EC9AA Term 3: Lectures on Economic Inequality

Debraj Ray, University of Warwick, Summer 2023

- **Supplement to Slides 2**: A General Model of Occupational Choice

A General Model of Occupational Choice

- **Production with capital and occupations**.
  - Population distribution on occupations $\boldsymbol{n}$ (endogenous).
  - Physical capital k.
  - Production function $y = F(k, \boldsymbol{n})$, CRS and strictly quasiconcave.

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- **Production with capital and occupations**.
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  - Physical capital k.
  - Production function $y = F(k, \boldsymbol{n})$, CRS and strictly quasiconcave.
- **Training cost function $\mathbf{x}$ on occupations**:
  - incurred up front.
  - parents pay directly, or bequeath and then children pay.

Prices

- **Perfect competition**.
  - Return on capital fixed at rate r (international k-mobility).
  - "Wage" vector $\mathbf{w} \equiv \{w(h)\}$ endogenously determined for each occupation h.
  - Together with r, $\mathbf{w}$ **supports profit-maximization**.

Supporting Profit Maximization

- $F(k, \boldsymbol{n})$ is associated with a **unit cost function** $c(\mathbf{w}, r)$.
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- If that unit cost $\neq$ output price:
  - $(\mathbf{w}, r)$ **cannot support profit maximization at positive output**.
  - Otherwise, it does.
  - **Note**: For any $\mathbf{w}$, there is a unique **scaling** $\mu > 0$ such that $(\mu\mathbf{w}, r)$ supports profit maximization.

Households

- Continuum of households, each with one agent per generation.
- $y = z + b + x(h)$

  wealth — consumption — fin. bequests — occ. choice
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    wealth consumption fin. bequests occ. choice
  - Child wealth $y' = (1+r)b + \mathbf{w}_{t+1}(h)$.
- Parent picks (b, h) to **maximize utility**.
  - No debt! $b \geq 0$.
  - Child grows up; back to the same cycle.

Preferences and Equilibrium

- **Preferences**: mix of income-based and nonpaternalistic

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- **Equilibrium**: wages $\mathbf{w}_t$, value functions V_t, occupational distributions $\boldsymbol{n}_t$ s.t.:
  - Each family i chooses $\{h_t(i), b_t(i)\}$ optimally
  - Occupational choices $\{h_t(i)\}$ aggregate to $\boldsymbol{n}_t$;
  - Firms willingly demand $\boldsymbol{n}_t$ at prices $(\mathbf{w}_t, r)$.
  - **Note**: physical capital willingly supplied to meet any demand.

Steady State

- A **steady state** is a stationary equilibrium with positive output and wages:
  - $\mathbf{w}_t = \mathbf{w} \gg 0$, and
  - $(k_t, \boldsymbol{n}_t) = (k, \boldsymbol{n})$ for all t, and $F(k, \boldsymbol{n}) > 0$.

Rich Occupational Structure

- **The richness assumption [R]**:

- The set of all training costs is a compact interval $[0, X]$.

- If n is zero on any positive interval of training costs, then $y = 0$.

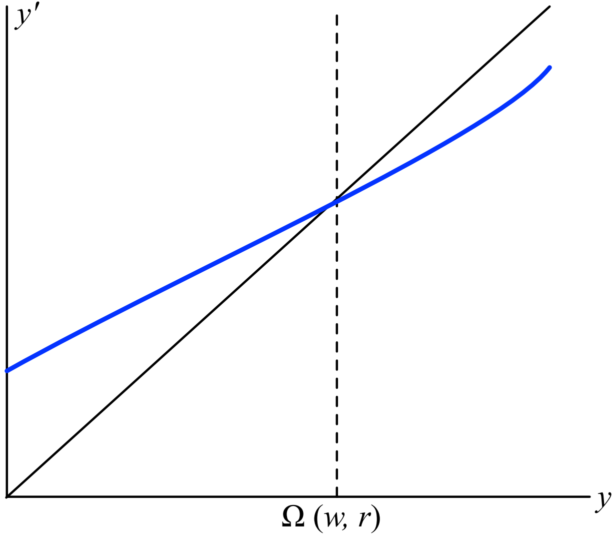
A Benchmark With No Occupational Choice

- **Financial bequests (at rate r) $+$ just one occupation (wage w).**
  - Parent with wealth y selects $b \geq 0$ to

