

# EC9AA Term 3: Lectures on Economic Inequality

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- [Slides 4](#): Measuring Upward Mobility

## Introduction

**Mobility** centrally important in current debates:

- In the United States and Europe

Chetty et al (2017), Alesina et al (2018), Manduca et al (2020)

- Connection to growth, inequality, aspirations etc.

Krueger (2012), Genicot and Ray (2017, 2020), Narayan (2018)

- The concept refers to:

- the ease of transition between various social categories;
- income, wealth, location, political persuasions ...

## What the Term Might Mean

### ■ Non-Directional:

- **Pure movement: off-diagonals in transition matrix.** Atkinson (1981), Bartholomew (1982), Conlisk (1974), Dardanoni (1993), Hart (1976), Prais (1955), Shorrocks (1978a,b) ...

### ■ Directional:

- **Movement up > movement down;** Chakravarty et al. (1985), Bénabou and Ok (2001), Chetty et al. (2014), Bhattacharya (2011), Fields and Ok (1996, 1999), Mitra and Ok (1998) ...

### ■ Relative:

- **Change relative to others;** Chakravarty et al. (1985), Bénabou and Ok (2001), Chetty et al. (2014), Fields (2007), Bhattacharya (2011)

### ■ Absolute:

- **Change per se: growth +/-;** Fields and Ok (1996, 1999), Mitra and Ok (1998), Chetty et al. (2017)

### ■ + all combinations of these ...

## A Large But Still Incomplete List

Name	Measure	Directional	Non-directional	Absolute	Relative
King (1983)	$M_K = 1 - \exp\left[-\frac{\tau}{n} \sum \frac{ z_i - y_i }{\mu_y}\right]$		✓		✓
Shorrocks index (1978)	$M_S = \frac{n - \text{Tr}(P)}{n-1}$		✓		✓
Variability of the eigenvalues	$\sigma(\gamma_i)$		✓		✓
Bartholomew (1982)	$M_B = \frac{1}{n-1} \sum_i \sum_j \pi_i p_{ij}  i - j $		✓		✓
IG Income Elasticity (IGE)	$\beta = \frac{\text{Cov}(S_{it}, S_{it-1})}{\text{Var}(S_{it-1})}$		✓	✓	
Correlation coefficient (CE)	$\rho_S = \frac{\text{Cov}(S_{it}, S_{it-1})}{\sqrt{\text{Var}(S_{it})} \sqrt{\text{Var}(S_{it-1})}}$		✓	✓	
Slope rank-rank	$\rho_{PR} = \text{Corr}(P_i, R_i)$		✓		✓
IG rank association (IRA)	$\beta = \frac{\text{Cov}(p_{it}^y, p_{it}^x)}{\text{Var}(p_{it}^x)}$		✓		✓
Mitra & Ok (1998)	$MO_\alpha(x, y) = \frac{1}{n} \gamma \left( \sum_i  y_i - x_i ^\alpha \right)^{1/\alpha}$		✓	✓	
Gini symmetric index of mobility	$GS = \frac{\sum_i (y_i - x_i)(F_{x_i} - F_{y_i})}{\sum_i (y_i - 1)F_{y_i} + \sum_i (x_i - 1)F_{x_i}}$		✓	✓	
Great Gatsby curve	$\text{Corr}(\text{Gini}, \text{IGE})$		✓	✓	
Bhattacharya (2011)	$\nu = Pr(F_1(Y_1) - F_0(Y_0) > \tau   s_1 \leq F_0(Y_0) \leq s_2, X = x)$	✓			✓
Absolute upward mobility (1)	$p_{25} = \mathbb{E}(Y   X \leq 25)$	✓			✓
Absolute upward mobility (2)	$A = \Phi\left(\frac{\mu_x - \mu_y}{\sqrt{\sigma_y^2 + \sigma_x^2 + 2\rho\sigma_y\sigma_x}}\right)$	✓			✓
Chetty et al (2017)	$\text{AM}(x, y) = \frac{1}{n} \sum_i (1_{y_i \geq x_i})$	✓		✓	
Rising up-up	$P_{20to100} = \mathbb{E}[Y = 100   X = 20]$	✓			✓
Bottom half mobility	$\mu_0^{50} = \mathbb{E}(y   x \in [0, 50])$	✓			✓
Fields & Ok (1999)	$\text{FO}(x, y) = \frac{1}{n} \sum_i (\ln(y_i) - \ln(x_i))$	✓		✓	
Card (2018)	$\mathbb{E}(y > 50   x \in [45, 70])$	✓		✓	
Pro-poor growth	$G = \sum_{k=1}^5 w_k g_k$	✓		✓	

## Why Another Measure?

### ■ Conceptual reasons

- Foundations unclear
- When clear, they are problematic

### ■ In particular:

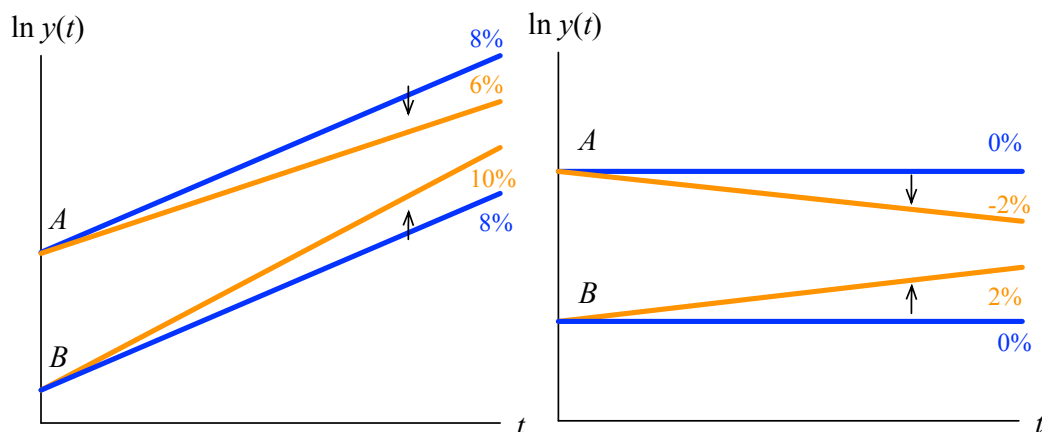
- “Mobility = mobility as pure movement + upward mobility”
- not interested in former component ([more later](#))

### ■ Data demands

- Existing measures rely heavily on panel data ([more discussion later](#)).
- This has held back empirical work, especially on developing countries.

