

Lectures on Economic Inequality

Warwick, Summer 2017, Slides 3

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- Overview: Convergence and Divergence
- Inequality and Divergence: Economic Factors, Part 1
- **Inequality and Divergence: Psychological Factors**
- Inequality, Polarization and Conflict
- Uneven Growth and Conflict

Aspirations and Inequality

- **Two-way interaction:**
- Aspirations → **inspiration or frustration**
 - economic investment
 - collective action
- Such actions shape growth and distribution.
- **Society** → aspirations
- Aspirations are shaped by the lives of others around us.

Aspirations Appadurai (2004), Ray (1998, 2006), Genicot-Ray (2015)

- A set of **reference points** to evaluate personal outcomes.

- Fundamentally social as well as personal:

$$a = \Psi(y, F),$$

where F = social distribution, y = personal characteristics.

- **Our Leading Example.**

y = personal income or **wealth**.

F = the ambient **distribution** of income or wealth

a = expressed in same units as wealth of one's children, or future self.

- More generally, aspirations are **multidimensional**:

- income, dignity, good health, recognition
- political power, or the urge to dominate others on religious or ethnic grounds.

- So are the **social influences on aspirations**:

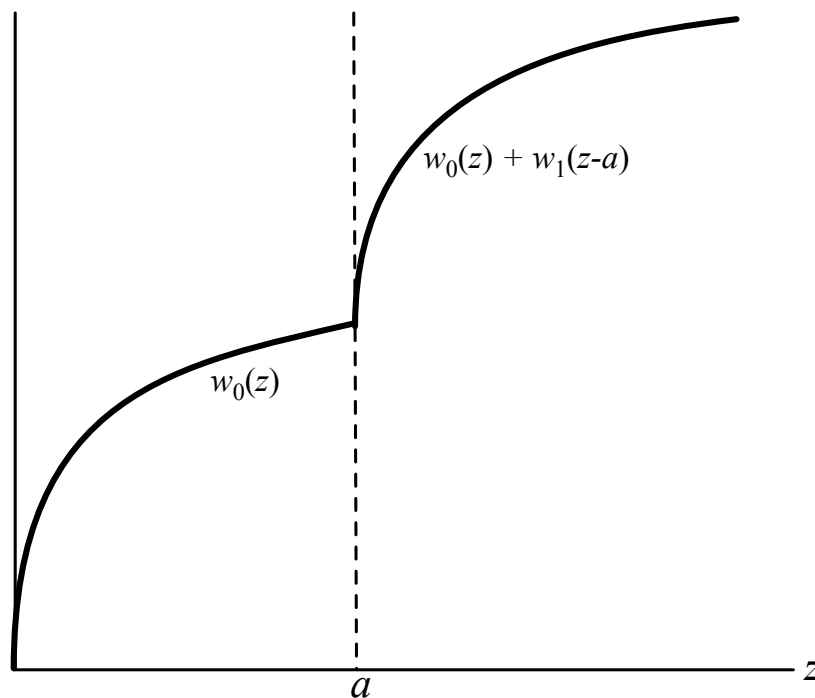
- the influence of peers or near-peers (Lewis 1958, Das Gupta 1994)
- sources of information (Wilson 1987)
- religion, caste, ethnicity (Munshi and Myaux 2001)
- statistical considerations (Munshi 1999)

How Aspirations Map Into Preferences

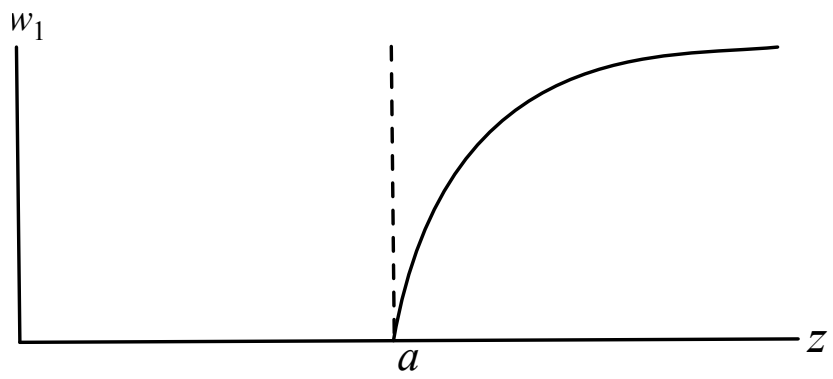
- In our leading example:

