# Lectures on Economic Inequality

Warwick, Summer 2016, Slides 2

Debraj Ray

- Overview: Convergence and Divergence
- Inequality and Divergence: Economic Factors, Part 1
- Inequality and Divergence: Psychological Factors
- Inequality, Polarization and Conflict
- Uneven Growth and Conflict

## The Basic Accumulation Equations

- Can view growth model as one of bequests (Becker-Tomes 1979, Loury 1981):

$$y_t = c_t + k_t,$$

- $y$ is income, $c$ is consumption, $k$ is bequest.

$$y_{t+1} = f(k_t) \text{ or } f(k_t, \alpha_t)$$

- Examples of $f$:
- Standard production function as in growth theory
- Competitive economy: $f(k) = w + (1 + r)k$.
- Returns to skills or occupations: for example,

$$\begin{aligned} f(k) &= \underline{w} \text{ for } k < \bar{x} \\ &= \bar{w} \text{ for } k > \bar{x}. \end{aligned}$$

- May be exogenous to individual, but endogenous to the economy

## Differential Savings Rates

- The simplest view: rich earn predominantly capital income

$$y_t = c_t + k_t$$

$$k_t = sy_t$$

$$y_{t+1} = rk_t$$

$$y(t) = y(0)(1 + sr)^t$$

- Say average rate of growth in the economy is $g$.
- So if initial rich share is $x(0)$, then $t$ periods later it will be

$$x(t) = x(0)\left(\frac{1 + sr}{1 + g}\right)^t$$

- Can back out $r$ if we know $s$ and $\{x(t)\}$:

$$r = \frac{[x(t)/x(0)]^{1/t}(1 + g) - 1}{s}$$

## Differential Savings Rates

- Do the rich save more than the poor?
- Unclear: out of lifetime income or current income?

Friedman (1957), see discussion in Dynan-Skinner-Zeldes (2004)

- Estimates from Survey of Consumer Finances (SCF):

| | 6-Yr Income Average | Instrumented By Vehicle Consumption |
|---|---|---|
| Quintile 1 | 1.4 | 2.8 |
| Quintile 2 | 9.0 | 14.0 |
| Quintile 3 | 11.1 | 13.4 |
| Quintile 4 | 17.3 | 17.3 |
| Quintile 5 | 23.6 | 28.6 |
| Top 5% | 37.2 | 50.5 |
| Top 1% | 51.2 | 35.6 |

Source: Dynan-Skinner-Zeldes (2004), they provide other estimates

$$r = \frac{[x(t)/x(0)]^{1/t}(1+g) - 1}{s}$$

- Some quick calculations for top 10% in the US:
  - $x_0 = 1/3$ in 1970, rises to $x_t = 47/100$ in 2000.
  - Estimate for $g$: 2% per year.
  - Estimate from Dynan et al for $s$: 35% (optimistic).
  - Can back out for $r$: $r = 9.7\%$.
- Inflation-adjusted rate of return on US stocks over 20th century: 6.5%
  - Much lower in the 1970s and 2000s, higher in the 1980s and 1990s.

$$r = \frac{[x(t)/x(0)]^{1/t}(1+g) - 1}{s}$$

- Similar calculations for top 1% in the US:
  - $x_0 = 8/100$ in 1980, rises to $x_t = 18/100$ in 2005.
  - Estimate for $g$: 2% per year.
  - Estimate from Dynan et al for $s$: 51%.
  - Can back out for $r$: $r = 10.5\%$.

$$r = \frac{[x(t)/x(0)]^{1/t}(1+g) - 1}{s}$$

- Try the top 0.1% for the United States:
  - $x_0 = 2.2/100$ in 1980, rises to $x_t = 8/100$ in 2007.
  - Estimate for $g$: 2% per year.
  - If these guys also save at 0.5, then $r = 14.4\%$!
  - If they save $3/4$ of their income, then $r = 9.6\%$.

$$r = \frac{[x(t)/x(0)]^{1/t}(1+g) - 1}{s}$$

- Slightly better job for Europe, but not much. Top 10%:
  - $x_0 = 29/100$ in 1980, rises to $x_t = 35/100$ in 2010.
  - Estimate for $g$: 2% per year.
  - Estimate from Dynan et al for $s$: 35%.
  - Can back out for $r$: $r = 7.5\%$.
- High relative to $r$ in Europe.
  - UK the highest at 5.3% over 20th century, others appreciably lower.

$$r = \frac{[x(t)/x(0)]^{1/t}(1+g) - 1}{s}$$

- Finally, top 1% for the UK:
  - $x_0 = 6/100$ in 1980, rises to $x_t = 15/100$ in 2005.
  - Estimate for $g$: 2% per year.
  - Estimate from Dynan et al for $s$: 51%.
  - Can back out for $r$: $r = 11.4\%$.

