

# EC9AA Term 3: Lectures on Economic Inequality

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- **Slides 3:** Functional Inequality: The Falling Labor Share

## The Fourth Fundamental Law of Capitalism

- We now downplay personal endowments and accumulation
  - Though still very much in the background
- Our focus: **the functional distribution across capital and labor**

## The Fourth Fundamental Law of Capitalism

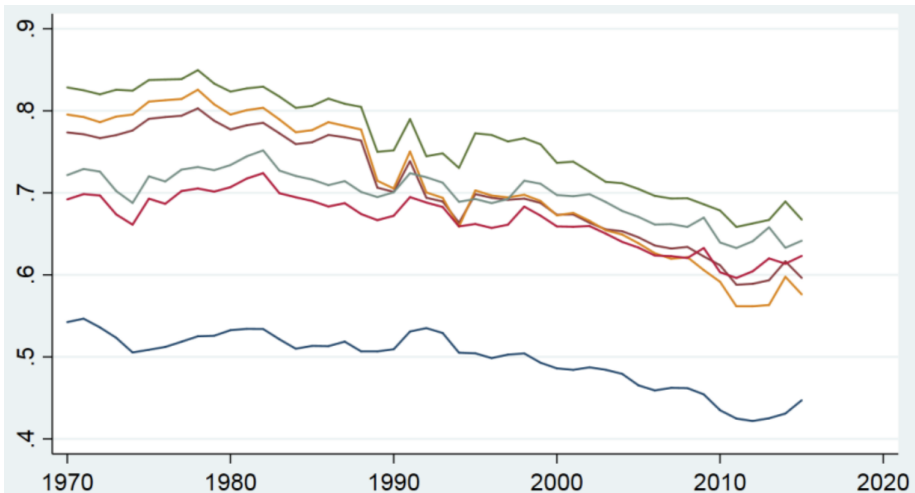
**With economic growth, capital displaces labor:**

*The labor share in national income must progressively vanish.*

- A **fundamental law**? You can't be serious.
- It isn't even testable (though stronger versions of it are)
- But it *is* a fundamental device for organizing our thoughts.

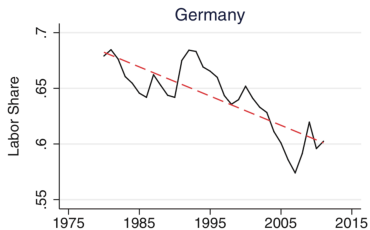
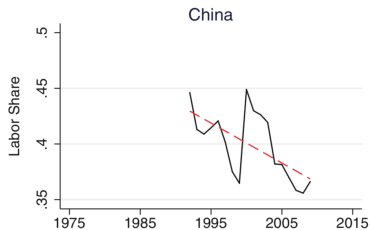
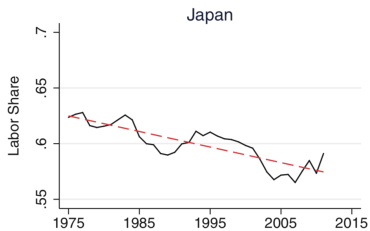
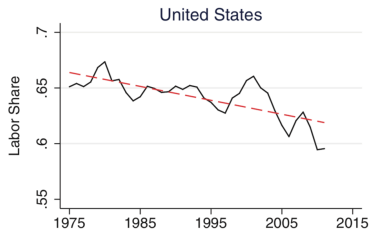
# Death of a Kaldor Fact

## The falling labor share:



# Death of a Kaldor Fact

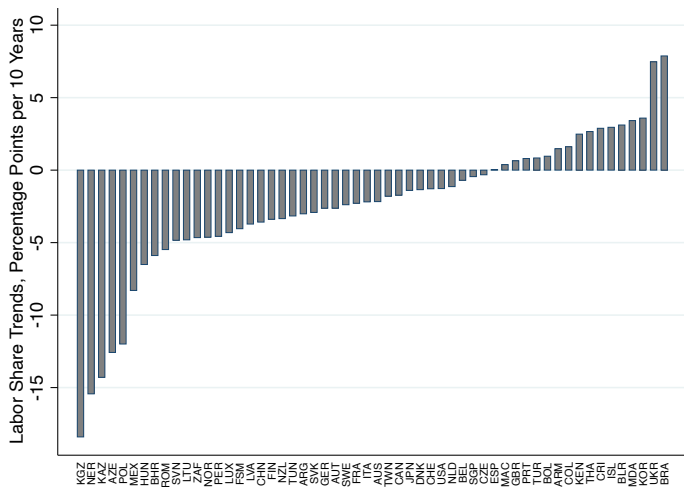
## The falling labor share:



