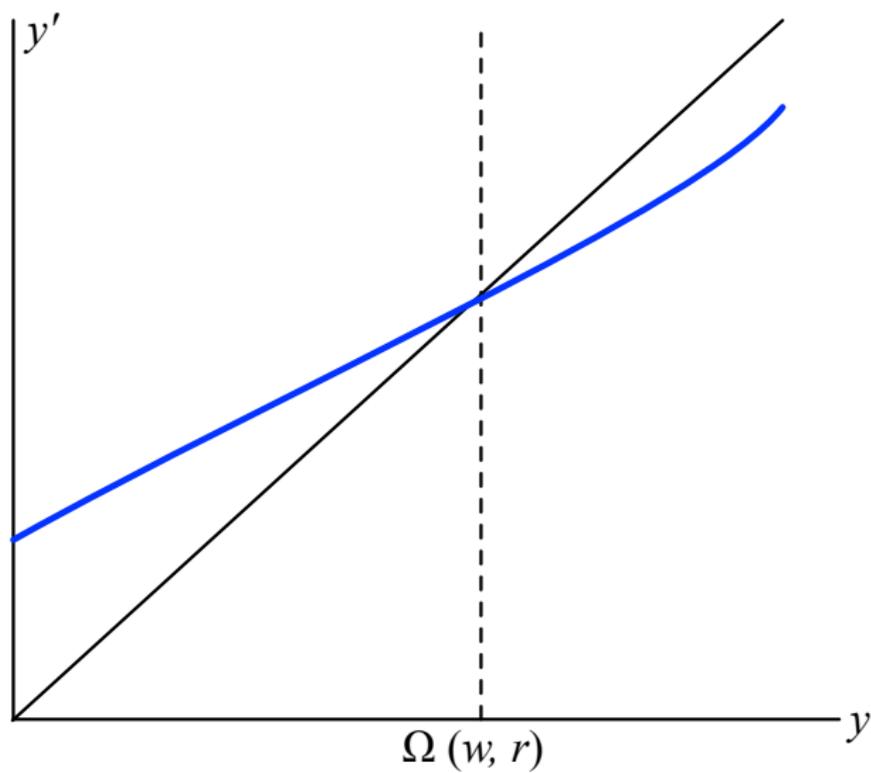
- **Financial bequests (at rate  $r$ ) + just one occupation (wage  $w$ ).**

- Parent with wealth  $y$  selects  $b \geq 0$  to

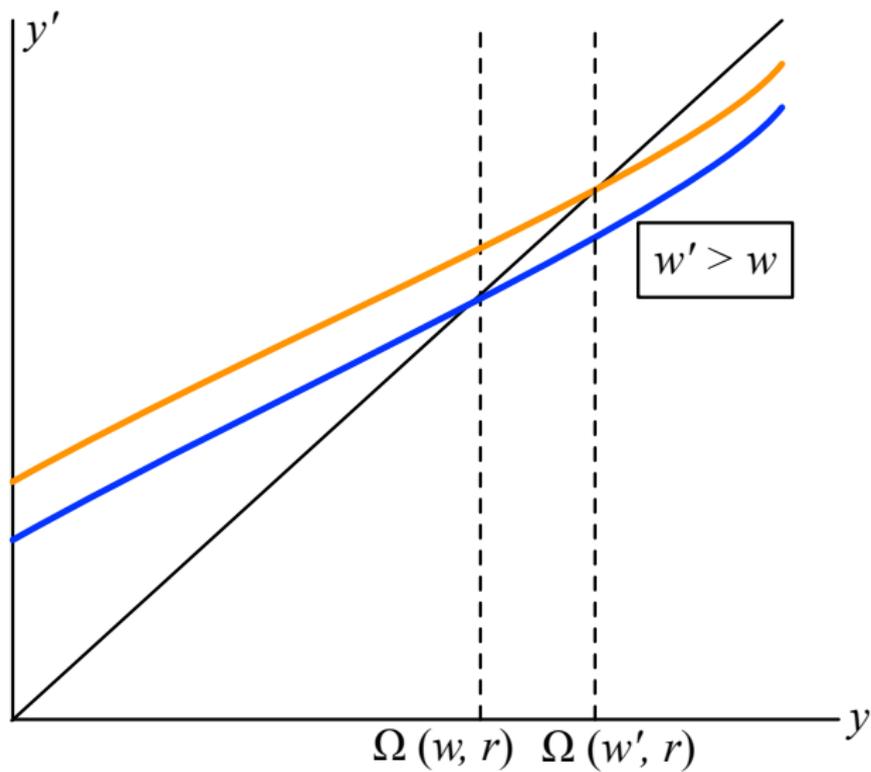
$$\max U(c) + \delta[\theta V(y') + (1 - \theta)P(y')].$$