- Summary

- Differential savings rates explain some of the inequality, but far from all of it.

## What Explains the High Rates of Return to the Rich?

- Two broad groups of answers:
  - The rich have physical access to better rates of return.
  - The rich have access to better information on rates of return.

- Begin with better physical access.

- The role of imperfect capital markets
  - Stocks (no problem)
  - Hedge funds?
  - Private unincorporated businesses (moral hazard, adverse selection)
  - Human capital (inalienability, holding children responsible for debt)

- Special aside on human capital:

- Should get more attention as a fundamental vehicle for inequality:

  - "Labor income inequality is as important or more important than all other income sources combined in explaining total income inequality." Fields (2004)

  - Even Piketty backs away when it comes to U.S. inequality:

"a very substantial and growing inequality of capital income since 1980 accounts for about one-third of the increase in inequality in the United States — a far from negligible amount." See also Saez and Zucman (2014)

  - Labor income inequality accounts for the bulk of it.

## Nonlinear $f$

- At an abstract level, varying rates of return captured by nonlinear $f$.

  - Growth model can be applied to individual dynastic households.

  - Lots of "mini growth models", one per household.

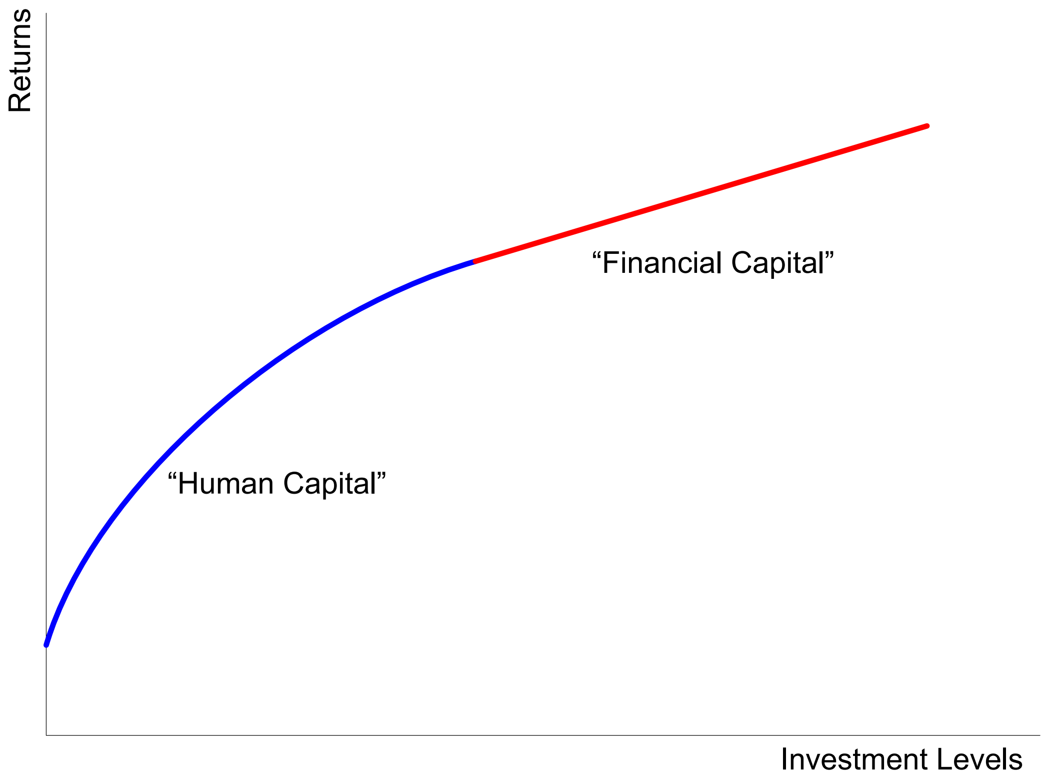
- Interpret $f$ as envelope of intergenerational investments:

  - Financial bequests

  - Occupational choice

  Becker and Tomes (1979, 1986), Loury (1981)

- So, not surprising that this literature looks like growth theory.




## Different Considerations

- The bequest decision
  - Fixed savings rate, intergenerational utility.
- The shape of $f$
  - Concave or convex?
- The endogeneity of $f$
  - New to standard growth models.

