EC9AA Term 3: Lectures on Economic Inequality

Debraj Ray, University of Warwick, Summer 2024

Slides 3: Functional Inequality: The Falling Labor Share

- We now downplay personal endowments and accumulation
- Though still very much in the background
- Our focus: the functional distribution across capital and labor

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- A **fundamental law**? You can't be serious.
- It isn't even testable (though stronger versions of it are)
- But it *is* a fundamental device for organizing our thoughts.

The falling labor share:



Guerriero (2019)

The falling labor share:



Karabarbounis and Neiman (2014). Also Harrison (2002) and Rodríguez and Jayadev (2010),

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- Covid-19

Capital-Labor Substitution

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Employment elasticities by sector, various regions. Kapsos (2005).

Region	Agriculture	Industry	Services
World	0.24	0.21	0.61
W. Europe	-1.08	-0.50	0.74
N. America	-0.02	0.26	0.60
Central/Eastern Europe	-0.51	0.11	0.51
East Asia (excl. Japan)	0.10	0.07	0.47
Japan	-2.04	-0.83	0.76
Australia/NZ	0.18	0.26	0.61
South-East Asia	0.01	0.82	1.08
South Asia	0.38	0.41	0.46
Latin America	-0.16	0.63	1.09
Sub-Saharan Africa	0.69	0.88	0.89

Capital-Labor Substitution

GDP and employment growth, some developing countries. An et al. (2017).

	Yearly, 1991–2000		Yearly, 2001–2015	
Country	GDP	EMP	GDP	EMP
Egypt	4.27	1.47	4.33	2.31
India	5.73	0.60	7.09	0.61
Indonesia	4.84	1.96	5.41	1.73
Kenya	2.09	2.20	4.38	2.00
Morocco	4.78	5.11	4.46	1.04
Nicaragua	3.17	5.61	3.66	3.19
Pakistan	4.48	1.99	4.29	2.84
Philippines	2.75	2.51	5.11	2.46
Tanzania	4.15	2.55	6.41	3.34
Vietnam	7.40	2.20	6.54	2.33

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But ...

- Net effect on labor share depends on the elasticity of substitution.
- E.g., dividing line: Cobb-Douglas production function.
- This is what I want to try and explore further.

Our Theory: Accumulation and Automation

Two pillars:

- I. Human-physical asymmetry
- II. Machine capital and robot capital

Mankiw-Romer-Weil 1992:

$$\dot{k}(t) = s_k y(t) - (n+\delta)k(t)$$
$$\dot{h}(t) = s_h y(t) - (n+\delta)h(t)$$

What does the second equation mean?

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- But human capital *cannot* be replicated in the same way.
- always in one physical self [inalienable].
- To some extent, scalable within occupation or sector
- But more fundamentally, scales across sectors.

Many sectors indexed by j:

 $y_j = f_j(k_j, \tau_j), \text{ [sector-specific, CRS]}$

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- So capital comes in two flavors:
- k: machines, complementary to labor.
- *r*: robots, substitutes for labor.

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But certainly a threat if the price is right:

"nothing humans do as a job is uniquely safe anymore. From hamburgers to healthcare, machines can be created to successfully perform such tasks with no need or less need for humans, and at lower costs than humans..." Scott Santens,

The Boston Globe, 2016

- Machine capital: $y_k = f_k(k_k, \tau_k)$, with $\tau_k = \tau_k(h_k, r_k)$.
- **Robot capital:** $y_r = f_r(k_r, \tau_r)$, with $\tau_r = \tau_r(h_r, r_r)$.
- Education: $y_e = f_e(k_e, \tau_e)$, with $\tau_e = \tau_e(h_e, r_e)$.
- All assumptions made earlier apply to these sectors as well.

- Raw labor is given (or normalized), but human capital grows endogenously.
- Initial allocation of humans across occupations.
- Individuals can move from sector to sector (or task to task).
- Educational $cost = e(i, j)p_e$, the endogenous price of education.

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- Asymptotic Homotheticity of Preferences:
- If $\mathbf{x}_m(\mathbf{p},z)$ is demand for goods by type m as function of current z, then

$$\lim_{z \to \infty} \frac{\mathbf{x}_m(\mathbf{p}, z)}{z} = \mathbf{d}_m(\mathbf{p}) \text{ for some function } \mathbf{d}_m(\mathbf{p}).$$

Price System

Competitive Pricing

- numeraire: rental rate on machine capital
- **p**: prices, includes (p_r, p_k, p_e)
- w: wages, includes (w_r, w_k, w_e)
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Unit cost function for output determines output price p_j by CRS:

$$p_j = c_j(1, q_j) = \min \{k_j + q_j \tau_j | f_j(k_j, \tau_j) = 1\}$$

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profit-maximization:

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automation index for each sector j and relative price $\zeta_j \equiv w_j/p_r$:

$$a_j(\zeta_j) \equiv \min_{(r_j,h_j)} \left\{ \frac{r_j}{h_j \zeta + r_j} \Big| (r_j,h_j) \text{ minimizes unit cost under } \zeta_j \right\} \in [0,1].$$

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consumption-savings choices pinned down by:

Interest rate
$$(t) = \frac{1 + (1 - \delta)p_k(t + 1)}{p_k(t)} - 1.$$

where $\delta \in (0,1)$ is the rate of depreciation.

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- Otherwise pretty standard:
- (Asymptotically) homothetic preferences
- Competitive price system;
- Condition for growth (patience relative to technology).

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Combining:

$$p_r \le c_r(1,\nu_r^{-1}p_r).$$

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 (relative to the normalized cost of machine rentals, set to 1)
- Depends on whether $c_r(1, \nu_r^{-1}p_r)$ goes below 45° line as $p^r \uparrow$.
- I.e., whether $c_r(1, \nu_r^{-1} p_r) < p_r$ for all large p_r .

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- Equivalent to $\nu_r > \lim_{\rho \to 0} c_r(\rho, 1)$.
- If this condition holds, then p_r must be bounded.

$\label{eq:rescaled} \quad \ \ \, {\rm If} \ \nu_r > \lim_{\rho \to 0} c_r(\rho,1) {\rm , then } \ p_r \ {\rm must \ be \ bounded.}$

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- Condition holds when $\nu_r > 1/2$, fails when $\nu_r \le 1/2$.
- Connection to self-replication in the robot sector (von Neumann).

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It bounds machine capital prices $p_k(t)$, and therefore the average interest rate

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- So under sufficient patience, the economy must grow.
- Human wages rise, robot prices bounded
- $\blacksquare \Rightarrow$ automation index $\rightarrow 1$ in every growing sector.

Automation and the Declining Labor Share

Theorem 1

 Assume (a) high patience among some subset of population, (b) asymptotically homothetic preferences, and (c) self replication. Then:

(i) Per-capita national income grows without bound: $Y(t) \rightarrow \infty$;

(ii) Each sector that grows without bound is asymptotically fully automated in the long run;

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Link to Piketty

Escape Hatches

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- Gets temporary patent protection, which she licenses to an active firm.
- After one period the advance goes public.
- Spillover fraction $\gamma > 0$ (public) for this factor in other sectors.

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- Enough patience for ongoing growth and capital accumulation.
- Self-replication: production of automata by means of automata.
- Under these conditions, labor income share $\rightarrow 0$:
- full automation in the long run ...
- ...despite wages rising over time (slow automation).

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Social Alternatives:

- universal basic income (e.g. Ideas for India special issue, Economic Survey)
- social stock portfolios (e.g., Ghosh and Ray 2020 on the India Fund)
- See Supplement to Slides 3.