$$\max U(c) + \delta[\theta V(y') + (1-\theta)P(y')].$$

  - Child wealth $y' \equiv w + (1+r)b$, increases in y.
- Converges to **limit wealth** $\Omega(w, r) < \infty$.
  - This needs $\theta < 1$.
  - Could depend on initial y (as in non-concave Ramsey model); we exclude that.

Limit Wealth in Benchmark Model



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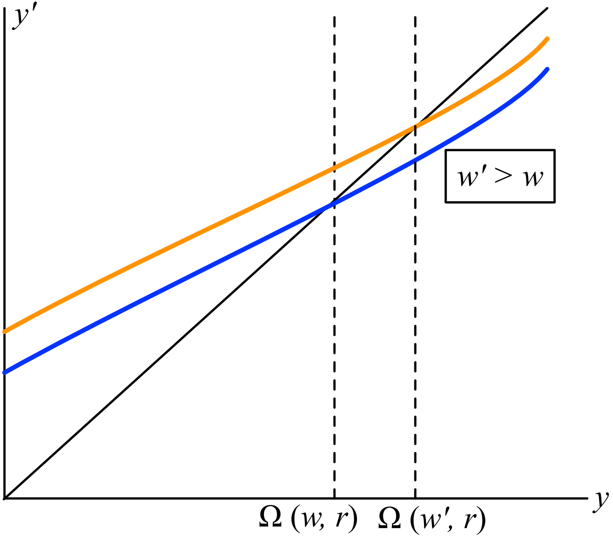
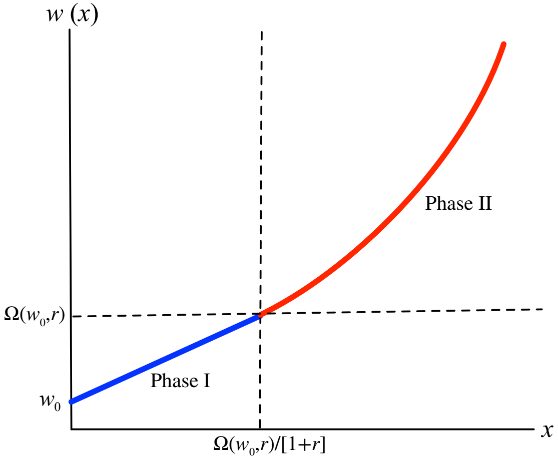


Back to Occupational Choice

Theorem 1

- *Every steady state* $\mathbf{w}$ *is fully described by a* ***two-phase property****:*



- **In Phase I w is linear in x**: there is $w_0 > 0$ such that

$$w(x) = w_0 + (1+r)x \text{ for all } x \leq \frac{\Omega(w_0, r)}{1+r}$$

  - All families in Phase I have the **same overall wealth $\Omega(w_0, r)$**.

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- **In Phase II, w follows the differential equation**

$$w'(x) = \frac{U'\left(w(x) - x\right)}{\delta[\theta U'\left(w(x) - x\right) + (1-\theta)P'(w(x))]}$$

  - with endpoint to patch with I: $w(x) = w_0 + (1+r)x$ as $x \downarrow \frac{\Omega(w_0,r)}{1+r}$.

- Families located in Phase II have **different wealths and lifetime consumptions**.