$$u(c) + w_0(z) + w_1(e),$$

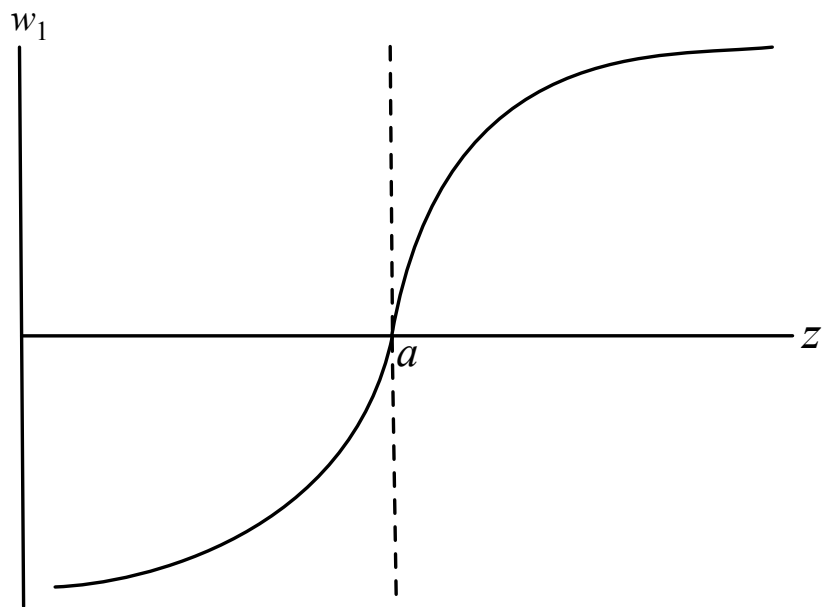
- where $e = \max\{z - a, 0\}$, and $z = \text{child's wealth}$.
- w_0 **intrinsic** utility; w_1 **aspirational** utility.
- Increasing, smooth, strictly concave.



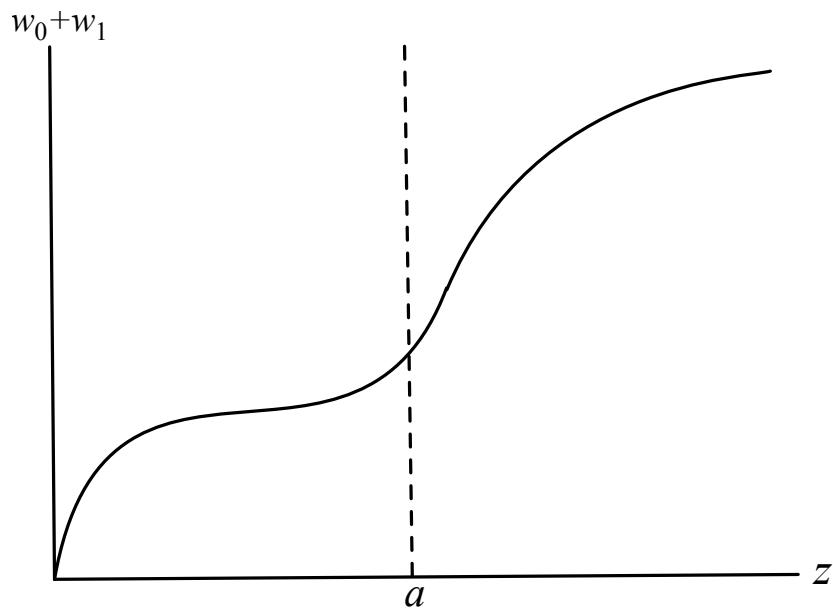
■ Remark 1. Alternatives for w_1 .



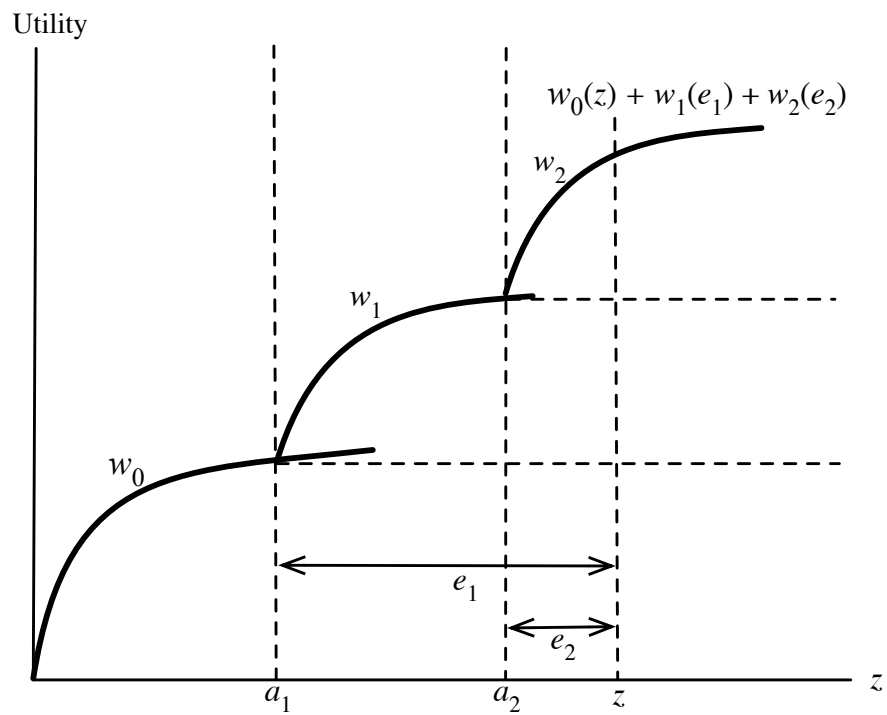
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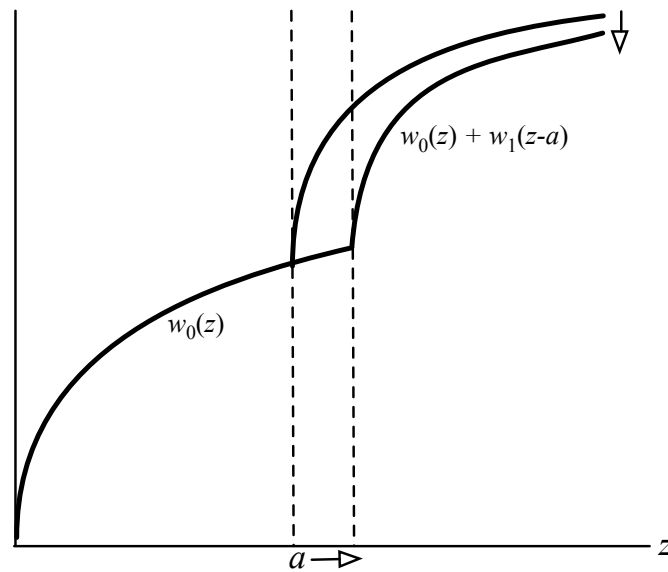