Karabarbounis and Neiman (2014). Also Harrison (2002) and Rodríguez and Jayadev (2010),

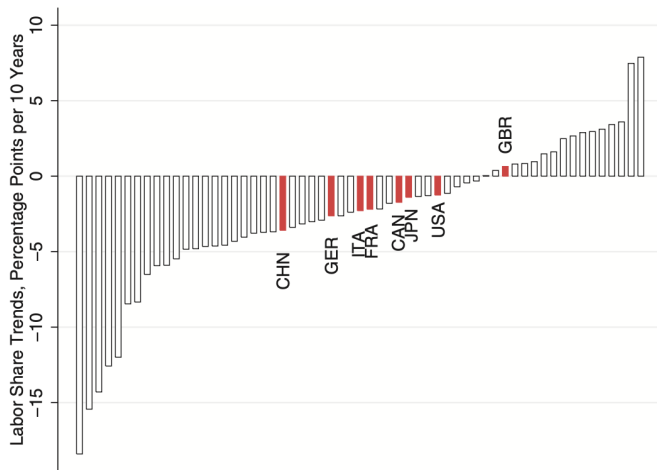
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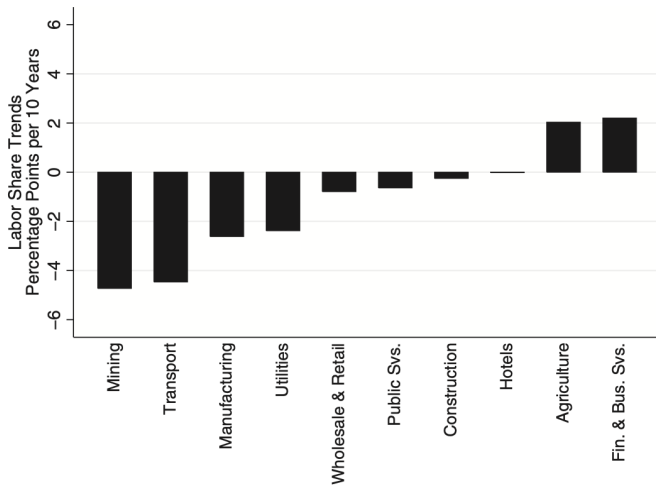
# Death of a Kaldor Fact

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# Death of a Kaldor Fact

## The falling labor share:





# Explanations

- **China Shock** Autor, Dorn and Hansen (2016)
  - globalization + cheap labor
  - but happening everywhere (e.g., jobless growth in India)
- **Concentration** Autor et al (2017), Azar and Vives (2019)
  - increasing product differentiation, gig economy, decaying unions
  - explanation, or outcome of declining labor power?
- **Technical progress** Acemoglu (1998, 2002), Acemoglu and Restrepo (2019)
  - robotics (hardware), machine learning (software)
  - But why is technical progress necessarily slanted to *displace* labor?
- **Covid-19**

# Explanations

## ■ Capital-Labor Substitution

- Employment elasticities by sector, various regions. Kapsos (2005).

Region	Agriculture	Industry	Services
World	0.24	0.21	0.61
W. Europe	-1.08	-0.50	0.74
N. America	-0.02	0.26	0.60
Central/Eastern Europe	-0.51	0.11	0.51
East Asia (excl. Japan)	0.10	0.07	0.47
Japan	-2.04	-0.83	0.76
Australia/NZ	0.18	0.26	0.61
South-East Asia	0.01	0.82	1.08
South Asia	0.38	0.41	0.46
Latin America	-0.16	0.63	1.09
Sub-Saharan Africa	0.69	0.88	0.89

# Explanations

## ■ Capital-Labor Substitution

- GDP and employment growth, some developing countries. An et al. (2017).