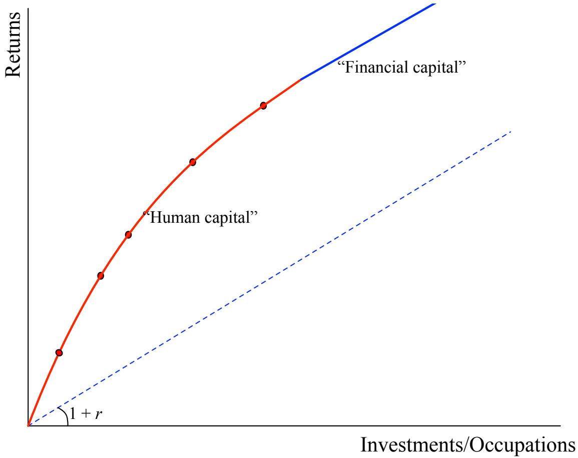
- **Closer look at Phase II**

$$w'(x) = \frac{U'\left(w(x) - x\right)}{\delta[\theta U'\left(w(x) - x\right) + (1 - \theta)P'(w(x))]}$$

- Shape comes from **Euler equation**:
  - depends fundamentally on preferences
  - **technology only serves to pin down baseline w_0** (remember remark on scaling)

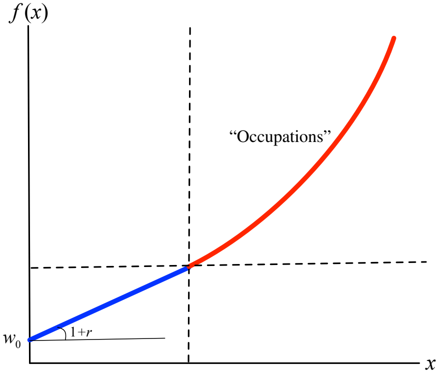
A Testable Implication

- **Recall standard model**. By assumption:



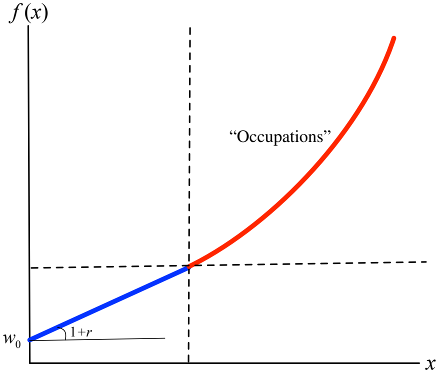
A Testable Implication

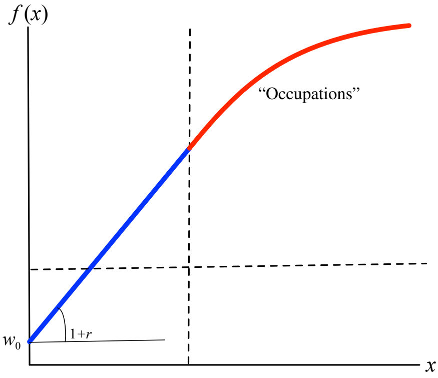
- **Compare to Phases I and II**: *This?*



A Testable Implication

- **Compare to Phases I and II**: *This?* *Or this?*





A Testable Implication

Theorem 2

The average return $\frac{w(x)-w_0}{x}$ *to occupational investment is flat in Phase I and strictly increasing in Phase II.*

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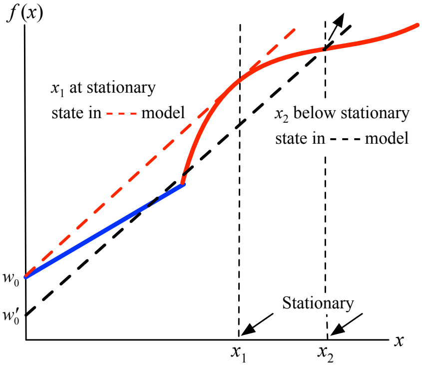

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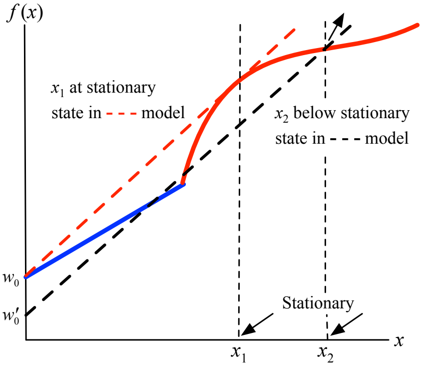

- Contradiction to unique limit wealth in benchmark, increasing in w.