- Remark 2. Extends to aspiration vectors.



■ Remark 3.

- Higher aspirations always bad for happiness in the short-run:



Restrictions on the Aspiration Formation Function

$$a = \Psi(y, F)$$

- Assume that Ψ is:
 - [regular] $\Psi(y, F)$ is continuous in (y, F) , nondecreasing in y .
 - [range-bound] $\min\{y, \min F\} \leq \Psi(y, F) \leq \max\{y, \max F\}$.
 - [scale-invariant] $\lambda\Psi(y, F) = \Psi(\lambda y, F^\lambda)$ for $\lambda > 0$. [$F^\lambda(\lambda y) = F(y)$.]
 - [socially monotone] $\Psi(y, F') > \Psi(y, F)$ when F' strictly FOSD F .

Aspirations and Distribution: Individual Incentives

- Start with F_t .
- Then $a_t = \Psi(y, F_t)$ for every $y \in \text{Supp } F_t$.

- At income y , choose $z \in [0, f(y)]$ to max

$$u(y - k(z)) + w_0(z) + w_1(\max\{z - a_t, 0\})$$

where $k(z) \equiv f^{-1}(z)$.

- F_{t+1} new distribution.
- From F_0 , recursively generates **equilibrium** sequence $\{F_t\}$:
- **Proposition 0.** An equilibrium (trivially) exists.

Tiny Digression: Benchmark Without Aspirations

- Choose $z \in [0, f(y)]$ to max

$$u(y - k(z)) + w_0(z)$$

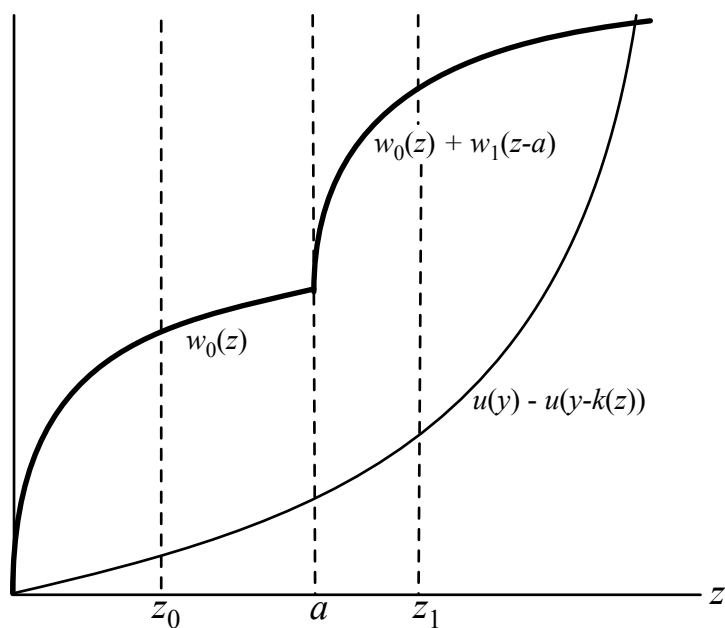
- **Steady state** condition:

$$d(y) \equiv -\frac{u'(y - k(y))}{f'(k(y))} + w_0'(y) = 0.$$

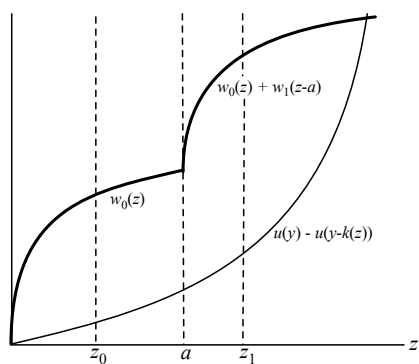
- **Assumption:** $d(y) > 0$ for some y and strictly decreasing if $d(y) \leq 0$.

