

EC9AA Term 3: Lectures on Economic Inequality

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- **Slides 2:** Occupational Choice and Inequality

Markets and Personal Inequality

- Two views:
- **Equalization**: Inequality an ongoing battle between convergence and “luck.”
 - Solow 1956, Brock-Mirman 1972, Becker-Tomes 1979, 1986, Loury 1981...
- **Disequalization**: Markets intrinsically create and maintain inequality.
 - Ray 1990, Banerjee-Newman 1993, Galor-Zeira 1993, Ljungqvist 1993, Freeman 1996, Mookherjee-Ray 2000, 2010...

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income consumption investment/bequest

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- Not surprising that this literature looks like growth theory.
- Lots of “mini growth models”, one per household.
- But f can have various interpretations.

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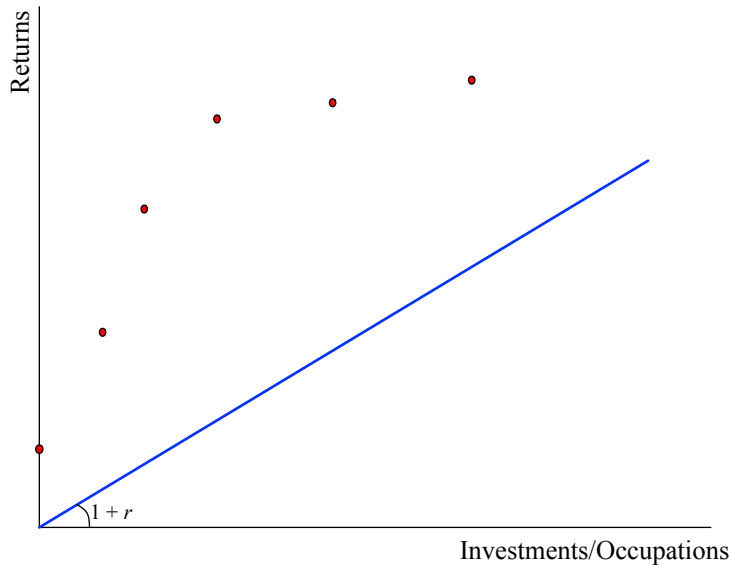
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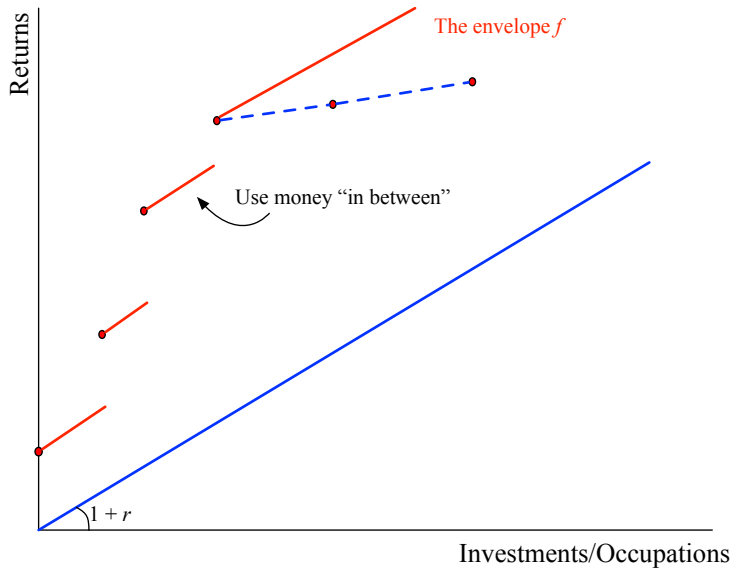
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- So interpret f as **envelope of intergenerational investments:**

- Financial bequests
- Occupational choice





Parental Preferences and Limited Mobility

- **Parental utility** $U(c) + W(y^+)$, where c = consumption and y^+ = child income, and:
 - U is increasing and strictly concave in c , and $W(y^+)$ is increasing in y^+ .

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Future utility

Bellman value

Exogenous value

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Theorem 1

- Let h describe all optimal choices of k for each y .
- Then if $y > y'$, $k \in h(y)$, and $k' \in h(y')$, it must be that $k \geq k'$.