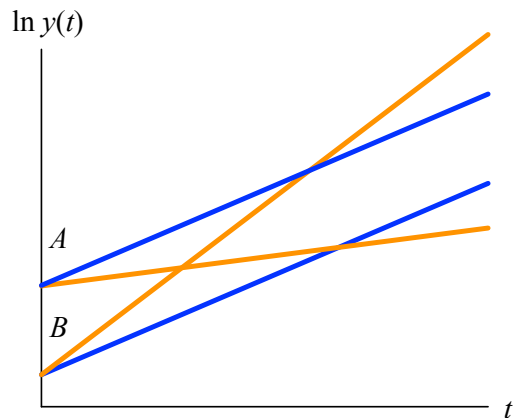
## Upward Mobility

- We propose a measure of **upward mobility** that is:
- **Directional**: rewards growth and punishes decline;
- at least for absolute measures ([more on relative measures later](#)).
- **Progressive**: higher if relatively poor enjoy faster growth.



## Upward Mobility

- We propose a measure of **upward mobility** that is:
  - **Directional**: rewards growth and punishes decline;
  - at least for absolute measures ([more on relative measures later](#)).
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## Snapshots and Trajectories

- We divide our approach into two parts:
  - An “instantaneous” measure or **upward mobility kernel** that is:
    - **directional** and **progressive**.
  - A **mobility measure on trajectories** that is:
    - based on the collection of instantaneous kernels.

## Instantaneous Upward Mobility

- **Central variable:**  $y$ , “income.”
  - state variable for individual well-being.
  - e.g., “permanent income” or a proxy, such as consumption
- **Data:** For each person:
  - $y_i > 0$  baseline income
  - $g_i = \dot{y}_i/y_i$  instantaneous growth rate.
  - $\mathbf{z}$  = the full collection  $\{z_i\}_{i=1}^n$ , where  $z_i = (y_i, g_i)$ .

## Instantaneous Upward Mobility

- **Upward mobility kernel:**  $M(\mathbf{z})$ , where  $\mathbf{z} = \{z_i\}_{i=1}^n$ , and  $z_i = (y_i, g_i)$ .
  - Anonymous, continuous.
  - Zero-growth normalization:  
 $g_i = 0$  all  $i \mapsto M(\mathbf{z}) = 0$ .
  - Consistency under population mergers.

Details

## Core Axiom

### ■ Examples:

- $\mathbf{y} = (5000, 10000) + \mathbf{g} = (8\%, 8\%) \succ \mathbf{y} = (5000, 10000) + \mathbf{g} = (6\%, 10\%)$ .
- $\mathbf{y} = (5000, 10000) + \mathbf{g} = (2\%, -2\%) \succ \mathbf{y} = (5000, 10000) + \mathbf{g} = (0\%, 0\%)$ .
- No crossings in continuous time.

### ■ Growth Progressivity.

- For any  $\mathbf{z}$ ,  $i$  and  $j$  with  $y_i < y_j$ , and  $\epsilon > 0$ , send  $g_i$  to  $g_i + \epsilon$  and  $g_j$  to  $g_j - \epsilon$ .
- Then  $M(\mathbf{z}') > M(\mathbf{z})$ .

### ■ Notes:

- Measure tolerates lower growth if poor can grow faster.
- Upward mobility  $\neq$  overall welfare.

## Upward Mobility Kernel

### Theorem 1

An upward mobility kernel is growth progressive if and only if it can be written as

$$M(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) g_i$$

for continuous permutation-invariant  $\{\phi_i\}$ , with  $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$  when  $y_i < y_j$ .

Proof Outline

## Sharpening the Kernel

- **Income Neutrality.**  $M(\mathbf{y}, \mathbf{g}) = M(\lambda \mathbf{y}, \mathbf{g})$  for all  $\lambda > 0$ .
- **Growth Alignment.**  $\mathbf{g} > \mathbf{g}' \Rightarrow M(\mathbf{y}, \mathbf{g}) > M(\mathbf{y}, \mathbf{g}')$  all  $\mathbf{y}$ .
- **Independent Pairwise Growth Tradeoffs:**

Is  $M((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij})) \geq M((y_i, g'_i), (y_j, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$ ?

Answer insensitive to  $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$ .

## Sharpening the Kernel

### Theorem 2

Under additional three axioms and  $n \geq 3$ ,  $M$  can be written as:

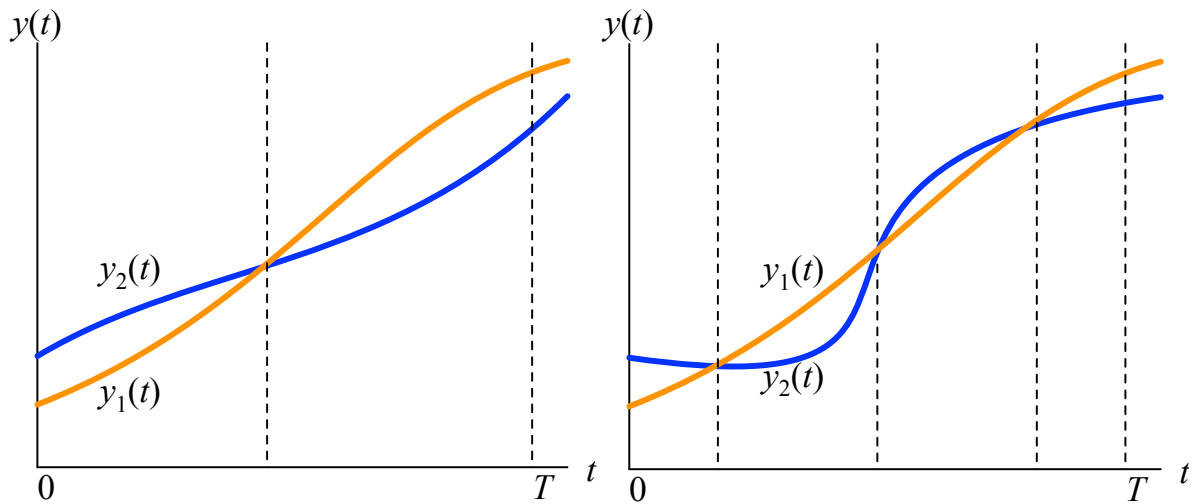
$$M_\alpha(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}}, \text{ for some } \alpha > 0.$$

- Proof employs a substantial extension of Gorman's separability theorem;

see Chatterjee (r) Ray (r) Sen (2021).