	Yearly, 1991–2000		Yearly, 2001–2015	
Country	GDP	EMP	GDP	EMP
Egypt	4.27	1.47	4.33	2.31
<b>India</b>	<b>5.73</b>	<b>0.60</b>	<b>7.09</b>	<b>0.61</b>
Indonesia	4.84	1.96	5.41	1.73
Kenya	2.09	2.20	4.38	2.00
Morocco	4.78	5.11	4.46	1.04
Nicaragua	3.17	5.61	3.66	3.19
Pakistan	4.48	1.99	4.29	2.84
Philippines	2.75	2.51	5.11	2.46
Tanzania	4.15	2.55	6.41	3.34
Vietnam	7.40	2.20	6.54	2.33

# Explanations

## ■ Capital-Labor Substitution

- Intuitively compelling:
- As growth occurs per-capita, capital becomes plentiful relative to labor
- So it makes sense that the relative prices of capital goods fall.

## ■ But ...

- Net effect on labor share depends on the **elasticity of substitution**.
- E.g., dividing line: Cobb-Douglas production function.
- This is what I want to try and explore further.

# Our Theory: Accumulation and Automation

- **Two pillars:**

- I. Human-physical asymmetry
- II. Machine capital and robot capital

## I. The Human-Physical Asymmetry

Mankiw-Romer-Weil 1992:

$$\dot{k}(t) = s_k y(t) - (n + \delta)k(t)$$

$$\dot{h}(t) = s_h y(t) - (n + \delta)h(t)$$

**What does the second equation mean?**

## I. The Human-Physical Asymmetry

- Physical capital can be indefinitely replicated:
  - And so can individual claims to them.
- But human capital *cannot* be replicated in the same way.
  - always in *one* physical self **[inalienable]**.
  - To some extent, scalable within occupation or sector
  - But more fundamentally, scales **across** sectors.

## II. Machines and Robots

- **Many sectors** indexed by  $j$ :

$$y_j = f_j(k_j, \tau_j), \text{ [sector-specific, CRS]}$$

- where  $k$  = machines and  $\tau$  = tasks.
- **Tasks produced by humans and/or robots:**  $\tau_i = \tau_i(h_i, r_i)$ .
- An intermediate “production function,” also CRS.
- More generally there could be many tasks per sector.
- **So capital comes in two flavors:**
  - $k$ : machines, complementary to labor.
  - $r$ : robots, substitutes for labor.



## II. Machines and Robots

### ■ The Feasibility of Automation

- Assume  $\tau_j(0, r) > 0$ .
- Which does not mean that automation is **optimal**
- Or that it will ever fully happen; e.g.:
- $\tau_j(h, r) = \nu_j r + \mu_j h + r^{\alpha_j} h^{1-\alpha_j}$  for  $\nu_j > 0$ ,  $\mu_j > 0$ , and  $\alpha_j \in (0, 1)$ .
- But certainly a threat if the price is right:

“nothing humans do as a job is uniquely safe anymore. From hamburgers to healthcare, machines can be created to successfully perform such tasks with no need or less need for humans, and at lower costs than humans...” Scott Santens,

*The Boston Globe*, 2016

## Three Special Sectors

- **Machine capital:**  $y_k = f_k(k_k, \tau_k)$ , with  $\tau_k = \tau_k(h_k, r_k)$ .
- **Robot capital:**  $y_r = f_r(k_r, \tau_r)$ , with  $\tau_r = \tau_r(h_r, r_r)$ .
- **Education:**  $y_e = f_e(k_e, \tau_e)$ , with  $\tau_e = \tau_e(h_e, r_e)$ .
- All assumptions made earlier **apply to these sectors** as well.

## A Bit More on Education

- Raw labor is given (or normalized), but **human capital grows endogenously**.
- Initial allocation of humans across occupations.
- Individuals can move from sector to sector (or task to task).
- Educational cost =  $e(i, j)p_e$ , the endogenous price of education.