Proof of Theorem 1

- Pick $y > y'$, $k \in h(y)$, and $k' \in h(y')$. **Suppose** $k' > k$.

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- k beats k' under y , so:

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■ Adding, rearranging:

$$U(y - k) - U(y - k') \geq U(y' - k) - U(y' - k'),$$

which **contradicts the strict concavity of U .**

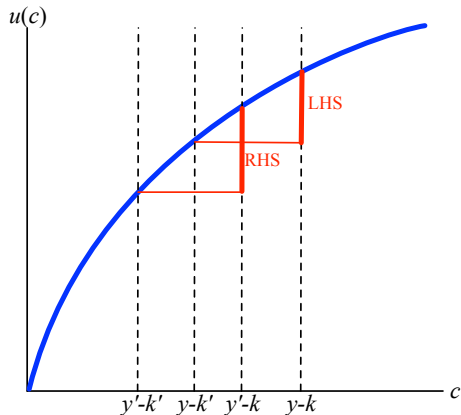


Illustration

For $y > y'$ and $k' > k$,

$$U(y - k) - U(y - k') \geq U(y' - k) - U(y' - k'),$$

contradicts this picture:



Remarks:

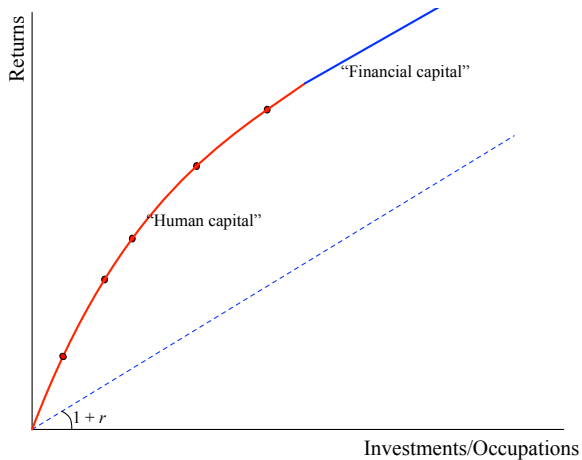
- h is “almost” a function.
- h can only jump up, not down.
- Same assertion is not true of optimal c .
- Note how curvature of U is important, that of W is unimportant.
- **Crucial for models in which f is endogenous** with uncontrolled curvature.

Standard Assumption

f is *exogenous*, and *concave*:

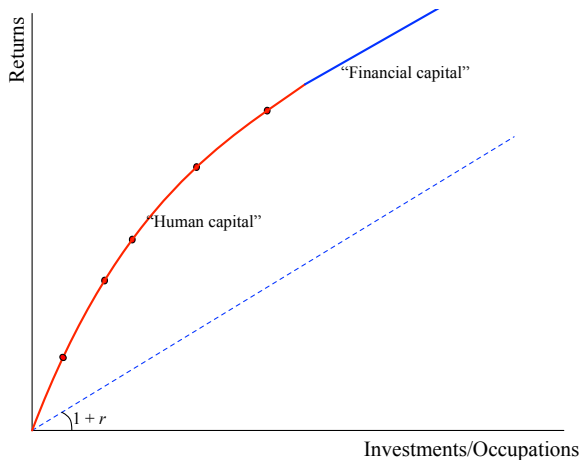
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- Generates convergence to unique steady state in the absence of uncertainty.

Convergence With Concavity: Intuition

- Look at **Bellman case** with no uncertainty:

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- But (1) + Envelope Theorem $\Rightarrow V'(y_{t+1}) = u'(c_{t+1})$, so:

$$U'(c_t) = \delta U'(c_{t+1}) f'(k_t). \quad (3)$$

- Theorem 1 + (3) imply **convergence to unique k^* , where $\delta f'(k^*) = 1$.**