## Income Trajectories

Towards a measure on trajectories:



- $\mathbf{y}[s, t] = \{y_i(\tau)_s^t\}_{i=1}^n$
- **Upward mobility measure:**  $\mu(\mathbf{y}[s, t])$ .

## Reducibility

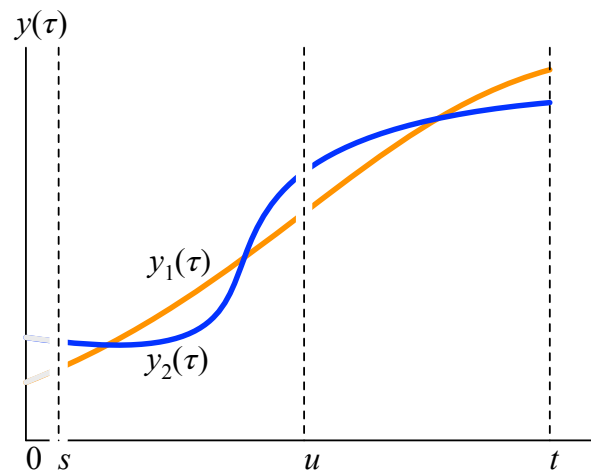
- Assume  $\mathbf{y}[s, t]$  continuously differentiable. Then:
  - Well-defined  $\mathbf{z}(\tau) = (\mathbf{y}(\tau), \mathbf{g}(\tau))$  for each  $\tau \in [s, t]$ .
  - Well-defined  $M(\mathbf{z}(\tau))$  for each  $\tau \in [s, t]$ .
- $\mu$  is **reducible** if it's expressible as a function of all these  $M$ 's:

$$\mu(\mathbf{y}[s, t]) = \Psi(\{M(\mathbf{z}(\tau))\}_s^t)$$

- with  $\mu(\mathbf{y}[s, t]) = m$  whenever  $M(\mathbf{z}(\tau)) = m$  for all  $\tau \in [s, t]$  (**normalization**)



## Additivity



- $\mu$  is **additive** if for all  $s < u < t$ ,
- $(t - s)\mu(\mathbf{y}[s, t]) = (u - s)\mu(\mathbf{y}[s, u]) + (t - u)\mu(\mathbf{y}[u, t])$ .

## Upward Mobility

### Theorem 3

Kernel axioms, reducibility, and additivity hold if and only if

$$\mu_{\alpha}(\mathbf{y}[s, t]) = \frac{1}{t - s} \ln \left[ \frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0.$$

- **Remark:** Can also use income categories and population shares (see paper).
- In what follows, we look at different aspects of this measure.

## Upward Mobility as Change in Welfare

### ■ Mobility measure:

$$\mu_{\alpha}(\mathbf{y}[s, t]) = \frac{1}{t - s} \ln \left[ \frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0.$$

### ■ Atkinson welfare function, or Atkinson equivalent income:

$$a_{\alpha}(\mathbf{y}) = \left( \frac{1}{n} \sum_{j=1}^n y_j^{-\alpha} \right)^{-\frac{1}{\alpha}},$$

for  $\alpha > 0$  (elasticity restricted).

### ■ $\mu_{\alpha}(\mathbf{y}[s, t]) =$ average growth of Atkinson equiv income on $[s, t]$ .

### ■ Not a measure of equality per se.

## Upward Mobility as Pro-Poor Growth

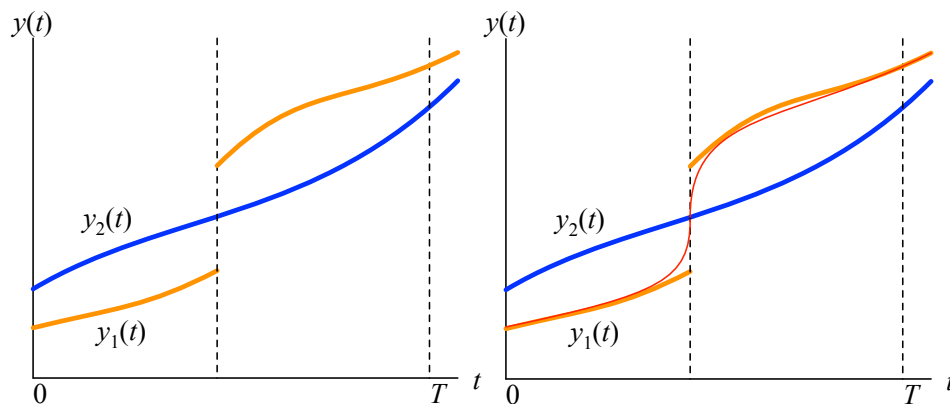
$$\begin{aligned} \cdot \quad \text{Upward Mobility} &= \frac{1}{t - s} \ln \left[ \frac{\sum_{j=1}^n y_j(t)^{-\alpha}}{\sum_{j=1}^m y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}} \\ \cdot \quad \text{Growth} &= \frac{1}{t - s} \ln \left[ \frac{\sum_{j=1}^n y_j(t)}{\sum_{j=1}^m y_j(s)} \right] \end{aligned}$$

## Upward Mobility as Pro-Poor Growth

- $$\text{Upward Mobility} = \frac{1}{t-s} \ln \left[ \frac{\sum_{j=1}^n y_j(t)^{-\alpha}}{\sum_{j=1}^m y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$$
- $$\text{Growth} = \frac{1}{t-s} \ln \left[ \frac{\sum_{j=1}^n y_j(t)}{\sum_{j=1}^m y_j(s)} \right] = \mu_{-1}(\mathbf{y}[s, t])$$
- Isn't even on our "boundary" as  $\alpha \rightarrow 0$ .
- Nevertheless, when all growth rates are the same,  $\mu_{\alpha} = \text{growth rate}$ .