(at most one, strictly positive steady state y^* in benchmark model)

From Aspirations and Wealth To Investment



- At most one “local” solution on either side of a .



Lower solution z_0 as in benchmark:

$$- [u'(y - k(z_0)) / f'(k(z_0))] + w'_0(z_0) = 0,$$

where $k(z) \equiv f^{-1}(z)$.

Upper solution z_1 :

$$- [u'(y - k(z_1)) / f'(k(z_1))] + w'_0(z_1) + w'_1(z_1 - a) = 0.$$

- Compare, and pick the one with the higher payoff.

■ An aspiration a is **satisfied** if optimum is no less than a .

It is **frustrated** if optimum strictly smaller than a .

■ **When is an aspiration satisfied, and when is it frustrated?**

■ useful partial equilibrium exercise, aspirations “exogenous.”

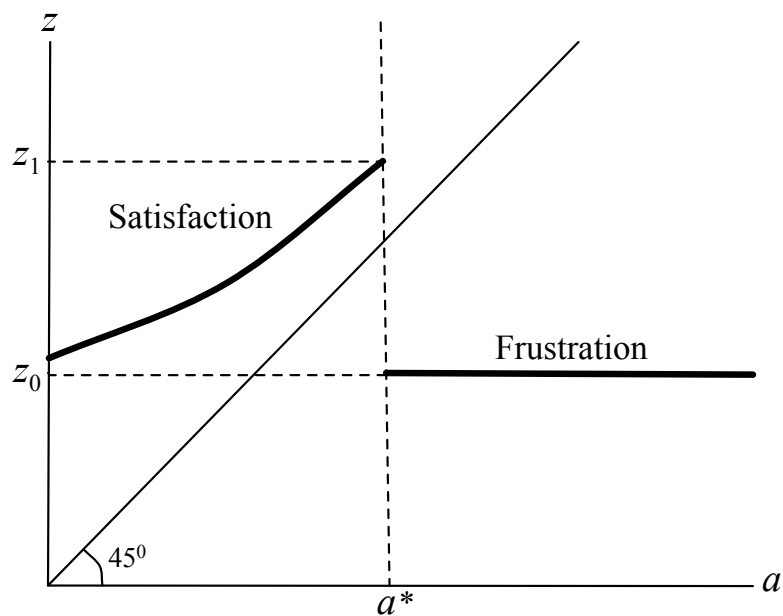
■ rise of television, advertising or the internet

■ change in the income distribution

■ **Proposition 1.** Fix current wealth.

■ There is a unique threshold value of aspirations below which aspirations are satisfied, and above which they are frustrated.

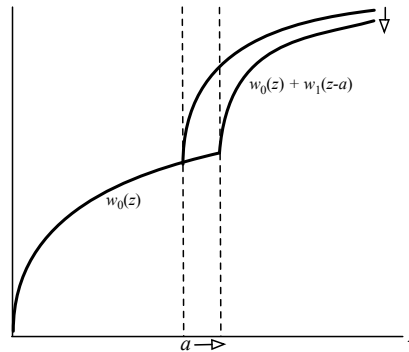
■ As long as aspirations are satisfied, chosen wealth grows with aspirations. Once aspirations are frustrated, chosen wealth becomes insensitive to aspirations.



■ Note the discontinuous jump-down.

On Frustration

- Recall that aspirational growth always lowers direct utility:



- If aspirations are frustrated, no inspirational role either:
- “The French found their position all the more intolerable as it became better.” de Tocqueville, 1856
- Lowered aspirations of low income students reduces school dropout Kearney-Levine, 2014, for the US; Goux-Gurgand-Maurin, 2014, for France

A Variant: Aspirations-Wealth Ratios and Investment

- Introducing the canonical linear model:
- Linear production: $f(k) = \rho k$.
- Constant-elasticity utility:

$$u(c) = c^{1-\sigma}, w_0(z) = \delta z^{1-\sigma}, \text{ and } w_1(e) = \delta \pi e^{1-\sigma}$$

- Investment choice: Given (y, a) , pick z to maximize

$$\left(y - \frac{z}{\rho}\right)^{1-\sigma} + \delta \left[z^{1-\sigma} + \pi (\max\{z - a, 0\})^{1-\sigma}\right].$$

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$$u(c) = c^{1-\sigma}, w_0(z) = \delta z^{1-\sigma}, \text{ and } w_1(e) = \delta \pi e^{1-\sigma}$$

- **Investment choice**: Given a/y , pick z/y to maximize

$$\left(1 - \frac{z/y}{\rho}\right)^{1-\sigma} + \delta \left[(z/y)^{1-\sigma} + \pi \left(\max \left\{ \frac{z}{y} - \frac{a}{y}, 0 \right\} \right)^{1-\sigma} \right].$$

(dividing through by y)

- **Investment choice**:

- **Aspirations ratio**: $r \equiv a/y$. Choose growth $g \equiv z/y$ to max

$$\left(1 - \frac{g}{\rho}\right)^{1-\sigma} + \delta \left[g^{1-\sigma} + \pi (\max \{g - r, 0\})^{1-\sigma} \right].$$

- **Failed aspiration**; solution \underline{g} independent of r :

$$\left(1 - \frac{\underline{g}}{\rho}\right)^{-\sigma} = \delta \rho \underline{g}^{-\sigma}$$

- **Satisfied aspiration**; solution $g(r)$ depends on r :

$$\left(1 - \frac{g(r)}{\rho}\right)^{-\sigma} = \delta \rho \left[g(r)^{-\sigma} + \pi (g(r) - r)^{-\sigma} \right].$$

■ Proposition 2.