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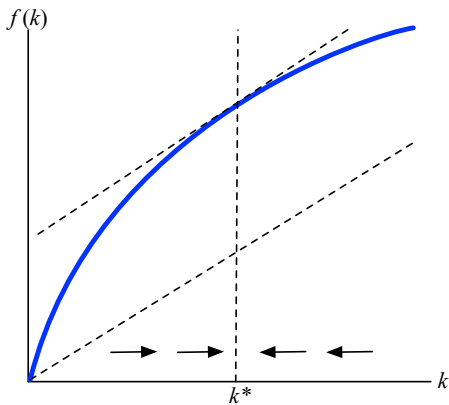
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- Envelope theorem *still works*, so:

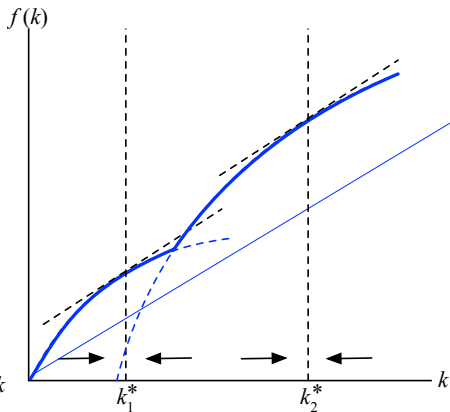
$$U'(c_t) = \delta U'(c_{t+1}) f'(k_t) . \quad (6)$$

- So again convergence to k^* , where $\delta f'(k^*) = 1$, **but now k^* is not unique.**

Comparison



$$f'(k^*) = 1$$



$$\delta f'(k_1^*) = \delta f'(k_2^*) = 1$$

Stochastic Shocks

- What happens to these models with stochastic shocks?
- Something weird, at least conceptually.

Theorem 2

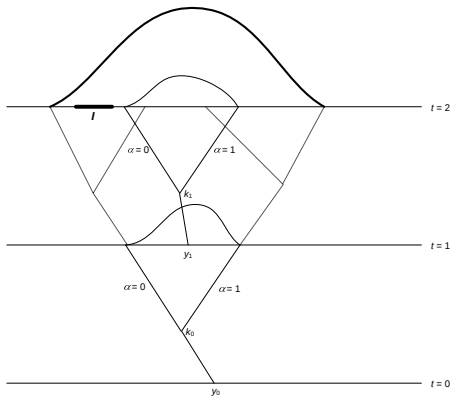
Brock-Mirman 1976, Becker-Tomes 1979, Loury 1981, extended to drop concavity

- Assume a mixing condition, such as $f(0, 1) > 0$ (*poor genius*) and $f(k, 0) < k$ for all $k > 0$ (*rich fool*).
- Then there exists a unique measure on incomes μ^* such that μ_t converges to μ^* as $t \rightarrow \infty$ from every μ_0 .

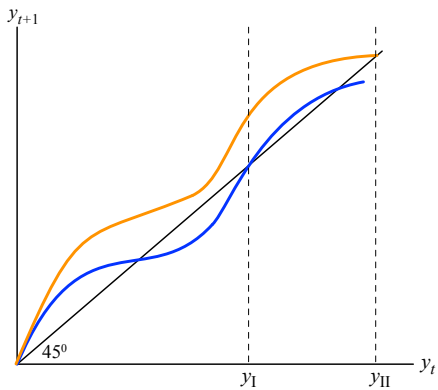
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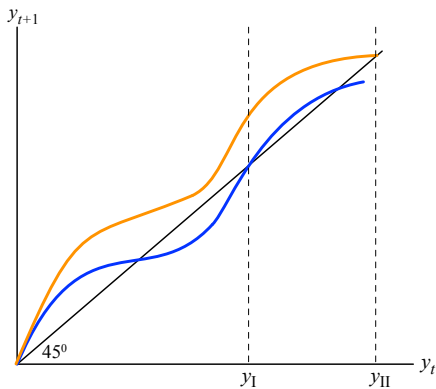


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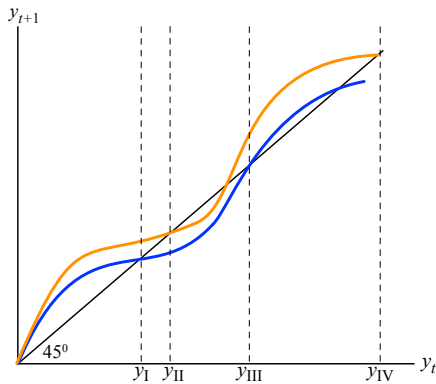


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■ III. The reliance on efficiency units.