## Discontinuous Trajectories

- If there are jumps, then mobility kernels aren't defined at some points.
- Examples: inheritance, job change, promotions ...



- Approximate by smooth functions and use continuity: **same answer**.

## Relative Upward Mobility

- **Relative upward mobility** nets out growth:

$$\begin{aligned}\rho_\alpha(\mathbf{y}[s, t]) &= \mu_\alpha(\mathbf{y}[s, t]) - \frac{1}{t-s} [\ln(\bar{y}(t)) - \ln(\bar{y}(s))] \\ &= \frac{1}{t-s} \ln \left[ \frac{\sum_{i=1}^n e_i(t)^{-\alpha}}{\sum_{i=1}^n e_i(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}\end{aligned}\quad (1)$$

- where  $e_i = y_i/\bar{y}$  is **excess growth factor** relative to per-capita income  $\bar{y}$ .
- $\rho_\alpha$  admissible under Theorem 1; can be further axiomatized.

## Upward Mobility and Panel Independence

- We now arrive at a central point of the paper:

- **Upward Mobility** =  $\frac{1}{t-s} \ln \left[ \frac{\sum_{j=1}^n y_j(t)^{-\alpha}}{\sum_{j=1}^m y_j(s)^{-\alpha}} \right]^{-\frac{1}{\alpha}}$  is **panel independent**.

## Upward Mobility and Panel Independence

1. Oh come on. Mobility is a construct for *dynasties* or *lineages*.

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- **Answer:** To assess a family's changing fortunes, *that* family must be tracked.
- But to assess upward mobility overall, it is *society* that must be tracked.
- A family receives **time-varying weights** depending on its relative location.
- The impact on overall mobility feeds through the impact on mobility kernels.
- Such nimble weight switches are central to our argument.

## Upward Mobility and Panel Independence

1. Oh come on. Mobility is a construct for *dynasties* or *lineages*.

---

- Specifically, study how the axioms work:
- **Growth Progressivity**  $\Rightarrow$  linearity of the kernel in growth rates.
- **Reducibility**  $\Rightarrow$

$$\mu(\mathbf{y}[s, t]) = \Psi \left( \left\{ \sum_{i=1}^n \phi_i(\mathbf{y}(\tau)) g_i(\tau) \right\}_s^t \right) = \Psi \left( \left\{ \sum_{i=1}^n \frac{\phi_i(\mathbf{y}(\tau))}{y_i(\tau)} \dot{y}_i(\tau) \right\}_s^t \right).$$

- **Additivity**  $\Rightarrow$

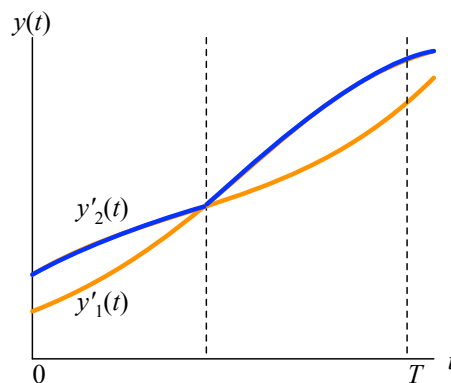
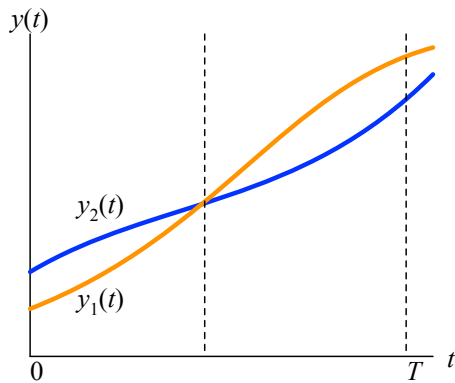
$$\mu(\mathbf{y}[s, t]) = \int_s^t \sum_{i=1}^n \frac{\phi_i(\mathbf{y}(\tau))}{y_i(\tau)} \dot{y}_i(\tau) d\tau.$$

- $\frac{\phi_i(\mathbf{y})}{y_i} = \frac{y_i^{-\alpha-1}}{\sum_j y_j^{-\alpha}}$ , which integrates out to Atkinson welfare. jumps?

## Upward Mobility and Panel Independence

2. But what about mobility as pure movement “back and forth”?

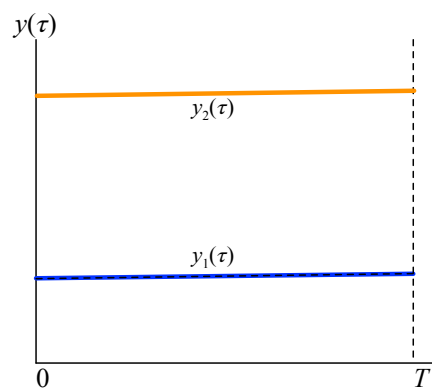
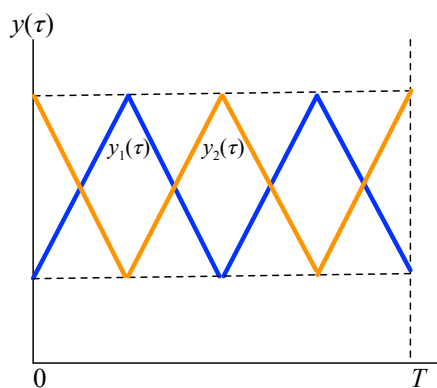
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## Upward Mobility and Panel Independence

2. But what about mobility as pure movement “back and forth”?

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- Different exchange mobility or pure movement. ✓
- Different inequalities. ✓
- But **upward** mobility in both panels is zero.