### III. Preferences and Neutrality

- People have (possibly different) utility functions  $u$  and discount factors  $\beta$ .
- Someone **starts** with financial wealth + wage income in sector  $j$ ;
- allocates current expenditure  $z(t)$  consumption
- gets educated [evolution of human capital];
- invests [evolution of financial capital, which are claims on physical capital];
- **Ends** with new wealth, maybe new sector. **Repeat.**
- **Asymptotic Homotheticity of Preferences:**
- If  $\mathbf{x}(\mathbf{p}, z)$  is demand for goods as function of current expenditure  $z$ , then

$$\lim_{z \rightarrow \infty} \frac{\mathbf{x}_m(\mathbf{p}, z)}{z} = \mathbf{d}_m(\mathbf{p}) \text{ for some function } \mathbf{d}_m(\mathbf{p}).$$

# Price System

## Competitive Pricing

- **numeraire**: rental rate on machine capital
- **p**: prices, includes  $(p_r, p_k, p_e)$
- **w**: wages, includes  $(w_r, w_k, w_e)$
- **Unit cost function for tasks** determines task price  $q_j$  by CRS:

$$q_j = q_j(w_j, p_r) = \min \{w_j h_j + p_r r_j \mid \tau_j(h_j, r_j) = 1\}.$$

- **Unit cost function for output** determines output price  $p_j$  by CRS:

$$p_j = c_j(1, q_j) = \min \{k_j + q_j \tau_j \mid f_j(k_j, \tau_j) = 1\}$$

# Price System

## Some Properties and Implications of Prices

- profit-maximization:

- $p_j \frac{\partial f_j(k_j, \tau_j)}{\partial \tau_j} = q_j$ ,  $p_j \frac{\partial f_j(k_j, \tau_j)}{\partial k_j} = 1$ , etc.

- automation index for each sector  $j$  and relative price  $\zeta_j \equiv w_j/p_r$ :

$$a_j(\zeta_j) \equiv \min_{(r_j, h_j)} \left\{ \frac{r_j}{h_j \zeta + r_j} \mid (r_j, h_j) \text{ minimizes unit cost under } \zeta_j \right\} \in [0, 1].$$

- consumption-savings choices pinned down by:

$$\text{Interest rate } (t) = \frac{1 + (1 - \delta)p_k(t + 1)}{p_k(t)} - 1.$$

where  $\delta \in (0, 1)$  is the rate of depreciation.

# Model Summary

## Summary of Ingredients

- Asymmetry in accumulation
  - Physical capital can scale within and across sectors
  - Human capital expands across tasks/sectors **Pillar I**
- The two faces of capital
  - machines and robots **Pillar II**
- Otherwise pretty standard:
  - Homothetic preferences
  - Competitive price system;
  - Condition for growth (patience relative to technology).

# The Critical Role Played by Robot Production

- **Robot production function** like any other:

$$y = f_r(k, \tau), \text{ where } \tau = \tau_r(h, r).$$

- **Robot price** comes from unit cost function:

$$p_r = c_r(1, q_r).$$

- **Task price** bounded by the feasibility of robot automation:

$$q_r = q_r(p_r, w_r) \leq \nu_r^{-1} p_r, \text{ where } \nu_r \equiv \tau_r(0, r)/r.$$

- **Combining:**

$$p_r \leq c_r(1, \nu_r^{-1} p_r).$$



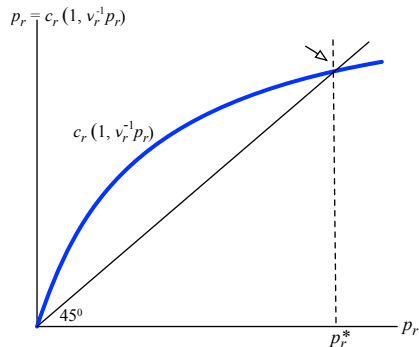
## The Critical Role Played by Robot Production

$$p_r \leq c_r(1, \nu_r^{-1} p_r).$$

- **Big question:** given this inequality, how high can robot prices go?  
(relative to the normalized cost of machine rentals, set to 1)
- Depends on whether  $c_r(1, \nu_r^{-1} p_r)$  goes below 45° line as  $p_r \uparrow$ .
- I.e., whether  $c_r(1, \nu_r^{-1} p_r) < p_r$  for all large  $p_r$ .

## The Critical Role Played by Robot Production

- $c_r(1, \nu_r^{-1} p_r) < p_r$  for all large  $p_r$ .