■ There is a unique ratio r^* such that for $r \equiv a/y > r^*$, wealth grows at rate \underline{g} , and for all $r \equiv a/y < r^*$, wealth grows at rate $g(r)$.

■ $g(r) \uparrow$ in r , but larger and bounded away from \underline{g} in r .

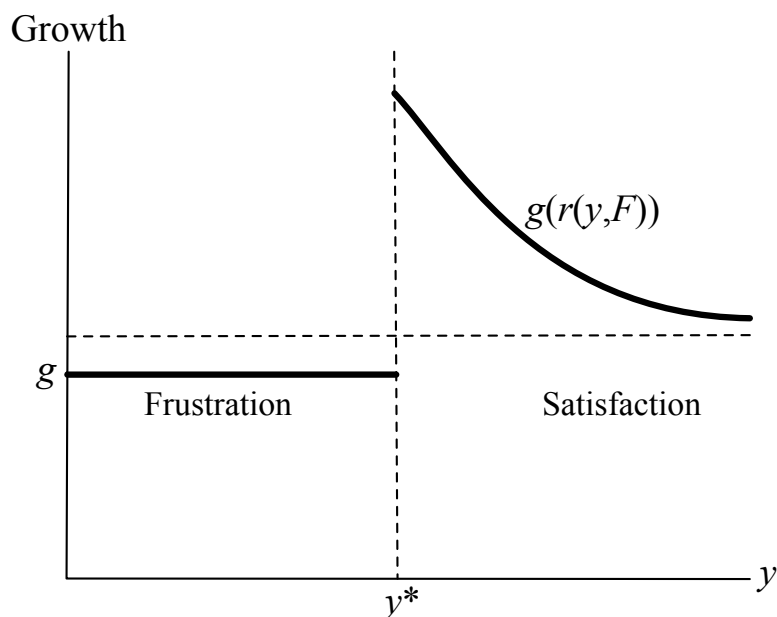
■ Moreover, we can link y to a/y :

■ Proposition 3.

■ Assume scale invariance and social monotonicity. Then for each F :

■ The aspirations ratio $r(y, F) \equiv \Psi(y, F)/y$ is decreasing in y .

■ Combining Propositions 2 and 3:



Society-Wide Evolution of Aspirations and Incomes

- Recall our recursive equilibrium notion:
 - Wealth distribution F_t at date t ; $a_t = \Psi(y, F_t)$.
 - Each person with wealth y chooses continuation z .
 - z is tomorrow's wealth, and F_{t+1} is new distribution.
 - From F_0 , recursively generate F_t and $a_t = \Psi(y, F_t)$ for all y and t .
- Questions:
 - Persistent or growing inequality, or convergence?
 - Connections between initial distribution and subsequent growth.

Steady States

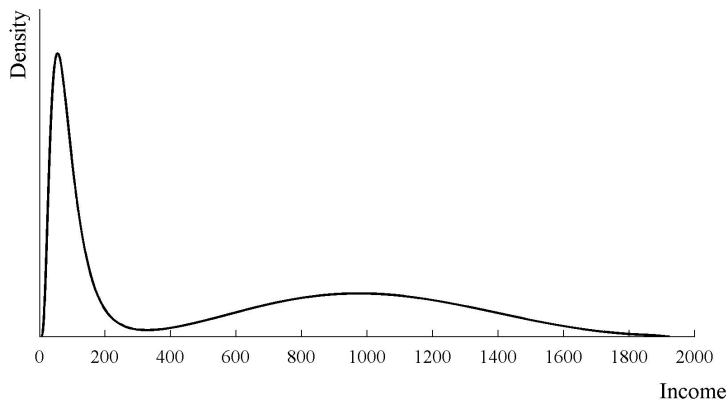
- Distribution F^* concentrated on strictly positive incomes:
 - $\{F^*, F^*, F^*, \dots\}$ equilibrium from F^* .
 - Natural setting: incomes in compact support, as in Solow model.
- Proposition 4.
 - There is no steady state with perfect equality.
- Argument.
 - Perfect equality implies concentration of y and a at same point.
 - Contradiction: everyone wants to move away from $y = a$.
 - Related to [symmetry-breaking](#).

Clustering in Steady State

- Proposition 5.
 - Assume range-bound, scale-invariant and socially monotone aspirations.
 - Then steady states must all be bimodal.
- Two-point distribution F^* . (y_ℓ, y_h, p) :
 - $y_\ell < y_h$, and p is population weight on y_ℓ .
 - Aspirations satisfy $a_i = \Psi(y_i, F^*)$ for $i = \ell, h$.
 - a_ℓ is a failed aspiration, so $d(y_\ell) = 0$.
 - And a_h is a satisfied aspiration, so $d(y_h) + w'_1(y_h - a_h) = 0$.