## Upward Mobility and Panel Independence

2. But what about mobility as pure movement “back and forth”?

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- Upward mobility **subtracts** downward movements from upward movement
- Exchange mobility **adds** them.

$$M_{\alpha}(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) g_i = M_{\alpha}^{+}(\mathbf{z}) - M_{\alpha}^{-}(\mathbf{z})$$

- where  $M_{\alpha}^{+}(\mathbf{z}) = \sum_{i=1}^n \phi_i^{+}(\mathbf{y}) \max\{g_i, 0\}$  and  $M_{\alpha}^{-}(\mathbf{z}) = \sum_{i=1}^n \phi_i^{-}(\mathbf{y}) \max\{-g_i, 0\}$ .

$$E_{\alpha}(\mathbf{z}) = \sum_{i=1}^n \phi_i(\mathbf{y}) |g_i| = M_{\alpha}^{+}(\mathbf{z}) + M_{\alpha}^{-}(\mathbf{z})$$

- Our preferred approach to exchange mobility.
- Such a measure would not be panel-independent.

## Upward Mobility and Panel Independence

3. Income isn't a sufficient statistic for lifetime welfare.

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- **Answer:** That's entirely possible.
- But imperfection of measurement is not an excuse for changing the measure.
- It is a reason to use the best data we have.
- Try **consumption** or **wealth** as proxies for a state variable; Deaton and Zaidi (2002)
- Or some other measure of permanent income (time-averaged?).
- Similar recommendations apply to poverty or inequality measurement.

## Upward Mobility and Panel Independence

4. But even then, individuals may belong to different **social** groups. How do we take that into account?

■ **Answer:**  $K$  social groups. Each person  $i$  belongs to one  $k(i) \in K$ .

• Data for kernel:  $(\mathbf{z}, \mathbf{w})$ , with  $z_i = (y_i, g_i)$ ,  $w_k$  the mean income of group  $k$ .

**Social Growth Progressivity.** For any  $\mathbf{z}$ ,  $i$  and  $j$  with  $(y_i, w_{k(i)}) \leq (y_j, w_{k(j)})$ , form  $\mathbf{z}'$  by altering  $g_i$  to  $g_i + \epsilon$  and  $g_j$  to  $g_j - \epsilon$ . Then  $M(\mathbf{z}') > M(\mathbf{z})$ .

**Social Income Neutrality.**  $M(\lambda \mathbf{y}, \mathbf{g}, \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$  &  $M(\mathbf{y}, \mathbf{g}, \lambda \mathbf{w}) = M(\mathbf{y}, \mathbf{g}, \mathbf{w})$ .

**Social Binary Growth Tradeoffs.** For any  $i, j$ , any  $(y_i, y_j, w_{k(i)}, w_{k(j)})$ , comparing  $((y_i, w_{k(i)}, g_i), (y_j, w_{k(j)}, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i), k(j)}))$  and  $((y_i, w_{k(i)}, g'_i), (y_j, w_{k(j)}, g'_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i), k(j)}))$  is insensitive to  $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij}, \mathbf{w}_{-k(i), k(j)})$ .

## Upward Mobility and Panel Independence

4, contd.

### Theorem 4

The above axioms hold if and only if for  $n \geq 3$  and groupings  $K$ ,

$$\mu_{\alpha, \beta}(\mathbf{y}[s, t], K) = \frac{1}{t - s} \left\{ \ln \left[ \frac{\sum_{i=1}^n y_i(t)^{-\alpha} w_{k(i)}(t)^{-\beta}}{\sum_{i=1}^n y_i(s)^{-\alpha} w_{k(i)}(s)^{-\beta}} \right]^{-1/\alpha} - \frac{\beta}{\alpha} \int_s^t \frac{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha} g_k(\tau)}{\sum_{k \in K} n_k w_k(\tau)^{-\beta} a_k(\tau)^{-\alpha}} d\tau \right\},$$

for some  $(\alpha, \beta) \gg 0$ , where  $a_k(\tau)$  is Atkinson equivalent group income.

- First term on RHS is panel-independent.
- Second term depends on trajectories, but **only at the group level**.
- Can approximate group Atkinson by standard inequality measures (see paper).



## Upward Mobility and Panel Independence

5. Anyway, we typically have panel data, don't we?

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■ **Answer:** No.

- For the United States, [Chetty et al \(2017\)](#) estimate:
  - % population share: children  $\succ$  parents (US birth cohorts, 1940–84).
  - Transitions estimated from a [unique panel of tax records](#)
  - $\oplus$  marginal income distributions from CPS and Census.
- Generally very hard to get hold of.
  - Though similar studies exist for other countries; e.g., [Acciari et al \(2021\)](#).

## Upward Mobility: Other Measures

Skip?

■ The [Chetty et al \(2017\)](#) measure (also [Berman 2021](#), [Acciari et al 2021](#)):

$$\mu^c(\mathbf{y}[0, 1]) = \sum_{i=1}^n I(y_i(0), y_i(1)).$$

- where  $I(y_i(0), y_i(1))$  is indicator for  $y_i(0) < y_i(1)$ .
- Population share for whom future  $\succ$  present.

■ The [Fields-Ok \(1999\)](#) measure:

$$\mu^{\text{FO}}(\mathbf{z}) = \frac{1}{n} \sum_{i=1}^n [\ln(y_i(0)) - \ln(y_i(1))] = \frac{1}{n} \sum_{i=1}^n \left[ \int_0^1 g_i(\tau) d\tau \right].$$

- Both must fail growth progressivity.

## Upward Mobility: Other Measures

### ■ Example for $\mu^c$ :

- Two persons at incomes \$10,000 and \$20,000.
- Growth rates 1% for both. Then  $\mu^c = 1$ .
- Transfer 2 points of growth from rich to poor. Then  $\mu^c = 1/2$ .
- But growth progressivity asks that mobility must rise.

## Upward Mobility: Other Measures

### ■ Rank-weighted measures:

- Such measures fail our axioms in a seemingly technical way:
- They are not continuous — and this isn't just a technicality.
- **Tiny changes** in incomes can generate **discrete jumps** in mobility.
- And worse: large changes in *relative* income could go unnoticed.
- **Our measure is indeed correlated with rank-based measures.**
- But is sensitive throughout, without being unduly affected by a rank switch.

