# EC9AA Term 3: Lectures on Economic Inequality

**Debraj Ray**, University of Warwick, Summer 2023

- Slides 2: Occupational Choice and Inequality

# Markets and Personal Inequality

- Two views:
- Equalization: Inequality an ongoing battle between convergence and "luck."
  - Solow 1956, Brock-Mirman 1972, Becker-Tomes 1979, 1986, Loury 1981…
- Disequalization: Markets intrinsically create and maintain inequality.
  - Ray 1990, Banerjee-Newman 1993, Galor-Zeira 1993, Ljungqvist 1993, Freeman 1996, Mookherjee-Ray 2000, 2010…

# Standard Accumulation Equations

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- Not surprising that this literature looks like growth theory.
  - Lots of “mini growth models”, one per household.
  - But $f$ can have various interpretations.

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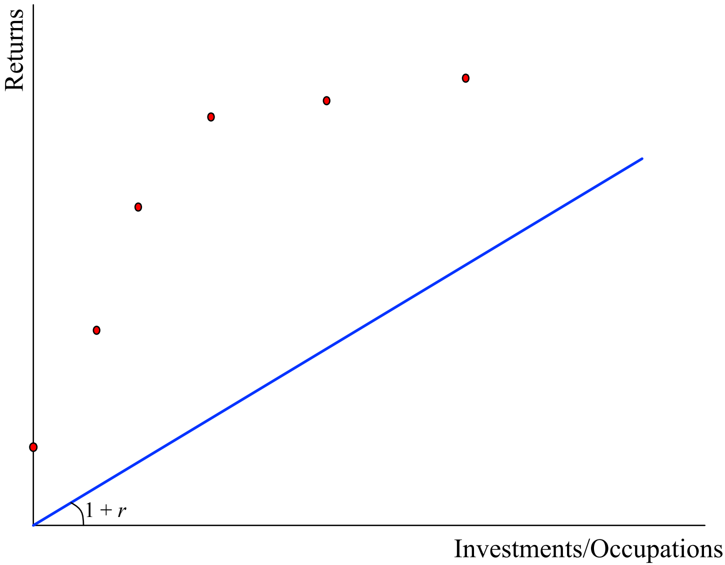
- So interpret $f$ as **envelope of intergenerational investments**:

- Financial bequests

- Occupational choice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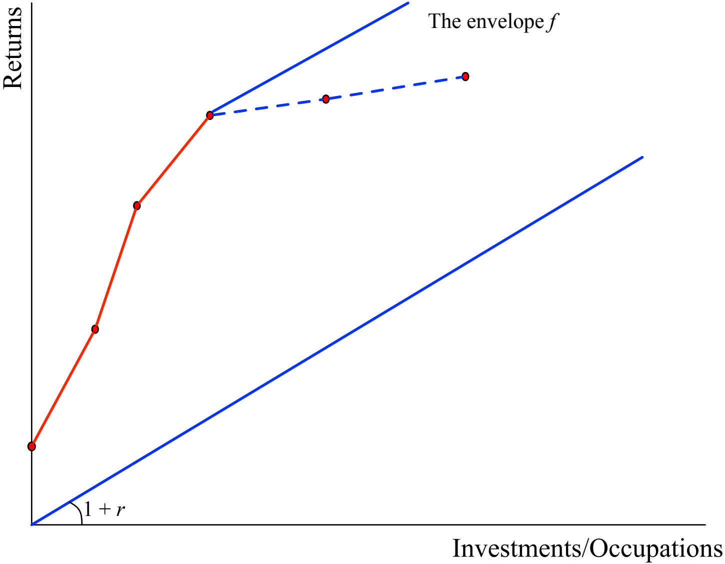
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**Theorem 1**

- *Let $h$ describe all optimal choices of $k$ for each $y$.*
- *Then if $y > y'$, $k \in h(y)$, and $k' \in h(y')$, it must be that $k \geq k'$.*

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  - $k$ beats $k'$ under $y$, so:

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  - Adding, rearranging:

$$U(y-k) - U(y-k') \geq U(y'-k) - U(y'-k'),$$

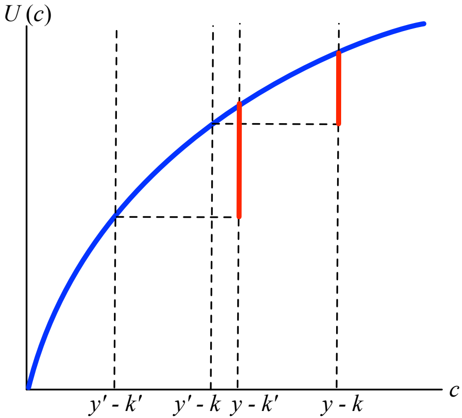
which **contradicts the strict concavity of $U$**. □

# Illustration

For $y > y'$ and $k' > k$,

$$U(y-k) - U(y-k') \geq U(y'-k) - U(y'-k'),$$

contradicts this picture:



**Remarks**:

- $h$ is “almost” a function.
- $h$ can only jump up, not down.
- Same assertion is not true of optimal $c$.
- Note how curvature of $U$ is important, that of $W$ is unimportant.
- **Crucial for models in which $f$ is endogenous** with uncontrolled curvature.

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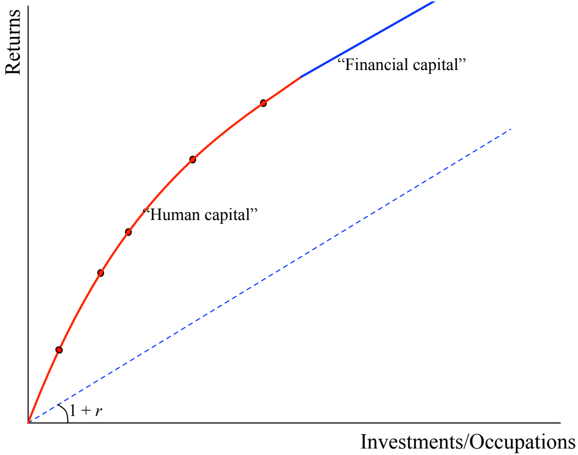

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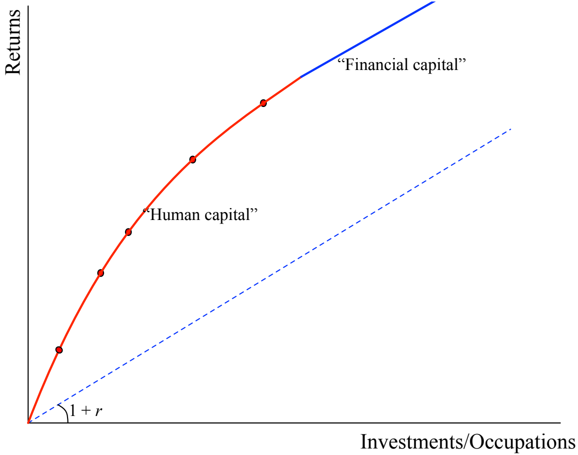


- Generates convergence to unique steady state in the absence of uncertainty.

# Convergence With Concavity: Intuition

- Look at **Bellman case** with no uncertainty:

$$V(y) \equiv \max_k \left[U(y-k) + \delta V(f(k))\right]. \tag{1}$$

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- But (1) + Envelope Theorem $\Rightarrow V'(y_{t+1}) = u'(c_{t+1})$, so:

$$U'(c_t) = \delta U'(c_{t+1}) f'(k_t). \tag{3}$$

- Theorem 1 + (3) imply **convergence to unique $k^*$, where $\delta f'(k^*) = 1$**.

# And Without Concavity?

- Without concavity: again, look at **Bellman case** with no uncertainty:

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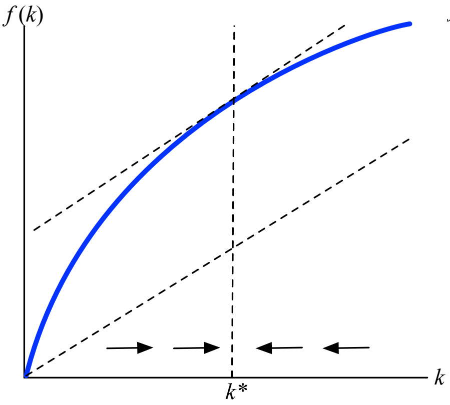
- Envelope theorem *still works*, so:

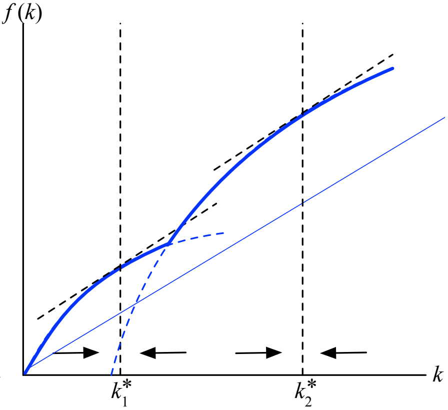
$$U'(c_t) = \delta U'(c_{t+1}) f'(k_t). \tag{6}$$

- So again convergence to $k^*$, where $\delta f'(k^*) = 1$, **but now $k^*$ is not unique**.

# Comparison




$$f'(k^*) = 1$$




$$\delta f'(k_1^*) = \delta f'(k_2^*) = 1$$

# Stochastic Shocks

- What happens to these models with stochastic shocks?
- Something weird, at least conceptually.

**Theorem 2**

*Brock-Mirman 1976, Becker-Tomes 1979, Loury 1981, extended to drop concavity*

- *Assume a mixing condition, such as $f(0,1) > 0$ (poor genius) and $f(k,0) < k$ for all $k > 0$ (rich fool).*
- *Then there exists a unique measure on incomes $\mu^*$ such that $\mu_t$ converges to $\mu^*$ as $t \to \infty$ from every $\mu_0$.*

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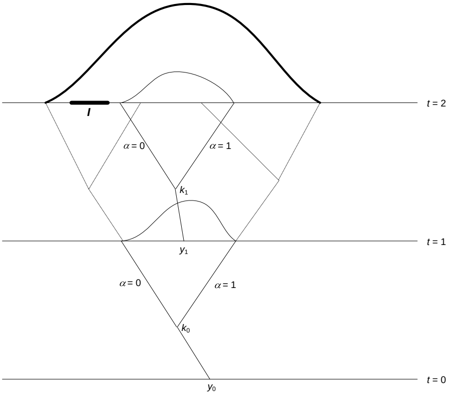
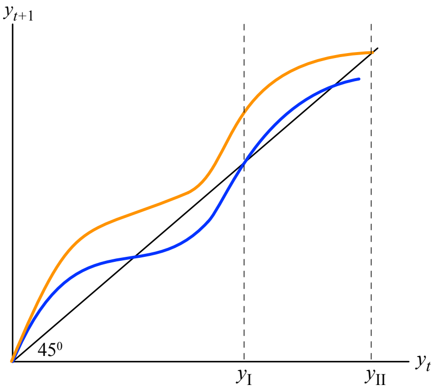


- **Core assumption**: a “mixing zone”:

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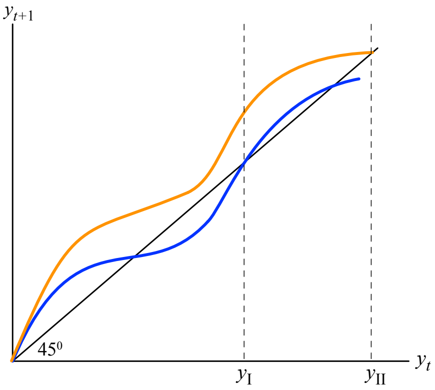


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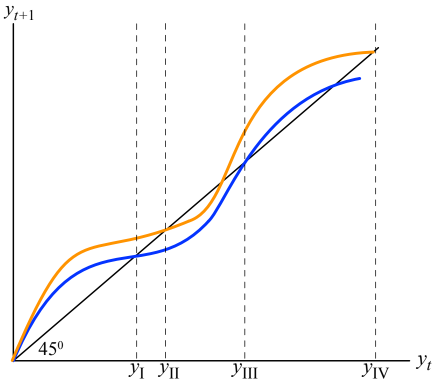


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- **II**. Disjoint supports.
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- **III**. The reliance on efficiency units.
  - No way to endogenize the returns to different occupations.
  - Whether $f$ concave at the household level **should depend on markets**.