Unique Steady State with Rich Occupational Structure

- We end with a **fundamental difference from two-occupation case**:

Theorem 3

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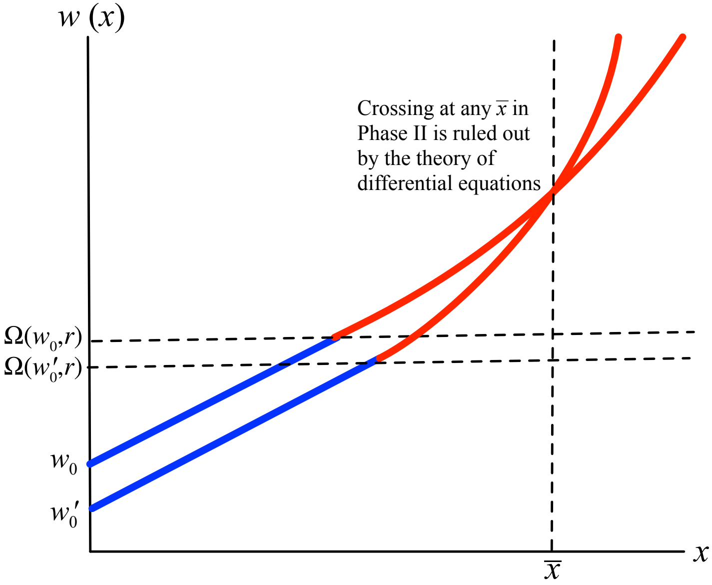
- Proof idea:

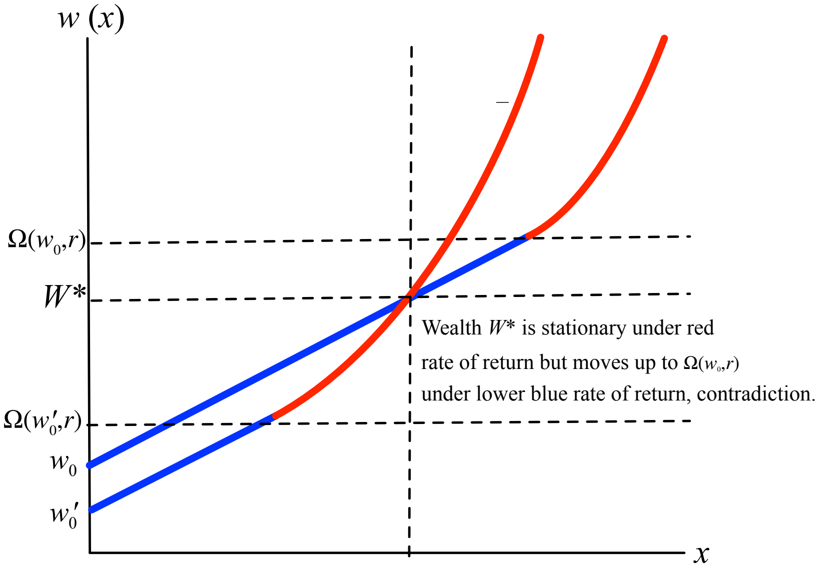
- No two members of the two-phase family (indexed only by w_0) can cross.
- Then **only one w_0 can support profit maximization** with positive output.

  (For all wages must co-move with intercept wage w_0.)

No-crossing argument, part I



No-crossing argument, part II



Three Remarks

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- Families at different occupations in Phase II **cannot have the same wealth**.
  - Thus, "most" inequality in this model comes from nonalienable capital.
- This focus will change when we consider automation in the next set of models and the decline in functional labor share.

Three Remarks

- **II. When is Phase II nonempty?**
- When there is a **large occupation span relative to bequest motive**:
  - Discounting.
  - Poverty, via TFP differences.
  - Growth in TFP, lowers effective bequest motive
  - World return on capital.
  - Globalization: new occupations.

Three Remarks

- **III. Two Notions of History-Dependence**
  - At the macro-level, history-dependence depends on occupational richness.
  - A lot of history-dependence at the individual level.
- The distribution as a whole is pinned down, but not who occupies which slot.