## Upward Mobility in the Data

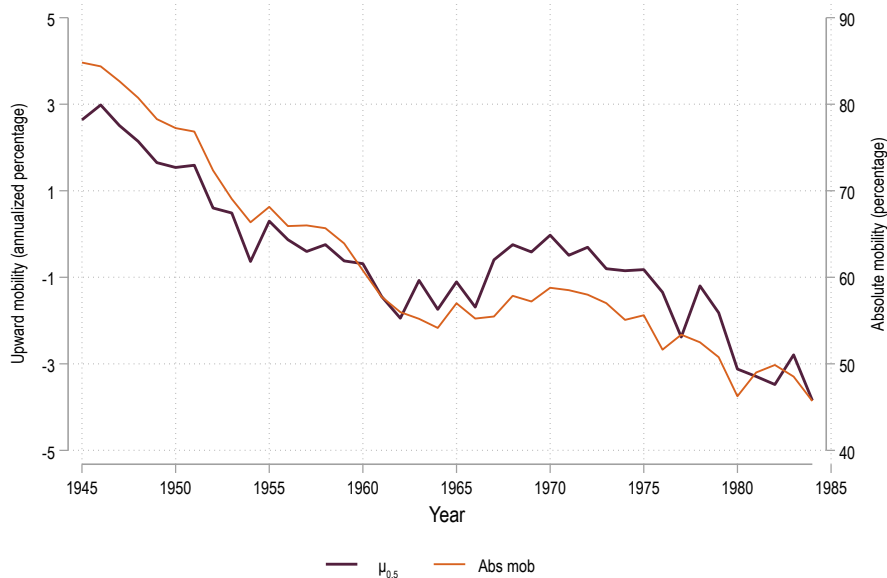
■ Chetty et al (2017) estimate  $M^I(z)$  for US birth cohorts, 1940–84.

■ They estimate a copula from a unique panel of tax records.

■ *In practice, the dependence on exact copulas seems limited*; Berman (2021)

“Estimating the absolute mobility in the United States with different copulas, some of which are very different from the one characterizing the United States, results in a similar evolution in time.”

## $\mu_\alpha$ Compared to Chetty et al (2017) for the United States



■ Robust to different  $\alpha$ .

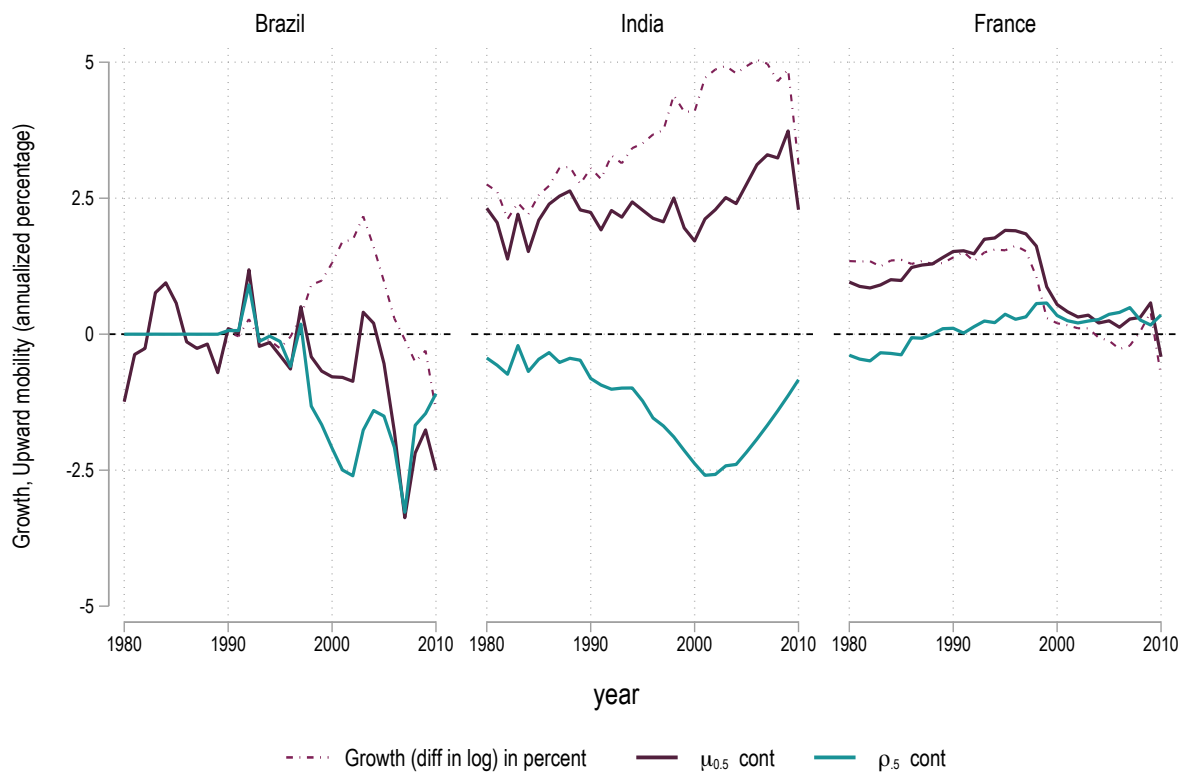
■ Robust to using other publicly available databases (e.g., WID).

## Upward Mobility in Brazil, India and France

### ■ Ten-year upward mobility in Brazil, India and France:

- Data from the World Inequality Database (repeated cross-sections).
- Measure  $\mu_{0.5}(\mathbf{y}[t, t + 10])$  and  $\rho_{0.5}(\mathbf{y}[t, t + 10])$ .
- Robust with respect to choice of  $\alpha$  (see paper).