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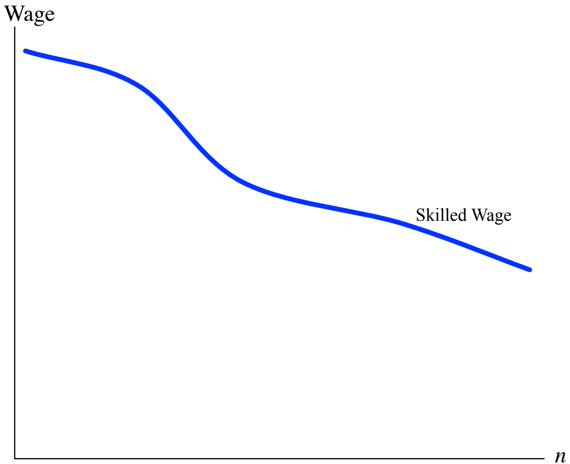
- Skilled wage: $w_s(n) \equiv f_1(n, 1-n)$
- Unskilled wage: $w_u(n) \equiv f_2(n, 1-n)$

# Skilled and Unskilled Wages



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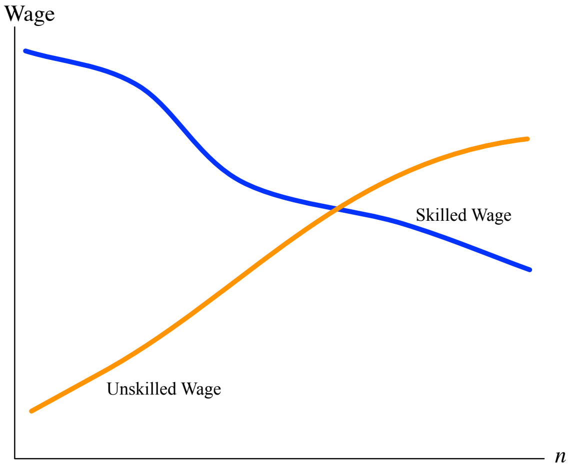

# Households

- **Continuum of households, each with one agent per generation**.
- Starting wealth $y$; $y = c + k$, where $k \in \{0, X\}$.
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  - Child wealth $y' = w$, where $w = w_s$ or $w_u$.
- **Parent maxes $U(c) + \delta V(y')$ (Bellman equation)**
  - No debt!
  - Child grows up; back to the same cycle.

## Equilibrium

- A sequence $\{n^t, w_s^t, w_u^t\}$ such that
  - $w_s^t = w_s(n^t)$ and $w_u^t = w_u(n^t)$ for every $t$.
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- **Steady states:**
  - A constant fraction $n$ are skilled
  - Wages are constant at $w_s = F_1(n, 1-n)$ and $w_u = F_2(n, 1-n)$
  - All parents keep replicating their skill status in their children.
  - Replication of skills follows from Theorem 1.

# Steady States in Occupational Choice

- **Conditions for $n$ to be a steady state:**

[Skilled parent] $$V(w_s) = \frac{u(w_s - X)}{1-\delta} \geq u(w_s) + \frac{\delta}{1-\delta}u(w_u)$$

[Unskilled parent] $$V(w_u) = \frac{u(w_u)}{1-\delta} \geq u(w_u - X) + \frac{\delta}{1-\delta}u(w_s - X)$$

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### Theorem 3

*Every $n$ with $w_s = F_1(n, 1-n)$ and $w_u = F_2(n, 1-n)$ such that*

$$\underbrace{u(w_u) - u(w_u - X)}_{\text{Unskilled Cost}} \geq \underbrace{\frac{\delta}{1-\delta}\left[u(w_s - X) - u(w_u)\right]}_{\text{Future Benefit}} \geq \underbrace{u(w_s) - u(w_s - X)}_{\text{Skilled Cost}}$$

*must be a steady state.*

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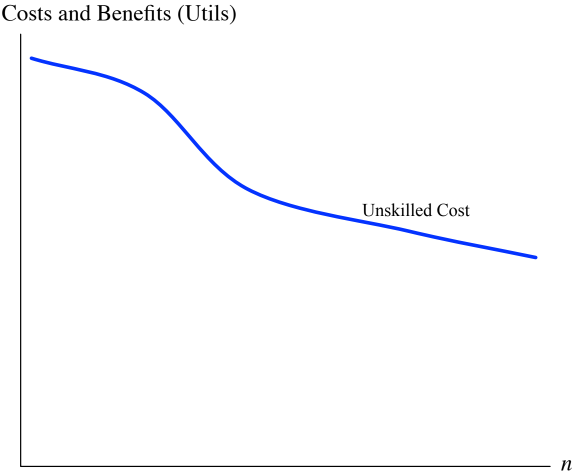