- No way to endogenize the returns to different occupations.
- Whether f concave at the household level **should depend on markets.**

Inequality and Markets

- Return to the **interpretation of f as occupational choice**.
- Dropping efficiency units creates movements in relative prices:
- f isn't "just technology" anymore.

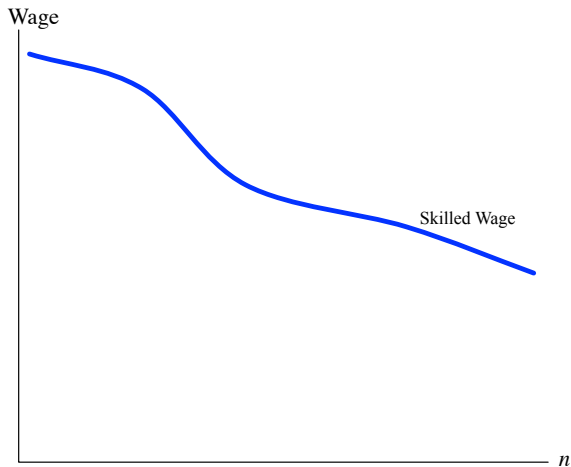
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- **An Extended Example** with just two occupations
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 - Population allocation $(n, 1 - n)$.
 - Output: $F(n, 1 - n)$, where F is a macro production function on skilled and unskilled labor:

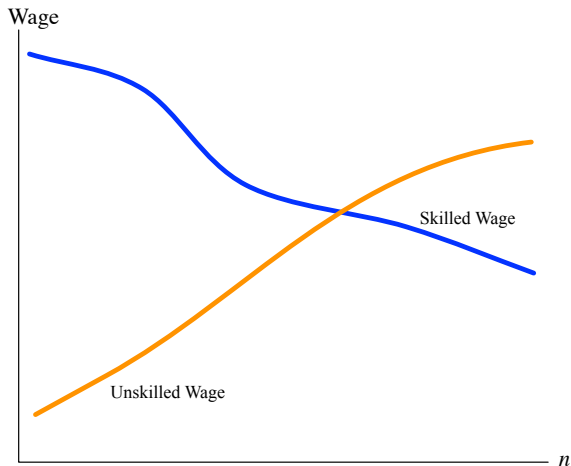
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Households

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- Starting wealth y ; $y = c + k$, where $k \in \{0, X\}$.
- Child wealth $y' = w$, where $w = w_s$ or w_u .
- **Parent maxes $U(c) + \delta V(y')$ (Bellman equation)**
- No debt!
- Child grows up; back to the same cycle.

Equilibrium

- A sequence $\{n^t, w_s^t, w_u^t\}$ such that
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- **Steady states:**
 - A constant fraction n are skilled
 - Wages are constant at $w_s = F_1(n, 1 - n)$ and $w_u = F_2(n, 1 - n)$
 - All parents keep replicating their skill status in their children.
 - Replication of skills follows from Theorem 1.

Steady States in Occupational Choice

■ Conditions for n to be a steady state:

[Skilled parent]
$$V(w_s) = \frac{u(w_s - X)}{1 - \delta} \geq u(w_s) + \frac{\delta}{1 - \delta} u(w_u)$$

[Unskilled parent]
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Theorem 3

Every n with $w_s = F_1(n, 1 - n)$ and $w_u = F_2(n, 1 - n)$ such that

$$\underbrace{u(w_u) - u(w_u - X)}_{\text{Unskilled Cost}} \geq \underbrace{\frac{\delta}{1 - \delta} [u(w_s - X) - u(w_u)]}_{\text{Future Benefit}} \geq \underbrace{u(w_s) - u(w_s - X)}_{\text{Skilled Cost}}$$

must be a steady state.