- **Child wealth**  $y' \equiv w + (1 + r)b$ , increases in  $y$ .
- Converges to **limit wealth**  $\Omega(w, r) < \infty$ .
- This needs  $\theta < 1$ .
- Could depend on initial  $y$  (as in non-concave Ramsey model); we exclude that.

## Limit Wealth in Benchmark Model



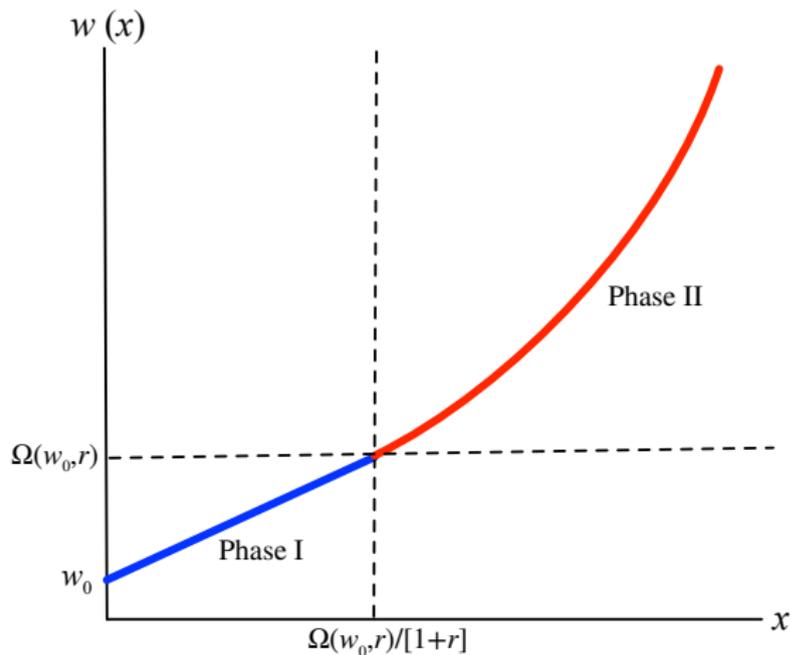
## Limit Wealth in Benchmark Model



# Back to Occupational Choice

## Theorem 1

- Every steady state  $w$  is fully described by a **two-phase property**:



- **In Phase I  $w$  is linear in  $x$ :** there is  $w_0 > 0$  such that

$$w(x) = w_0 + (1 + r)x \text{ for all } x \leq \frac{\Omega(w_0, r)}{1 + r}$$

- All families in Phase I have the **same overall wealth**  $\Omega(w_0, r)$ .