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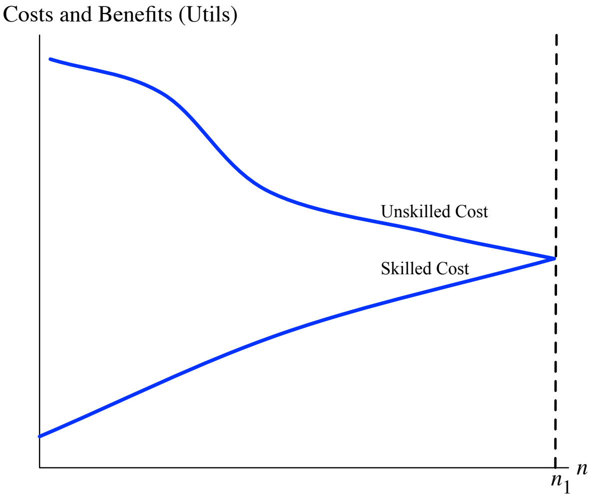

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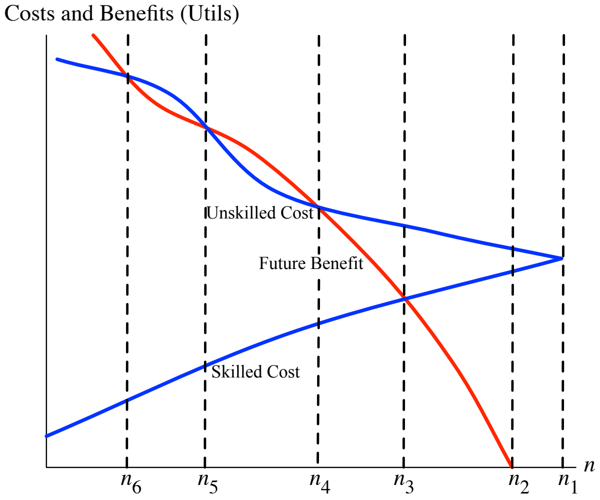

# Features of the Two-Occupation Model

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3. **No convergence**; persistent inequality in *utilities*.

   - Symmetry-breaking argument.

# Features of the Two-Occupation Model

4. **Dynamics and history-dependence**.

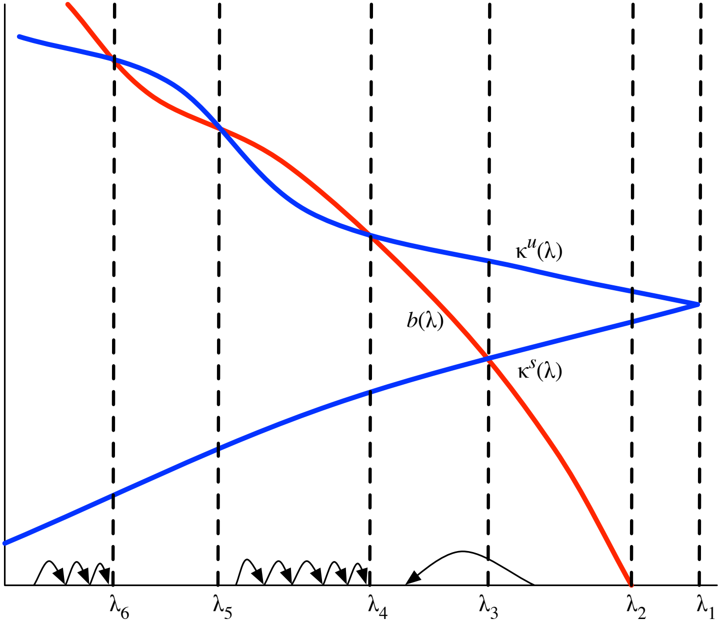
**Theorem 4**

*(i) From any initial $n$ that is a steady state, the system remains there: $n_t = n$ for all $t$.*