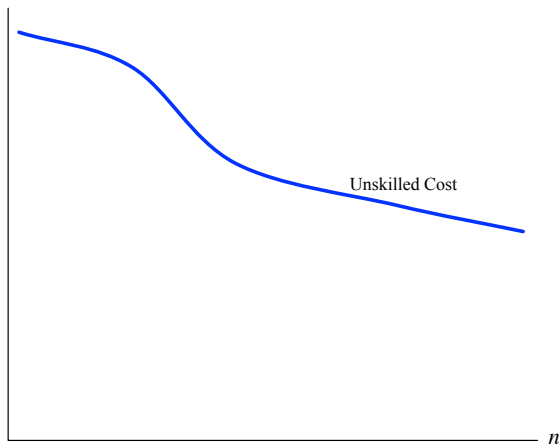
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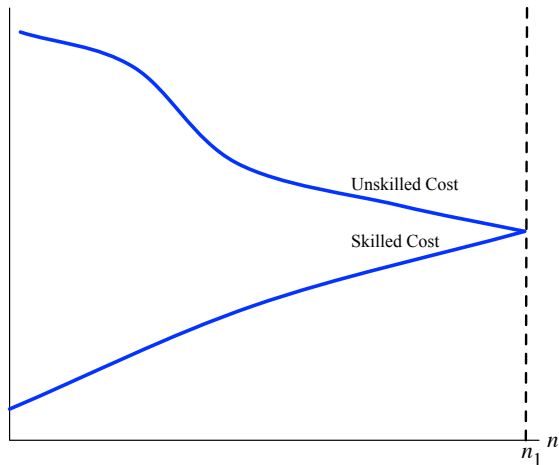
Costs and Benefits (Utils)



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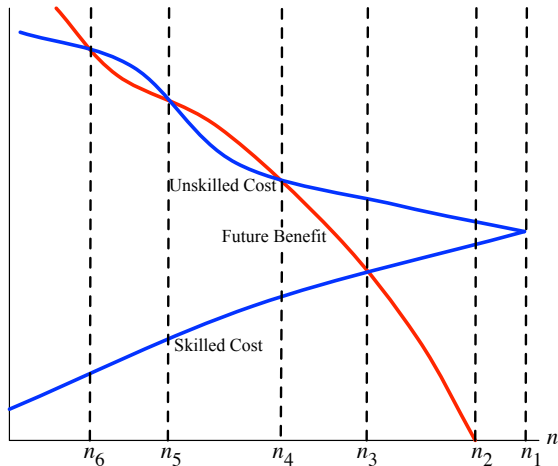
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3. **No convergence**; persistent inequality in *utilities*.

- Symmetry-breaking argument.

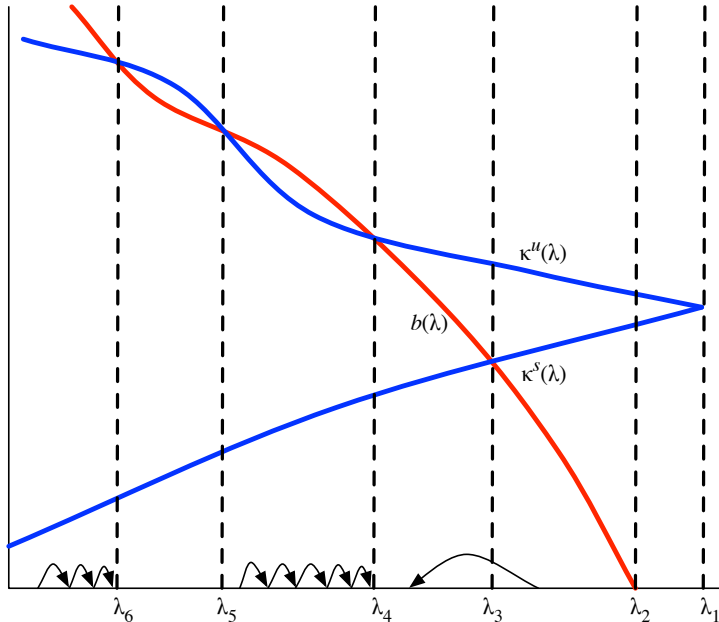
Features of the Two-Occupation Model

4. Dynamics and history-dependence.

Theorem 4

- (i) From any initial n that is a steady state, the system remains there: $n_t = n$ for all t .
- (ii) From any initial n that is not a steady state, but with some steady state $n' > n$, n_t converges monotonically up to the **smallest** steady state exceeding n .
- (iii) From any initial n that is larger than any steady state, n_t converges down in **one period** to some steady state.