- Equivalent to  $\nu_r > \lim_{\rho \rightarrow 0} c_r(\rho, 1)$ .
- **If this condition holds, then  $p_r$  must be bounded.**

## The Critical Role Played by Robot Production

- If  $\nu_r > \lim_{\rho \rightarrow 0} c_r(\rho, 1)$ , then  $p_r$  must be bounded.
- Condition automatically holds for Cobb-Douglas production
- Or for all CES production with elasticity of substitution no less than 1.
- Could fail if elasticity of substitution is below 1.
- **Example:**  $y_r = \left[ \frac{1}{2}k_r^{-1} + \frac{1}{2}\tau_r^{-1} \right]^{-1}$
- Condition holds when  $\nu_r > 1/2$ , fails when  $\nu_r \leq 1/2$ .
- Connection to self-replication in the robot sector (von Neumann).

## The Critical Role Played by Robot Production

- **This boundedness of robot prices is key.**

- It bounds machine capital prices  $p_k(t)$ , and therefore the average interest rate

$$\text{Interest rate } (t) = \frac{1 + (1 - \delta)p_k(t + 1)}{p_k(t)} - 1.$$

- So under sufficient patience, the economy must grow.
- Human wages rise, robot prices bounded
- $\Rightarrow$  **automation index  $\rightarrow 1$  in every growing sector.**

# Automation and the Declining Labor Share

## Theorem 1

■ Assume (a) high patience among some subset of population, (b) asymptotically homothetic preferences, and (c) self replication. Then:

(i) Per-capita national income grows without bound:  $Y(t) \rightarrow \infty$ ;

(ii) Each sector that grows without bound is asymptotically fully automated in the long run;

(iii) The share of human labor in national income must converge to zero.

**A relative, not absolute crisis:** If education costs are bounded and there is a sequence of sectors such that  $\nu_i \rightarrow 0$ , then every human wage goes to infinity.

[Link to Piketty](#)

# Escape Hatches

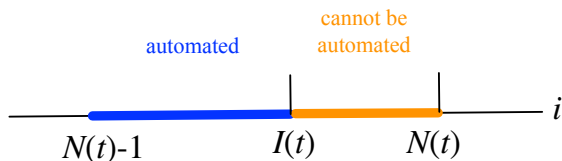
## Four escape routes

- No growth:
  - With a positive measure of highly patient agents, this cannot happen.
- No self-replication:
  - This is an empirical question. It could happen.
- No homotheticity:
  - Again, an empirical question.
  - But homotheticity will need to fail *in a particular way*.
- Technical progress.

# Directed Technical Progress

## Directed technical progress to the rescue?

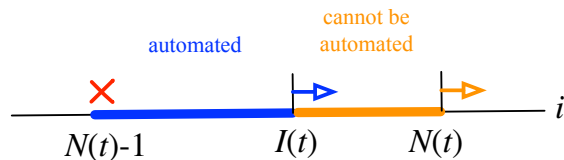
- That depends on what you choose to assume.
- Extensive Margin:
  - Acemoglu and Restrepo (2018): new goods are **un-automatable**



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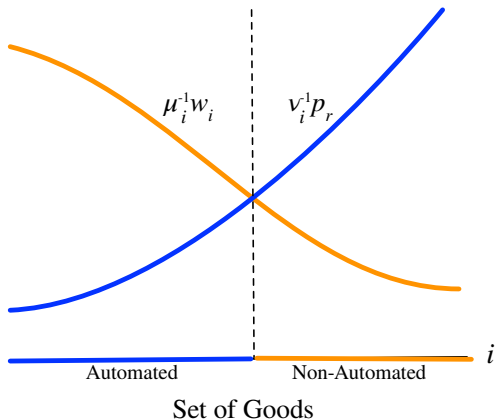




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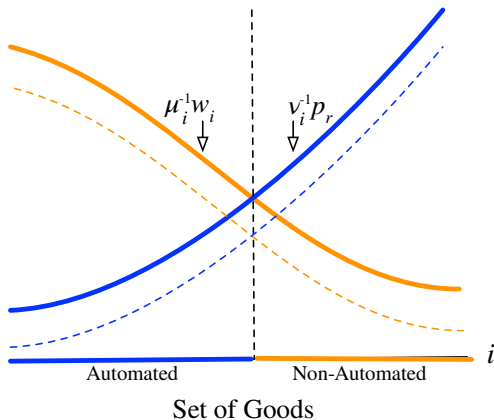
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# Directed Technical Progress