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- **In Phase II,  $w$  follows the differential equation**

$$w'(x) = \frac{U'(w(x) - x)}{\delta[\theta U'(w(x) - x) + (1 - \theta)P'(w(x))]}$$

- with endpoint to patch with I:  $w(x) = w_0 + (1 + r)x$  as  $x \downarrow \frac{\Omega(w_0, r)}{1 + r}$ .
- Families located in Phase II have **different wealths and lifetime consumptions.**

## ■ Closer look at Phase II

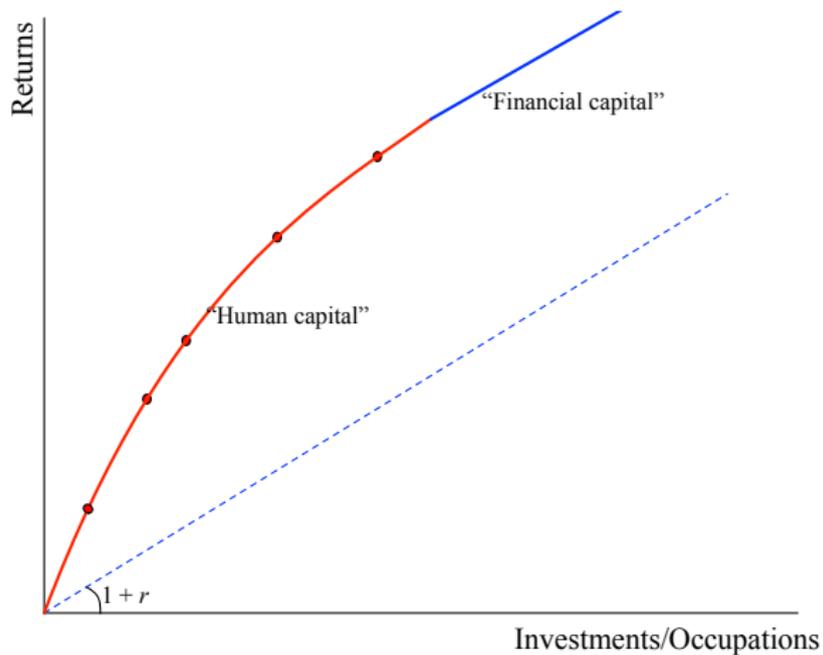
$$w'(x) = \frac{U'(w(x) - x)}{\delta[\theta U'(w(x) - x) + (1 - \theta)P'(w(x))]}$$

## ■ Shape comes from **Euler equation**:

- depends fundamentally on preferences
- **technology only serves to pin down baseline**  $w_0$  (remember remark on scaling)

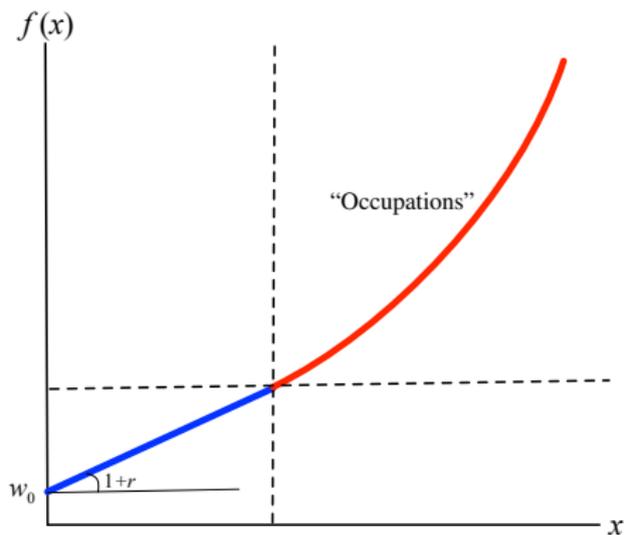
# A Testable Implication

- **Recall standard model.** By assumption:



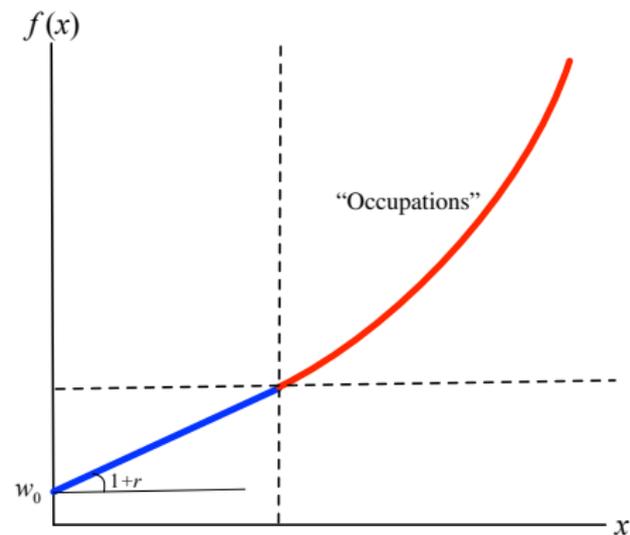
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- Compare to Phases I and II: *This?*

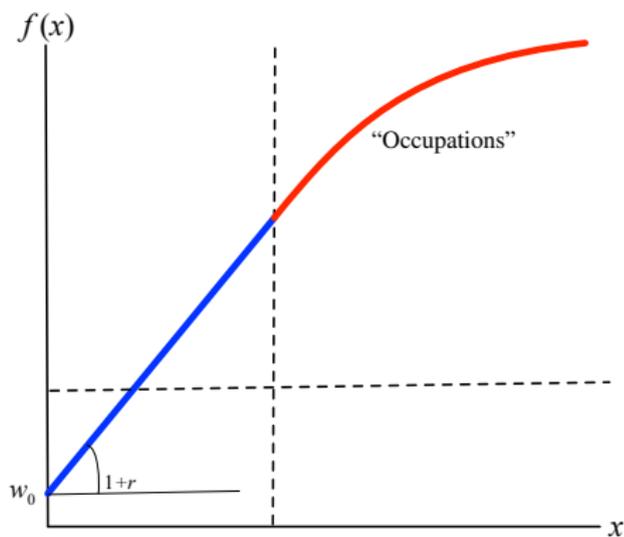


# A Testable Implication

■ Compare to Phases I and II: *This?*



*Or this?*



## A Testable Implication

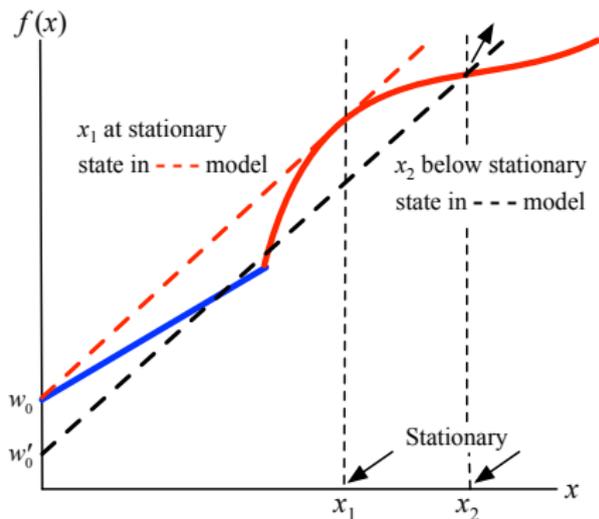
### Theorem 2

*The average return  $\frac{w(x)-w_0}{x}$  to occupational investment is flat in Phase I and strictly increasing in Phase II.*

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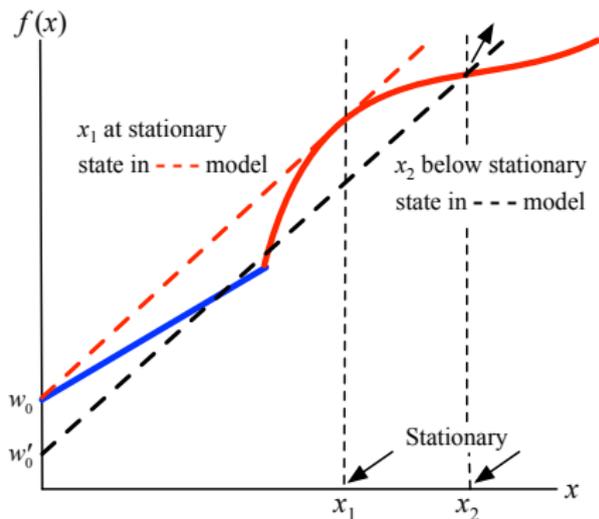
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- Contradiction to unique limit wealth in benchmark, increasing in  $w$ .