## The Bequest Decision: Preferences

- Warm Glow. $U(c_t, k_t)$.
  - See, e.g., Banerjee and Newman (1993).
- Consumption-Based. $U(c_t, c_{t+1})$.
  - See, e.g., Arrow (1973), Bernheim and Ray (1986).
- Income-Based. $U(c_t, y_{t+1})$.
  - See, e.g., Becker and Tomes (1979, 1981).
- NonPaternalistic. $U(c_t, V_{t+1})$, where $V_{t+1}$ is lifetime utility of gen $t+1$.
  - See, e.g., Barro (1978) and Loury (1981).
- A general form: $\mathrm{E}_\alpha U(c_t, \Psi_t(k_t, \alpha))$.

## A General Principle for Limited Mobility

- Parental utility

$$\mathrm{E}_\alpha U(c, \Psi(k, \alpha)),$$

  - where $\Psi(k, \alpha)$ is some nondecreasing map.
- Assume $U$ strictly concave in $c$, and $c$ and $\Psi$ are complements:
  - $U(c, \Psi) - U(c', \Psi)$ nondecreasing in $\Psi$ whenever $c > c'$.
- "Reduced-form" maximization problem:

$$\max w(c, k) \equiv \mathrm{E}_\alpha U(c, \Psi(k, \alpha))$$

  - subject to $y = c + k$.
  - (Includes dynamic programming)

- **Theorem.**

- Let $h$ describe all optimal choices of $k$ for each $y$.
- Then if $y > y'$, $k \in h(y)$, and $k' \in h(y')$, it must be that $k \geq k'$.

- **Remarks:**

- $h$ is "almost" a function.
- $h$ can only jump up, not down.
- Same assertion is not true of optimal $c$.
- Note how curvature of $w$ in $c$ is important, in $k$ is unimportant.
- Comes in handy again and again in different models.

- **Proof.**

- Suppose not. Then for some $y > y'$, $k \in h(y)$, we have $k' > k$.
- $k'$ feasible for $y$ and $k$ feasible for $y'$ (why?). So

  $w(y-k,k) \geq w(y-k',k')$ and $w(y'-k',k') \geq w(y'-k,k)$

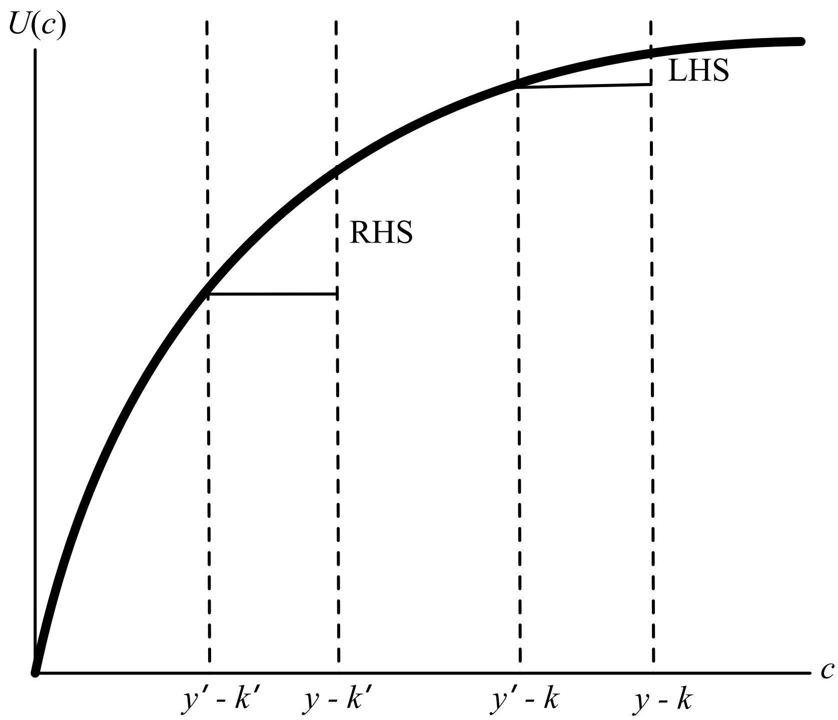
- Adding and transposing:

$$w(y-k,k) - w(y'-k,k) \geq w(y-k',k') - w(y'-k',k').$$

- Replace second entry $k'$ on RHS with $k$, RHS $\downarrow$ by complementarity:

$$w(y-k,k) - w(y'-k,k) \geq w(y-k',k) - w(y'-k',k).$$

- Now draw a diagram: strict concavity of $w$ is contradicted.