## Upward Mobility in Brazil, India and France



## Ongoing Research: Distribution and Mobility

Esteban, Genicot, Mayoral, Ray (in preparation)

### ■ How does distribution affect subsequent mobility?

#### ■ Distribution $\oplus$ future mobility?

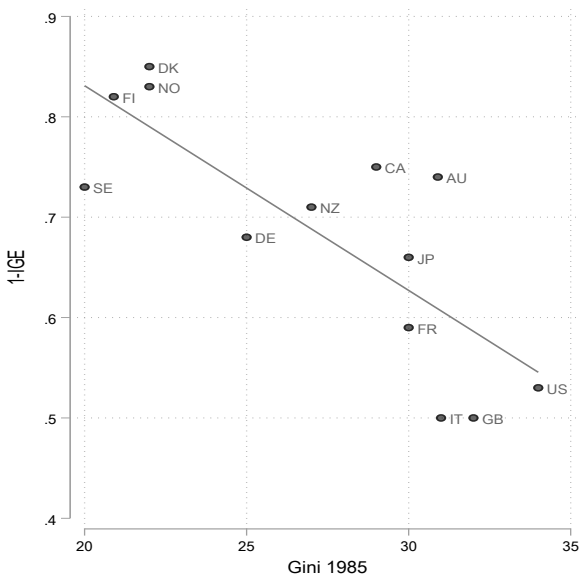
- Mechanical mean reversion
- Classical convergence: convex technology

#### ■ Distribution $\ominus$ future mobility?

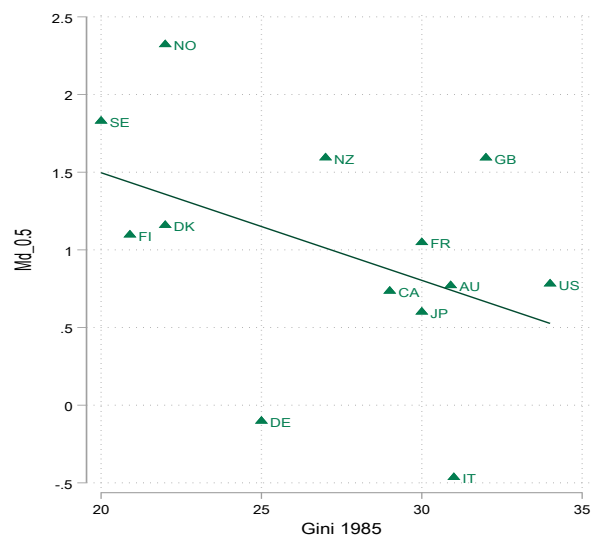
- Classical poverty traps: missing credit markets, nonconvexities.
- Psychological traps:  $\beta$ - $\delta$ , aspirations failure

## The Great Gatsby Curve

- High inequality is correlated with low mobility Krueger (2012)



Krueger (2021) / Corak (2013)

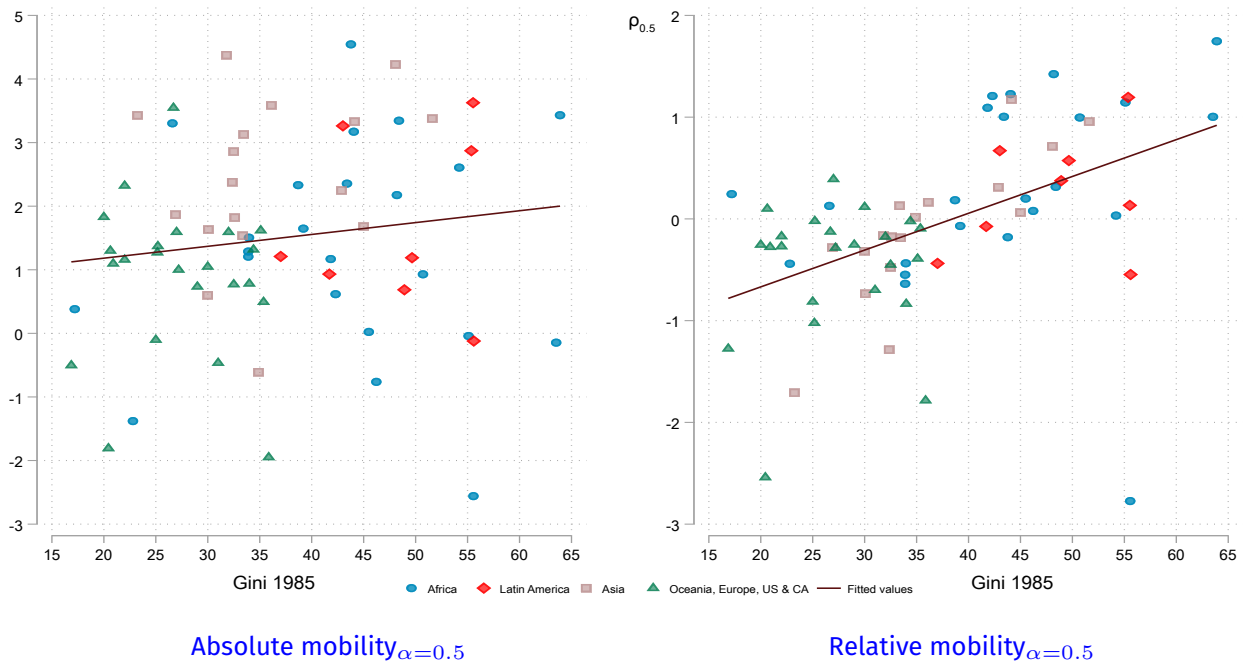


Using  $\mu_{0.5}$

## The Great Gatsby Curve

- Does the cross-section hold up? No.

86 countries (WID); 1985-2015: Genicot (P) Ray (P) Concha-Arriagada



## The Great Gatsby Curve

- But the expansion of data allows us to **exploit panel structure**.
- Preliminary:** 4-period panel (1980, 1990, 2000, 2010), 174 countries (WID)

Absolute Upward Mobility, $\alpha = 0.5$ [t, t+10]				
	[1]	[2]	[3]	[4]
GINI	1.875 (0.000)	2.391 (0.000)		
ATKINSON			1.881 (0.000)	2.299 (0.000)
LOG(INCOME) <sub>t</sub>		-6.879 (0.000)		-6.873 (0.000)
c	-10.096 (0.000)	5.795 (0.033)	-12.489 (0.000)	3.414 (0.226)
R <sup>2</sup>	0.096	0.404	0.104	0.411
Obs	696	696	696	696
Estimation	FE	FE	FE	FE

All regressions with year effects and country FE. Standard errors clustered at the country level. *p*-values in parentheses.

