EC9AA Term 3: Lectures on Economic Inequality

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- **Supplement 1 to Slides 1**: Piketty's Laws

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- **First Fundamental Law**:

$$\frac{\text{Capital Income}}{\text{Total Income}} = \frac{\text{Capital Income}}{\text{Capital Stock}} \times \frac{\text{Capital Stock}}{\text{Total Income}}.$$

  - Accounting identity

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$$K(t+1) = K(t) + I(t) = [1 - \delta(t)]K(t) + s(t)Y(t)$$

  - Convert to growth rates:

$$G(t) = \frac{s(t)}{\theta(t)},$$

where $G(t) = [K(t+1) - K(t))]/K(t)$ and $\theta(t) = K(t)/Y(t)$.

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  - Approximate per-capita version: subtract population growth rate:

$$g(t) \simeq \frac{s(t)}{\theta(t)} - \delta(t) - n(t),$$

  - **Note**: *Not* a theory unless you take a stand on one or more of the variables.

Backwards: “Explaining” Capital-Output Ratios Using Growth Rates!

- Piketty:

“If one now combines variations in growth rates with variations in savings rate, it is easy to explain why different countries accumulate very different quantities of capital…One particularly clear case is that of Japan: with a savings rate close to 15 percent a year and a growth rate barely above 2 percent, it is hardly surprising that Japan has over the long run accumulated a capital stock worth six to seven years of national income. This is an automatic consequence of the [second] dynamic law of accumulation.” (p.175)

“The very sharp increase in private wealth observed in the rich countries, and especially in Europe and Japan, between 1970 and 2010 thus can be explained largely by slower growth coupled with continued high savings, using the [second] law …” (p. 183)

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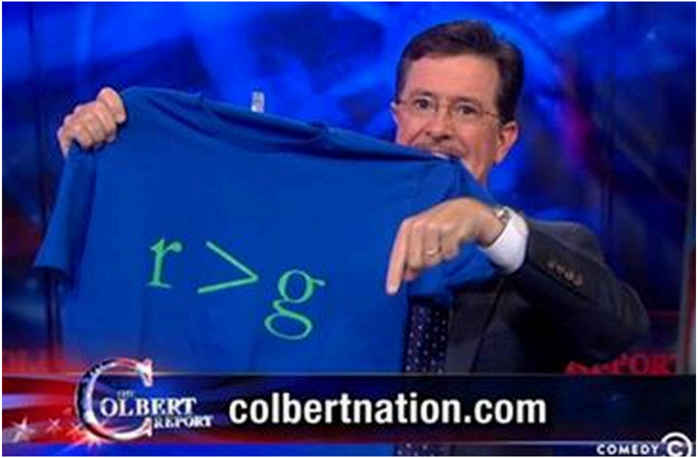
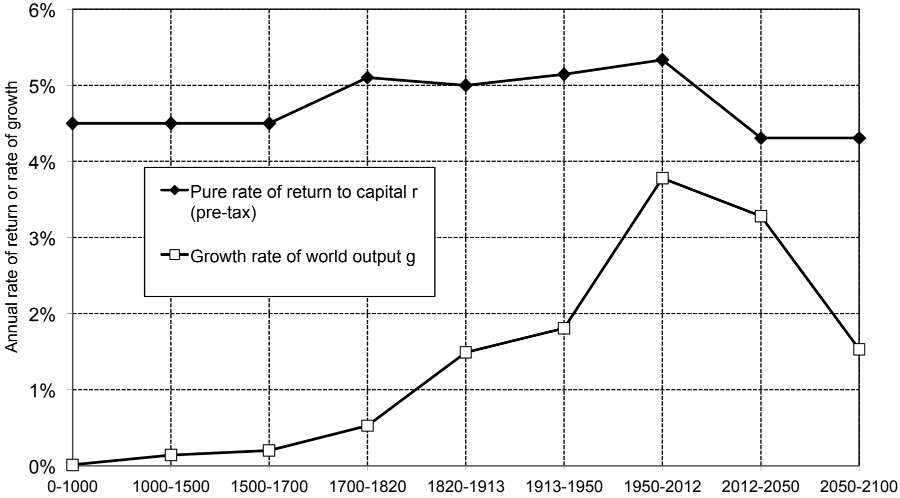


$r > g$: "The Central Contradiction of Capitalism"

"Whenever the rate of return on capital is significantly and durably higher than the growth rate of the economy, …wealth originating in the past automatically grows more rapidly than wealth stemming from work."

"This inequality expresses a fundamental logical contradiction …the past devours the future …the consequences are potentially terrifying, etc."

- $r > g$ in the data.



Not a Tautology, True, But an Efficiency Condition

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  - Impose $s(t) = s$, $\delta(t) = \delta$, and L_t growing at rate n to get:

$$y_t = Ak_t^{\theta}$$

  - and

$$(1+n)(1+\gamma)k_{t+1} = (1-\delta)k_t + sAk_t^{\theta}$$

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- So far: $y_t = Ak_t^\theta$ and $(1+n)(1+\gamma)k_{t+1} = (1-\delta)k_t + sAk_t^\theta$, so that

$$k_t \to k^* \simeq \left[\frac{sA}{n+\gamma+\delta}\right]^{1/(1-\theta)}$$

- and

$$y_t \to y^* \simeq A^{1/(1-\theta)} \left[\frac{s}{n+\gamma+\delta}\right]^{\theta/(1-\theta)} .$$

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- On the other hand, r is given by the marginal product:

$$\begin{aligned} r_t &= \theta A \left[K_t/(1+\gamma)^t L_t\right]^{\theta-1} \\ &= \theta A k_t^{\theta-1} \\ &\to \theta A \left[\frac{sA}{n+\gamma+\delta}\right]^{-1} \\ &= \frac{\theta}{s}[n+\gamma+\delta], \end{aligned}$$

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- So down to comparing $r = \frac{\theta}{s}[n + \gamma + \delta]$ with $g = n + \gamma$.
  - $\Rightarrow r > g$ if $\theta \geq s$ (surely true empirically, but also for deeper reasons):

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- s is **inefficient** if consumption can be improved in all periods.
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- Recall that Y_t/L_t converges to

$$A^{1/(1-\theta)}(1+\gamma)^t \left(\frac{s}{n+\gamma+\delta}\right)^{\theta/(1-\theta)}$$

  - and per-capita consumption converges to the path

$$A^{1/(1-\theta)}(1+\gamma)^t \left(\frac{s}{n+\gamma+\delta}\right)^{\theta/(1-\theta)}(1-s).$$

- It follows that if $s > \theta$, the growth path is inefficient.
  - **So efficiency implies** $r > g$ — not the central contradiction of capitalism!