Debraj Ray, University of Warwick, Summer 2020

Supplement to Slides 1
Recall our question:

What explains the high rates of return to the rich?

Two broad groups of answers:

- The rich have access to better information on rates of return
- The rich have physical access to better rates of return.
A theory of individual-specific $r$:

- Higher individual wealth $\Rightarrow$ higher rate of return on it.
- More effort spent on gathering information.

Compare/contrast with “efficiency wage” models:

- Deliberate investment in information yields the higher rate
  unlike nutrition-efficiency, but similar to dynamic incentives
- Payoff is multiplicative (on $r$) as opposed to additive
  other “efficiency-wage” models generate level effects
A Model of Investing in Investment

- Individuals with more financial wealth will spend more effort finding good rates of return on it.

- Simplest model of this:

  \[
  \sum_{t=0}^{\infty} \delta^t \frac{c_t^{1-\theta} - 1}{1-\theta},
  \]

  where \( \theta > 0 \), and

  \[
  c_t = (1 + r_{t-1})F_{t-1} + w(1 - e_t) - F_t,
  \]

  and

  \[
  r_t = \Psi(e_t)
  \]

- \( F \): financial wealth, \( w \): wage rate, and \( e \): informational effort.

- \( \Psi \) concave.
A Model of Investing in Investment

- Familiar Euler equation for choice of $F_t$:
  \[
  \left( \frac{c_{t+1}}{c_t} \right)^\theta = \delta r_t
  \]

- Slightly less familiar Euler equation for choice of $e_t$:
  \[
  \left( \frac{c_{t+1}}{c_t} \right)^\theta = \delta \frac{F_t}{w} \Psi'(e_t).
  \]

- Proposition. Individuals with a higher ratio of $F$ to $w$ earn a higher rate of return, and grow faster, even if the effect on their savings rate is ambiguous.

  - Proof. Combine the two Euler equations and definition of $r$ to see that
    \[
    r_t = \frac{F_t}{w} \Psi'(e_t) = \Psi(e_t)
    \]
    for all $t$. Now prove the proposition by contradiction.

- Note: $s$ and $r$ reinforce each other when $\theta < 1$. 

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A Model of Investing in Investment

- Or you can have your cake and eat it too. Consider

\[ c_t = r_{t-1}F_{t-1} + w - z_t - F_t, \]

where \( r_t = \Phi(z_t) \) (e.g., paying an expert to do your research).

- Then Euler equation for \( z \) is given by

\[ \left( \frac{c_{t+1}}{c_t} \right)^\theta = \delta F_t \Phi'(z_t), \]

- **Proposition.** Those with higher \( F \) earn higher rates of return.

- PS: Contrast the two propositions.