## The Great Gatsby Curve

- But the expansion of data allows us to **exploit panel structure**.
- **Preliminary:** 4-period panel (1980, 1990, 2000, 2010), 174 countries (WID)

	<b>Relative Upward Mobility, <math>\alpha = 0.5</math> [t, t+10]</b>			
	[1]	[2]	[3]	[4]
GINI <sub>t</sub>	1.505 (0.000)	1.511 (0.000)		
ATKINSON <sub>t</sub>			1.567 (0.000)	1.572 (0.000)
LOG(INCOME) <sub>t</sub>		-0.074 (0.532)		-0.081 (0.523)
c	-8.324 (0.000)	-8.154 (0.000)	-10.640 (0.000)	-10.452 (0.000)
R <sup>2</sup>	0.164	0.164	0.213	0.213
Obs	696	696	696	696
Estimation	FE	FE	FE	FE

All regressions with year effects and country FE. Standard errors clustered at the country level.  
*p*-values in parentheses.

## Measuring Upward Mobility: A Summary

- A **bewildering variety** of mobility indices:
  - directional/non-directional; absolute/relative.
- We axiomatize a **class of upward mobility measures**
  - At the core is the **growth progressivity axiom**.
  - Analogue of the Lorenz criterion for inequality measurement
- Our **trajectory-based measure** is pinned down by two conditions
  - **reducibility** and **additivity**.
  - It is **panel-independent**
- If convincing, this **significantly expands the scope of empirical inquiry**

## Population Consistency

**Given:**  $\mathbf{z} = (y_1, g_1, \dots, y_k, g_k, \dots, y_n, g_n)$

$$\mathbf{z}' = (y_1, g_1, \dots, y_k, g_k - \epsilon, \dots, y_n, g_n) \quad |$$

$$\mathbf{z}'' = (y_1, g_1, \dots, y_k, g_k + \epsilon, \dots, y_n, g_n)$$

**and**  $\mathbf{z}'$  and  $\mathbf{z}''$  have average mobility distinct from  $\mathbf{z}$ :  $\frac{1}{2}[M(\mathbf{z}') + M(\mathbf{z}'')] \neq M(\mathbf{z})$ ,

**Then:**  $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$ .

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## Appendix: Proof of Theorem 1

■ **Step 1. For every  $k$ ,  $m(g_k) \equiv M(g_k | \mathbf{y}, \mathbf{g}_{-k})$  is affine in  $g_k$ , or equivalently:**

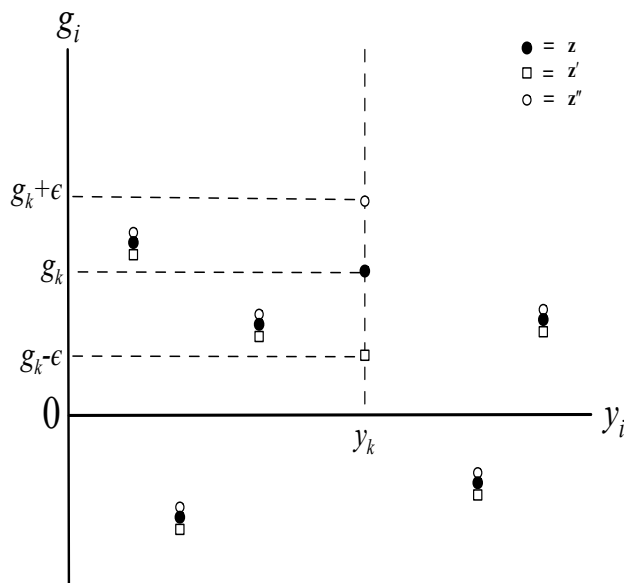
$$m(g_k) = \frac{1}{2} [m(g_k - \epsilon) + m(g_k + \epsilon)] \text{ for every } \epsilon > 0.$$

■ Suppose false for some  $g_k$  and  $\epsilon$ .

- Define  $\mathbf{z} = (\mathbf{y}, \mathbf{g}_{-k}, g_k)$ ,  $\mathbf{z}' = (\mathbf{y}, \mathbf{g}_{-k}, g_k - \epsilon)$ , and  $\mathbf{z}'' = (\mathbf{y}, \mathbf{g}_{-k}, g_k + \epsilon)$ .
- Then  $M(\mathbf{z}') + M(\mathbf{z}'') \neq M(\mathbf{z}) + M(\mathbf{z})$ .
- By Local Merge,  $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$ .
- Say  $M(\mathbf{z}' \oplus \mathbf{z}'') > M(\mathbf{z} \oplus \mathbf{z})$ .



## Appendix: Proof of Theorem 1



## Appendix: Proof of Theorem 1

- **Step 2. (Gallier 1999)  $M(\mathbf{z})$  multiaffine so can be written as:**

$$M(\mathbf{z}) = \sum_S \phi_S(\mathbf{y}) \left[ \prod_{j \in S} g_j \right].$$

for a collection  $\{\phi_S\}$  defined for every  $\emptyset \neq S \subset \{1, \dots, n\}$ .

- **Step 3. All nontrivial product terms above must have zero coefficients.**

Suppose  $\{i, j\} \subset S$  for some  $S$  with  $\phi_S(\mathbf{y}) \neq 0$ . We will only move  $g_i$  and  $g_j$  but with  $g_i + g_j = G$ , so hold all else fixed and write

$$\begin{aligned} M(\mathbf{y}, \mathbf{g}) &= \alpha g_i(G - g_i) + \beta g_i + \gamma(G - g_i) + \delta. \\ \Rightarrow \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_i} - \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_j} &= \alpha G - 2\alpha g_i + \beta - \gamma. \end{aligned}$$

Choose  $G$  and  $g_i$  to violate Growth Progressivity. [back](#)