*(ii) From any initial $n$ that is not a steady state, but with some steady state $n' > n$, $n_t$ converges monotonically up to the* ***smallest*** *steady state exceeding $n$.*

*(iii) (ii) From any initial $n$ that is larger than any steady state, $n_t$ converges down in* ***one period*** *to some steady state.*

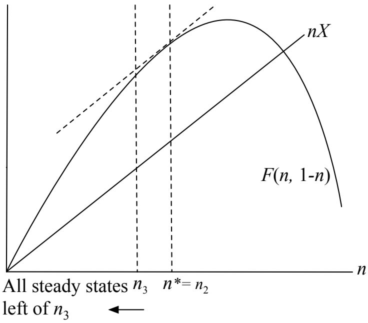
# Dynamics



# Features of the Two-Occupation Model

5. **Every steady state is inefficient**.

- Efficient allocation maximizes $F(n, 1-n) - nX$:




$F_1(n^*, 1-n^*) - F_2(n^*, 1-n^*) = X$, $\Rightarrow w_s^* - X = w_u^* \Rightarrow n^* = n_2$. But every steady state is to the left of $n_3$ (see steady state diagram).

# Features of the Two-Occupation Model

6. **Can easily embed other models here, such as entrepreneurship**.

- Reinterpret $s$ as entrepreneur, $u$ as worker.
- $X$ is setup cost for industrialization.
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- $X$ is setup cost for industrialization.
- $F(s, u) = sf\left(\frac{u}{s}\right)$

- Then:

- $F_2(s, u) = f'\left(\frac{u}{s}\right) = w$, and
- $F_1(s, u) = f\left(\frac{u}{s}\right) - \frac{u}{s}f'\left(\frac{u}{s}\right) = f\left(\frac{u}{s}\right) - \frac{u}{s}w$ = profit.

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7. **Policy questions, such as conditionality in educational subsidies**

- Recall social's planner's $n^*$ had higher net output than any steady state:
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- **Unconditional**: give equally to currently unskilled parents:

$$T_t = \frac{n_t \tau}{1 - n_t} w_s(n_t).$$

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7. **Policy questions, such as conditionality in educational subsidies**

- Recall social's planner's $n^*$ had higher net output than any steady state:
  - So there could be a role for **educational subsidies**.
  - Assume all subsidies funded by taxing $w_s$ at rate $\tau$.
- **Unconditional**: give equally to currently unskilled parents:

$$T_t = \frac{n_t \tau}{1 - n_t} w_s(n_t).$$

- **Conditional**: give to *all* parents conditional on educating children.

$$Z_t = \frac{n_t \tau}{n_{t+1}} w_s(n_t).$$

(can contemplate other obvious variants with similar results)

# Features of the Two-Occupation Model

**Theorem 5**

- *With unconditional subsidies, every left-edge steady state declines, lowering the proportion of skilled labor and increasing pre-tax inequality, which undoes some or all of the initial subsidy.*

- *With conditional subsidies, every left-edge steady state goes up, increasing the proportion of skilled labor. In steady state, no direct transfer occurs from skilled to unskilled, yet unskilled incomes go up and skilled incomes fall.*

- *Conditional subsidies therefore generate superior macroeconomic performance (per capita skill ratio, output and consumption).*

## Other Applications

- **Trade theory** in which autarkic inequality determines comparative advantage.
- **Country-level specialization** when national infrastructure is goods-specific.
- **Fertility patterns** in models of occupational choice.

# A General Model with Financial Bequests and Occupational Choice

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  - New insights

    Are there multiple steady states as in the two-occupation model?

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$$\begin{aligned} u'(w(x) - x) &= \delta[\theta V'(w(x)) + (1-\theta)P'(w(x))]w'(x) \\ &= \delta[\theta u'(w(x) - x) + (1-\theta)P'(w(x))]w'(x). \end{aligned}$$

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- Obtain a **differential equation for the wage function**:

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  - A unique solution, and typically *not concave*.
- Endogenous inequality, but no multiplicity of steady states.
  - Macro- versus micro-history-dependence.

# Luck versus Markets: Philosophy of Inequality

**Equalization**: Inequality an ongoing battle between convergence and "luck"

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- It remains to be seen if this is the right view of the world.