## Production Function:

$$y_j = f_j(\theta_{jt}k_{jt}, \mu_{jt}h_{jt} + \nu_{jt}r_{jt})$$

- Productivities  $\theta_{jt}, \mu_{jt}, \nu_{jt}$  all affected by R&D

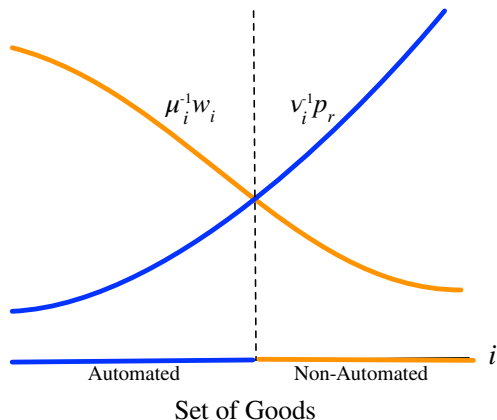
## R&D:

- Inventor can advance productivity at rate  $\rho$  for any chosen factor-sector pair:
- Cost  $\kappa(\rho)$  increasing, convex, prohibitive at  $\bar{\rho}$ , same for every factor and sector.
- Gets temporary patent protection, which she licenses to an active firm.
- After one period the advance goes public.
- Spillover fraction  $\gamma > 0$  (public) for this factor in other sectors.

## Directed Technical Progress

### Theorem 2 (The Extended Dismal Scenario)

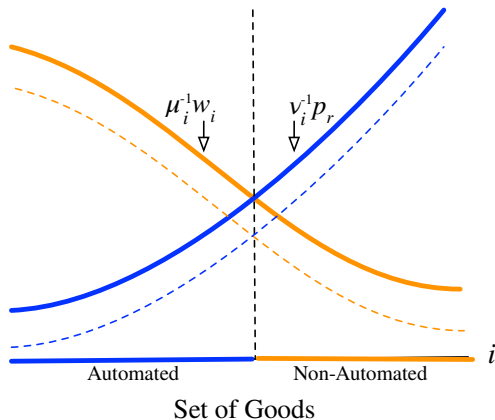
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- Then in any equilibrium with capital growth, the income share of human labor must converge to zero as  $t \rightarrow \infty$ .



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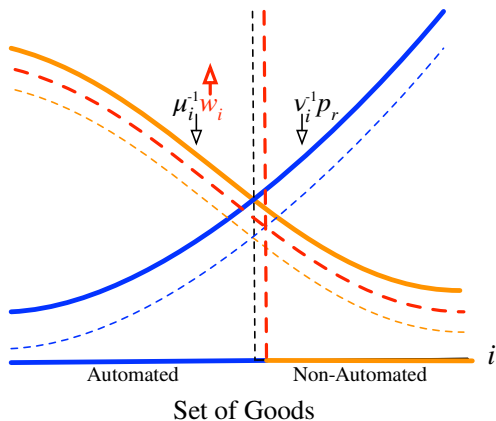
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# Summary

## The Falling Labor Share:

- A natural consequence of any theory that rests on **two pillars**:
  - An **asymmetry** in physical and human capital accumulation
  - A recognition that physical capital can be **machine-like** or **robot-like**.
- **Other important features**:
  - **Preference neutrality** with respect to human-friendly or robot-friendly goods.
  - **Enough patience** for ongoing growth and capital accumulation.
  - **Self-replication**: production of automata by means of automata.
- **Under these conditions, labor income share**  $\rightarrow 0$ :
  - full automation in the long run ...
  - ...despite wages rising over time (slow automation).

## Summary

**An age-old anxiety:** that “capital” will inherit the earth.

- But the underlying worry is about the **personal distribution of income**.
- *that* will depend on how much people save, and in what *form* they save.
- **Financial education** is fundamentally important.
- I'm pessimistic about the prospects of intelligent, informed savings in equity
- but probably this is the only way to avoid a long-run crisis

### **Social Alternatives:**

- universal basic income (e.g. *Ideas for India* special issue, *Economic Survey*)
- social stock portfolios (e.g., Ghosh and Ray 2020 on the **India Fund**)
- See **Supplement to Slides 3**.