Remarks on Clustering

- Of course, convergence to **degenerate** poles is an artifact (akin to single steady-state income in Solow model.)
- With stochastic shocks (e.g., Brock-Mirman 1972): smoothly dispersed.



Const-elasticity u : $\sigma = 0.8$, $\delta = 0.8$ and $\pi = 1$; $f(k, \theta) = \theta(A/\beta)k^\beta$, where $\beta = 0.8$, $A = 4$ and θ lognormal mean 1. a = average of own y and mean y .

Multimodality in the Literature

- [US] Pittau-Zelli 2004, Sala-i-Martin 2006, Zhu 2005

- [world] convergence clubs

Durlauf-Johnson 1995, Quah 1993, 1996, Durlauf-Quah, 1999

- Quah uses the term “twin peaks.”
- Bimodality also a feature of polarized distributions

Esteban-Ray 1994, Wolfson 1994.

Aspirations, Inequality and Endogenous Growth

- Return to canonical linear model:
 - constant-elasticity utility, linear production.
- Recall **aspirations ratio** $r = a/y$.
- And recall Proposition 3: there is r^* such that:
 - if aspirations too high ($r > r^*$), grow at \underline{g}
 - otherwise (if $r \leq r^*$), grow at $g(r)$.

Ultimate Equality | Perpetually Widening Inequality

■ **Proposition 6.** Assume aspirations are range-bound, scale-invariant and socially monotone. Let F_0 be initial distribution of with compact support. Then there are just two possibilities:

I. **Convergence to Perfect Equality.** There is $g^* > 1$ such that y_t/g^{*t} converges to a single point independent of $y_0 \in \text{Supp } F_0$; or

II. **Persistent Divergence.** F_t “separates” into two components defined by threshold $y^* \in \text{int Range } F_0$:

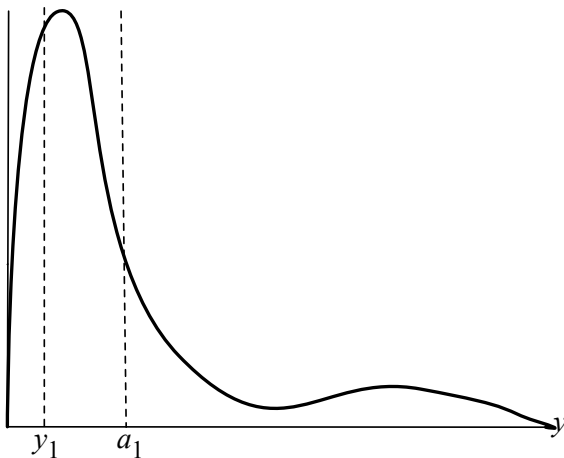
- If $y < y^*$, income grows forever after at \underline{g} .
- If $y > y^*$, income has asymptotic growth $\bar{g} > \underline{g}$, with $\bar{g} - 1 > 0$, and y_t/\bar{g}^t has the same limit independent of y_0 , as long as y_0 exceeds y^* .
- $\underline{g} < \bar{g} \leq g^*$: equality exhibits faster growth.
- In Case II, relative inequality never settles, it perpetually widens.

Social Monotonicity and the Inequality Proposition

■ Social monotonicity \Rightarrow If $y_1 < y_2$, then $r(y_1, F) > r(y_2, F)$.

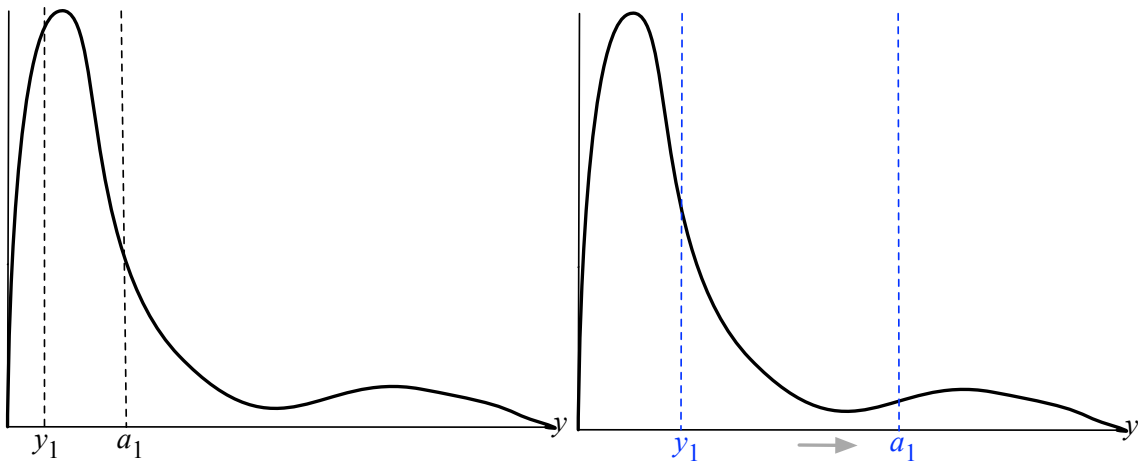
■ The argument for the equality-inequality proposition hinges on this.