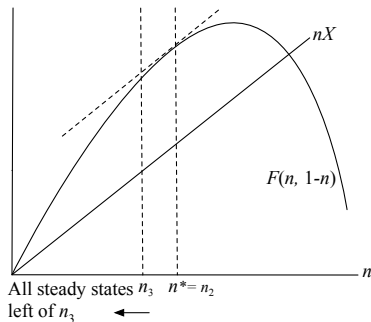
Dynamics



Features of the Two-Occupation Model

5. Every steady state is inefficient.

- Efficient allocation maximizes $F(n, 1 - n) - nX$:



$F_1(n^*, 1 - n^*) - F_2(n^*, 1 - n^*) = X, \Rightarrow w_s^* - X = w_u^* \Rightarrow n^* = n_2$. But every steady state is to the left of n_3 (see steady state diagram).

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- $F(s, u) = sf\left(\frac{u}{s}\right)$
- Then:
 - $F_2(s, u) = f'\left(\frac{u}{s}\right) = w$, and
 - $F_1(s, u) = f\left(\frac{u}{s}\right) - \frac{u}{s}f'\left(\frac{u}{s}\right) = f\left(\frac{u}{s}\right) - \frac{u}{s}w = \text{profit}.$

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- Recall social's planner's n^* had higher net output than any steady state:
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- **Unconditional**: give equally to currently unskilled parents:

$$T_t = \frac{n_t \tau}{1 - n_t} w_s(n_t).$$

- **Conditional**: give to *all* parents conditional on educating children.

$$Z_t = \frac{n_t \tau}{n_{t+1}} w_s(n_t).$$

(can contemplate other obvious variants with similar results)

Features of the Two-Occupation Model

Theorem 5

- *With unconditional subsidies, every left-edge steady state declines, lowering the proportion of skilled labor and increasing pre-tax inequality, which undoes some or all of the initial subsidy.*
- *With conditional subsidies, every left-edge steady state goes up, increasing the proportion of skilled labor. In steady state, no direct transfer occurs from skilled to unskilled, yet unskilled incomes go up and skilled incomes fall.*
- *Conditional subsidies therefore generate superior macroeconomic performance (per capita skill ratio, output and consumption).*

Other Applications

- **Trade theory** in which autarkic inequality determines comparative advantage.
- **Country-level specialization** when national infrastructure is goods-specific.
- **Fertility patterns** in models of occupational choice.

A General Model with Occupational Choice

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The Wage Function in Steady State

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- Fully defined by preferences, except for $w(0)$, the unskilled wage.
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- Endogenous inequality, **but no multiplicity of steady states**.
- Macro- versus micro-history-dependence.

How are Wages Pinned Down?

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- Try out the pure value function model with $\theta = 1$:
- Then $w'(x) = (1/\delta)x$, or $w(x) = w(0) + (1/\delta)x$.
- The production function determines **quantities of labor** in different skills.
E.g., if:

$$y = A \left[\int_x a(x) \lambda(x)^\rho \right]^{1/\rho},$$

- then first-order condition for profit-maximization is

$$w(0) + \frac{1}{\delta}x = A^\rho y^{1-\rho} a(x) \lambda(x)^{\rho-1}.$$

- which pins down $w(0)$, y and every $\lambda(x)$.

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- Is this function (= household “production function”) concave?
 - In the Bellman case, yes, because it is affine.
 - Otherwise, no.

Luck versus Markets: Philosophy of Inequality

Equalization: Inequality an ongoing battle between convergence and “luck”

Disequalization: Markets intrinsically create and maintain inequality

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■ It remains to be seen if this is the right view of the world.