## Unique Steady State with Rich Occupational Structure

- We end with a **fundamental difference from two-occupation case**:

### **Theorem 3**

*There is at most one steady state.*

## Unique Steady State with Rich Occupational Structure

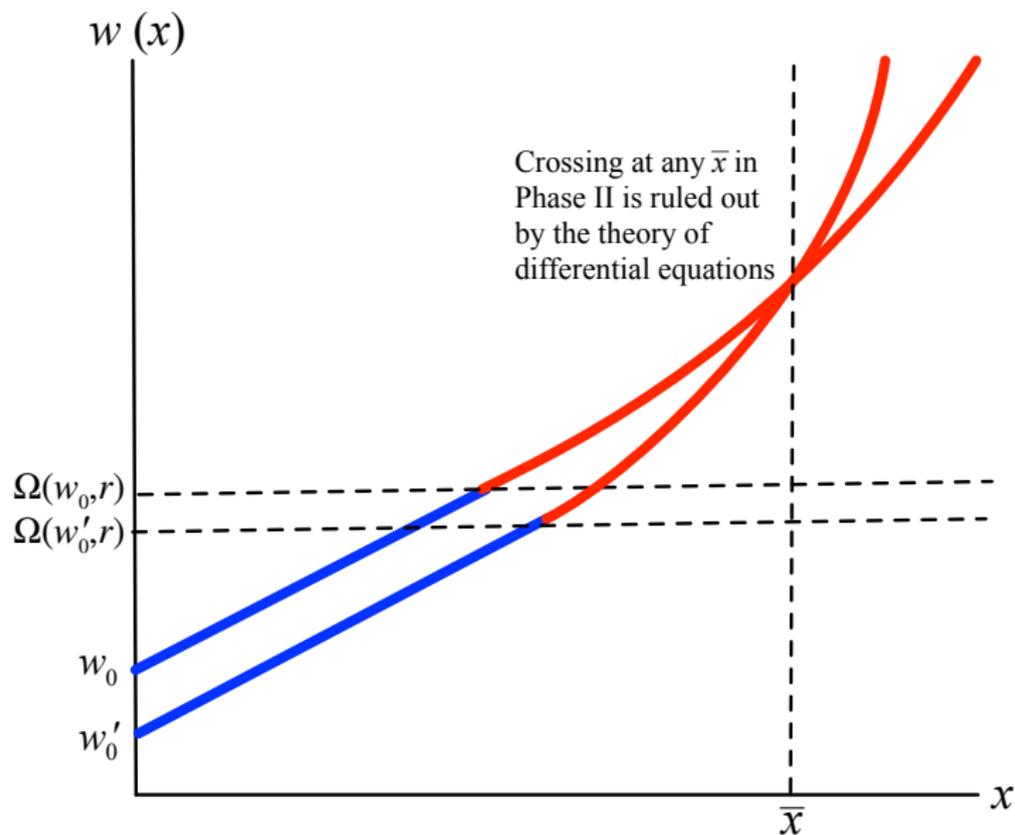
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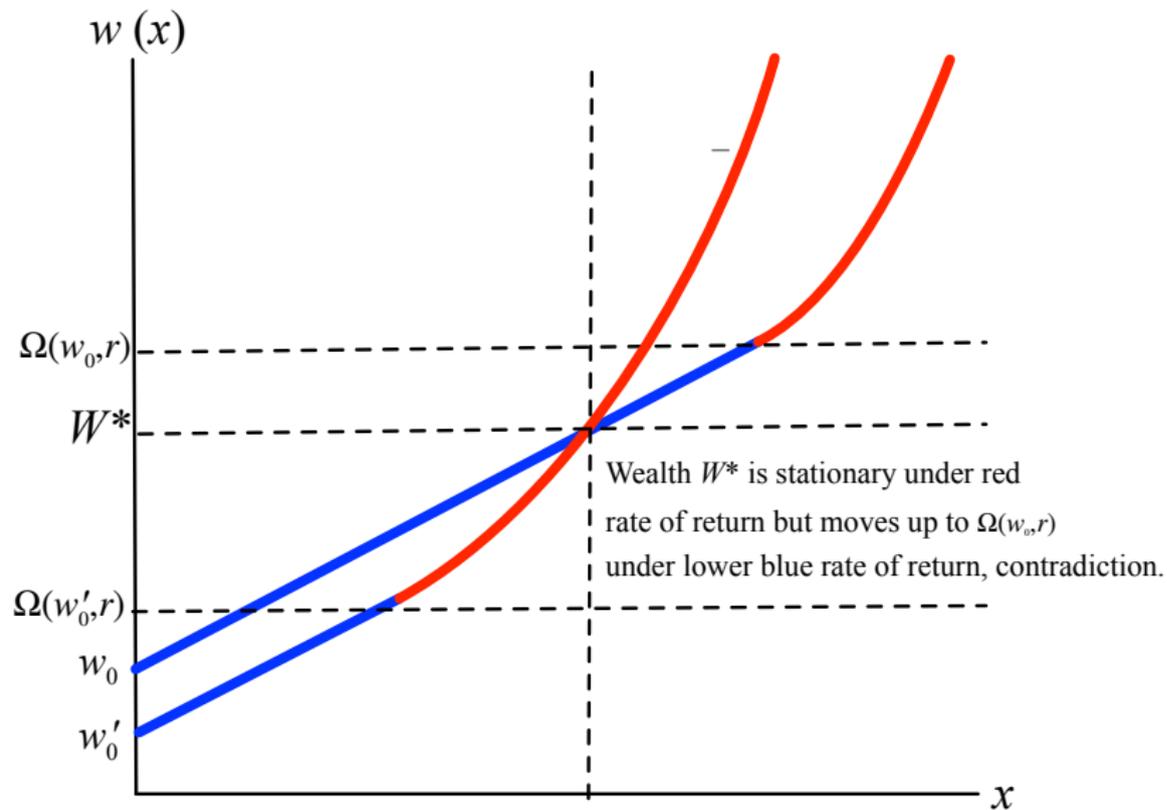
*There is at most one steady state.*

- Proof idea:
  - No two members of the two-phase family (indexed only by  $w_0$ ) can cross.
  - Then **only one**  $w_0$  **can support profit maximization** with positive output.  
(For all wages must co-move with intercept wage  $w_0$ .)

## ■ No-crossing argument, part I



## ■ No-crossing argument, part II



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- In Phase I, there is **perfect equality** of overall wealth.
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- Families at different occupations in Phase II **cannot have the same wealth**.
- Thus, “most” inequality in this model comes from nonalienable capital.
- This focus will change when we consider automation in the next set of models and the decline in functional labor share.

## Three Remarks

- **II. When is Phase II nonempty?**
- When there is a **large occupation span relative to bequest motive**:
  - Discounting.
  - Poverty, via TFP differences.
  - Growth in TFP, lowers effective bequest motive
  - World return on capital.
  - Globalization: new occupations.

## Three Remarks

### ■ III. Two Notions of History-Dependence

- At the macro-level, history-dependence depends on occupational richness.
- A lot of history-dependence at the individual level.
- The distribution **as a whole** is pinned down, but not who occupies which slot.