■ **Example.** Aspirations = conditional mean above income.



Social Monotonicity and the Inequality Proposition

- Social monotonicity \Rightarrow If $y_1 < y_2$, then $r(y_1, F) > r(y_2, F)$.
- The argument for the equality-inequality proposition hinges on this.
- **Example.** Aspirations = conditional mean above income.



The Proposition Without Social Monotonicity

- **Example 1.** Balanced Growth with Unbounded Distribution.
- Aspirations = conditional mean above income.
- **Observation.** There are balanced growth paths with all incomes satisfied and non-degenerate inequality. The accompanying income distribution must be Pareto.

$$F\left(\frac{y}{[1+g]^t}\right) \equiv F(w) = 1 - (A/w)^{r/(r-1)}$$

for all $w \geq A$ and (A, r) such that $r \in (1, r^*]$.

- Everyone has the same aspirations ratios.

The Proposition Without Social Monotonicity

- **Example 2.** Balanced Growth with Compact Distribution.
- Aspirations determined by incomes in some range around own incomes:
 - For some $\beta > 0$, $\Psi(y, F)$ is insensitive to $y' \notin [y(1 - \beta), y(1 + \beta)]$.
- **Observation.** Beginning with any distribution with compact support, the income distribution converges to a set of isolated income clusters, each growing at a factor strictly larger than \underline{g} .
- Balanced growth even under distributions with bounded support ...
- ... but at the expense of assuming the possibility of *total* isolation.

The Proposition Without Social Monotonicity

- **Example 3.** Infinite Crossings.
- Distribution with just three mass points $y^1 > y^2 > y^3$.
- The poorest at y^3 don't "see" the rich, but are inspired by y^2 , **grow at \bar{g}**
- The middle class "see" only the rich, but are frustrated by them, **grow at \underline{g}**
- The rich just "see" themselves, **grow at g**
- Assume $y^2 \underline{g} = y^3 g$ and $y^3 \bar{g} = y^2 g$:
- Then balanced growth with 2 and 3 perennially crossing.

Extension of Equality-Inequality Proposition

■ Minimally monotone aspirations:

- Distribution F and $y \in \text{Supp } F$,
- F' weakly FOSD F , but $F'(y) = F(y)$ (no crossings over y “from below.”)
- $\Rightarrow \Psi(y, F') \geq \Psi(y, F)$.

Extension of Equality-Inequality Proposition

■ **Proposition 7.** Assume aspirations are range-bound, scale-invariant and minimally monotone. Let F_0 be initial distribution with compact support. Then just three possibilities:

I. Convergence to Perfect Equality. There is $g^* > 1$ such that y_t/g^{*t} converges to a **single point** independent of $y_0 \in \text{Supp } F_0$; or

II. Persistent Divergence. Over time, F_t “separates” into two components:

- Frustrated individuals converging to a positive limit measure. Their incomes grow at rate $\underline{g} - 1$.
- Satisfied individuals with common asymptotic growth factor $\bar{g} > \underline{g}$, with $\bar{g} - 1 > 0$. y_t/\bar{g}^t converges to the same limit irrespective of y_0 .

III. Infinite Crossings. Income ranks cross and re-cross ad infinitum:

- Compatible with constant or increasing inequality (but not convergence).

Summary So Far

- A theory of **aspirations formation**.
- Emphasizes the **social** foundations of individual aspirations
- Relates those aspirations to investment and growth.
- Such behavior can be aggregated, thus closing the model.
- Central feature: aspirations can both incentivize and frustrate.
- This approach is tractable and may be useful in other contexts.

Aspirations and Distribution: Collective Action

- Two reactions to social change:
 - **Individual** investment (or disinvestment)
 - **Collective** action
- Collective action of two kinds:
 - For promoting the self-interest of “my group”
 - For attacking “other groups.”
- Groups?
 - Based on class, religion, ethnicity, immigrant status . . .

Aspirations and Groups

- Consider two groups, h and m . For any person i in group h (say):

$$a_h(i) = \Psi_h(y(i), \mu_h, \mu_m).$$

- where $y(i)$ is own characteristic, and μ_j is mean for group j .
- **Example** [conformity]: only μ_j matters for person in j .
- Munshi and Myaux (2006) on fertility norms in Bangladesh.
- **Example** [rivalry]: only μ_{-j} matters for person in j .
- Mitra and Ray (2014) on religious violence in India.

Rivalry and Collective Action

$$a_j(i) = \Psi_h(y(i), \mu_{-j}).$$

- A fundamental asymmetry:
- **Individual action** to improve own lot ...
- **Collective action** to reduce aspirations and envy.
- And only possible with cross-group aspirations (targeting).

Rivalry and Collective Action

$$a_j(i) = \Psi_h(y(i), \mu_{-j}).$$

- Budget constraint: $y = k + ty + c$
- t : fraction time spent in violence, collectively chosen, reduces μ_{-j} .
- k investment, privately chosen by individual, $z = f(k)$ as before.
- Proposition 8.
 - An increase in rival income increases violence directed against rival group.
 - An increase in own income reduces violence directed against rival group.
 - In both cases, the effect on individual investments is ambiguous.

Distribution and Conflict: Hindu-Muslim Violence

- Recurrent episodes of religious violence
 - Partition era of the 1940s, and earlier
 - Continuing through the second half of the twentieth century.
- ~ 1,200 riots, 7,000 deaths, 30,000 injuries over 1950–2000.

Some Ethnographic Literature

- Thakore (1993) on Bombay riots [[land](#)]
- Das (2000) on Calcutta riots [[land](#)]
- Rajgopal (1987), Khan (1992) on Bhiwandi and Meerut riots [[textiles](#)]
- Engineer (1994), Khan (1991) on Jabbalpur, Kanpur, Moradabad [[bidis](#), [brass-ware](#)]
- Upadhyaya (1992) on Varanasi riots [[sari dealers](#)]
- Wilkinson (2004) on Varanasi [[wholesale silk](#)]
- Field et al (2009) on Ahmedabad [[housing](#)]

- **Example:** Engineer (1987) on [Meerut riots](#):

“If [religious zeal] is coupled with economic prosperity, as has happened in Meerut, it has a multiplying effect on the Hindu psyche. The ferocity with which business establishments have been destroyed in Meerut bears testimony to this observation. Entire rows of shops belonging to Muslims . . . were reduced to ashes.”

- And yet. . .

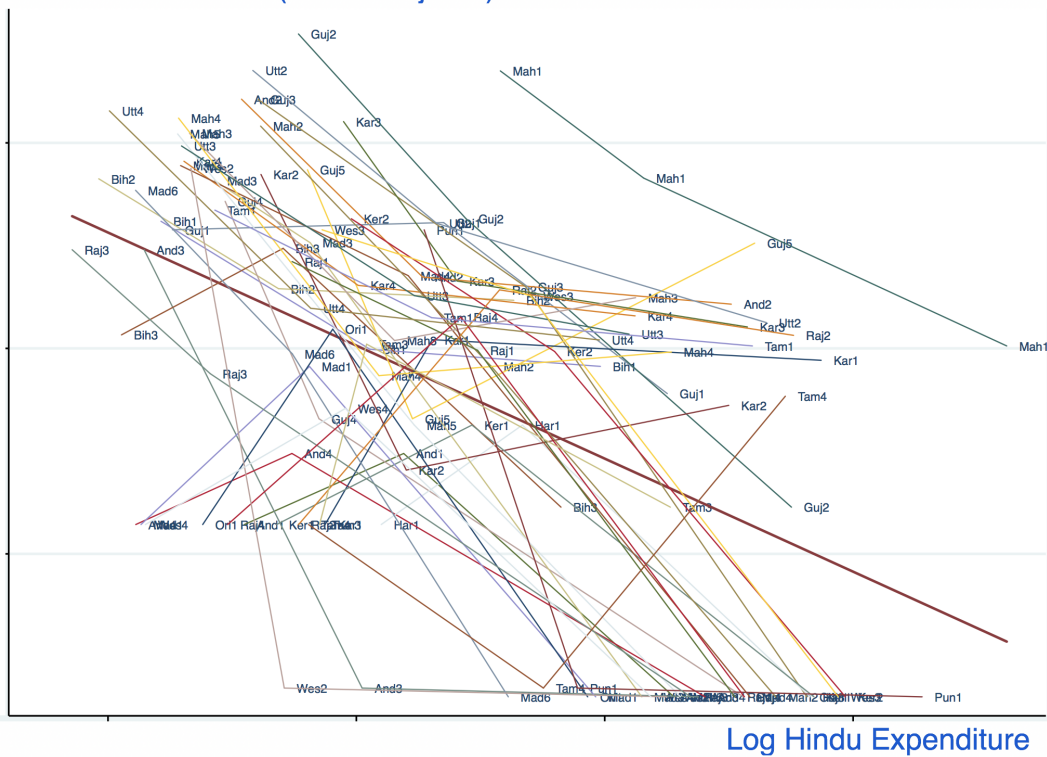
- Wilkinson (2004):

“Despite the disparate impact of riots on Hindus and Muslims, however, little hard evidence suggests that Hindu merchants and financial interests are fomenting anti-Muslim riots for economic gain. . . .”

- Horowitz (2001, p. 211):

“The role that commercial competition is said to play is said to be a covert, behind-the-scenes role, which makes proof or disproof very difficult.”

Log Residual Casualties (Killed + Injured)



Summary

- Preferences are fundamentally **social constructions**.
- (A truism, but economists are often blind to truisms)
- **Core idea**: interactive model of aspirations and development.
- The same direction of change can be inspiring, then frustrating.
- **Two applications**:
 - The evolution of economic inequality
 - Uneven growth and conflict
- **Ongoing work** with Joan Esteban, Eugenio Rojas, Raimundo Undurraga:
 - Aspirations and mobility in polarized societies.