$w(y-k,k) - w(y'-k,k) \geq w(y-k',k) - w(y'-k',k)$, contradiction!

## Solving The Standard Model

extension of Brock-Mirman (1972) and Loury (1981).

- Production:
  - $f$ exogenous, crosses $45^0$ for every $\alpha$.
- Preferences:
  - $u(c) + \delta \mathrm{E} V$, where $\delta \in (0,1)$ and $V$ is the value function.
- Maximization: For every $y$, parent chooses $k \in [0, y]$ to

$$\max u(y-k) + \delta \mathrm{E}_\alpha V\left(f(k,\alpha)\right)$$

- Bellman equation:

$$V(y) = \max_{0 \leq k \leq y} \left[u(y-k) + \delta \mathrm{E}_\alpha V\left(f(k,\alpha)\right)\right].$$

  - Existence and continuity of $V$ proved using standard arguments.

$$V(y) = \max_{0 \leq k \leq y} \left[ u(y-k) + \delta \mathrm{E}_{\alpha} V \left( f(k, \alpha) \right) \right].$$

- First order condition:

$$u'(y-k) = \delta \mathrm{E}_{\alpha} V' \left( f(k, \alpha) \right) f_k(k, \alpha)$$

- Using the envelope theorem this gives us the well-known Euler equation:

$$\begin{aligned} u'(c_t) = u'(y_t - k_t) &= \delta \mathrm{E} \left[ V' \left( y_{t+1} \right) f_k(k_t, \alpha) | k_t \right] \\ &= \delta \mathrm{E} \left[ u' \left( c_{t+1} \right) f_k(k_t, \alpha) | k_t \right]. \end{aligned}$$

## No Uncertainty

- $\alpha$ degenerate. Write production function as $f(k)$.

■ Theorem. From any initial condition, capital stocks and incomes converge monotonically. All positive limits $k^*$ solve

$$\delta f'(k^*) = 1.$$

- Limit is independent of history if $f$ has diminishing returns.

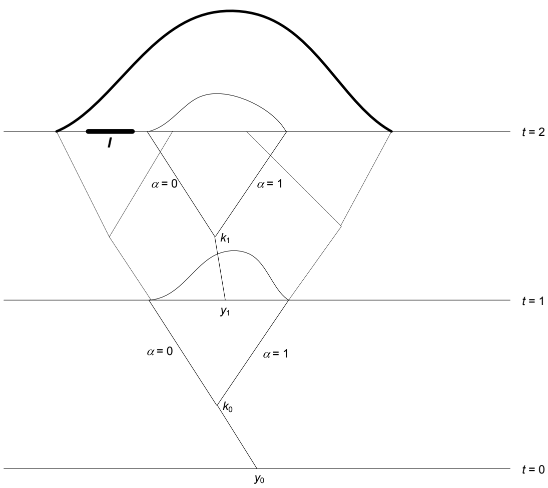
- Otherwise could be history-dependent.

- Proof follows directly from applying the limited mobility theorem.

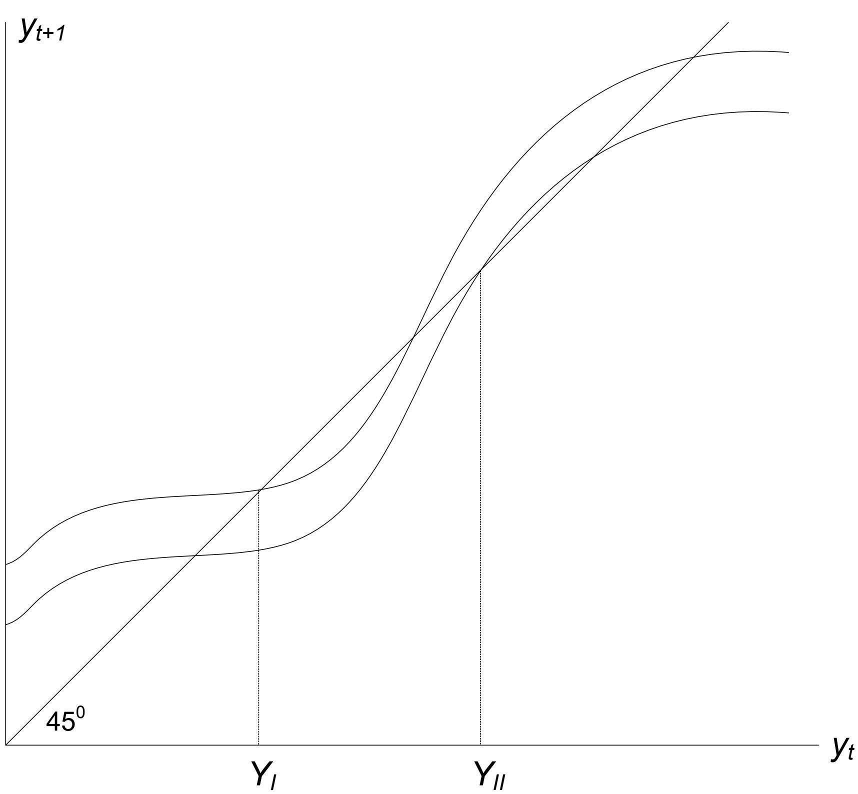
## Uncertainty and History Make two assumptions:

- Poor Genius. $f(0,1) > 0$.
- Rich Fool. $f(k,0) < k$ for all $k > 0$.

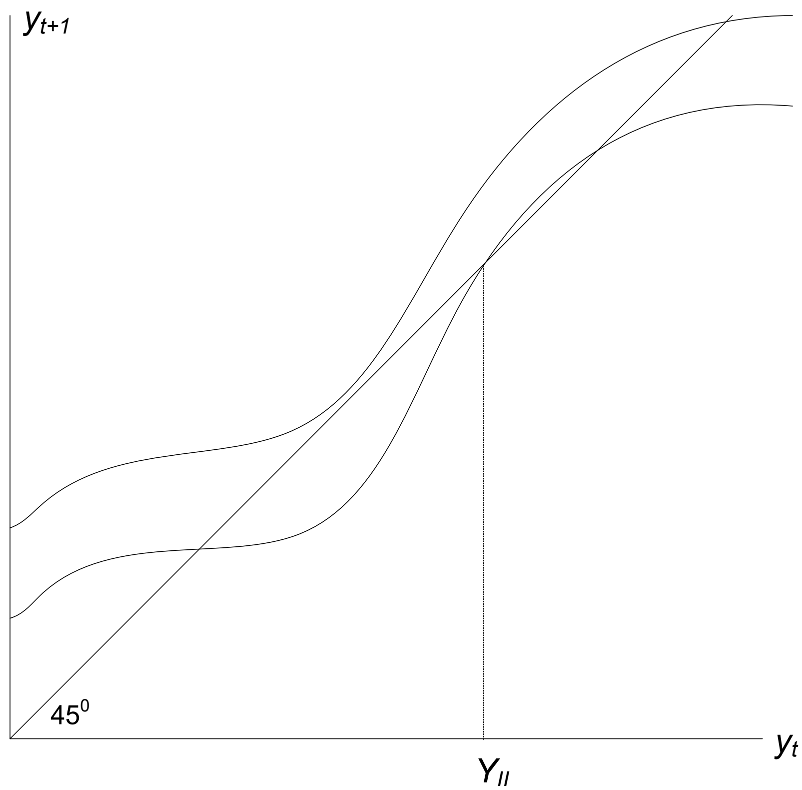
■ Theorem. Then there exists a unique measure on incomes $\mu^*$ such that $\mu_t$ converges to $\mu^*$ as $t \to \infty$ from every $\mu_0$.




■ Core assumption: a "mixing zone". In this case, it fails:



- Here's a case where a mixing zone exists:




- Two major drawbacks of this model:
- The reliance on stochastic shocks
  - Ergodicity could be a long time coming, so misleading
  - E.g., New York State lottery ⇒ mixing.
- If there is no mixing, then multiple steady states:
  - But must have disjoint supports, which is weird.

## Inequality and Markets

- Return to the interpretation of $f$ as occupational choice.
- Dropping efficiency units creates movements in relative prices:
- $f$ isn't "just technology" anymore.
- An Extended Example with just two occupations
- Two occupations, skilled $S$ and unskilled $U$. Training cost $x$.
- Population allocation $(\lambda, 1 - \lambda)$.
- Output: $f(\lambda, 1 - \lambda)$
- Skilled wage: $w_s(\lambda) \equiv f_1(\lambda, 1 - \lambda)$
- Unskilled wage: $w_u(\lambda) \equiv f_2(\lambda, 1 - \lambda)$

## Households

- Continuum of households, each with one agent per generation.
- Starting wealth $y$; $y = c + k$, where $k \in \{0, x\}$.
- Child wealth $y' = w$, where $w = w_s$ or $w_u$.
- Parent makes choice to max utility.
- No debt!
- Child grows up; back to the same cycle.

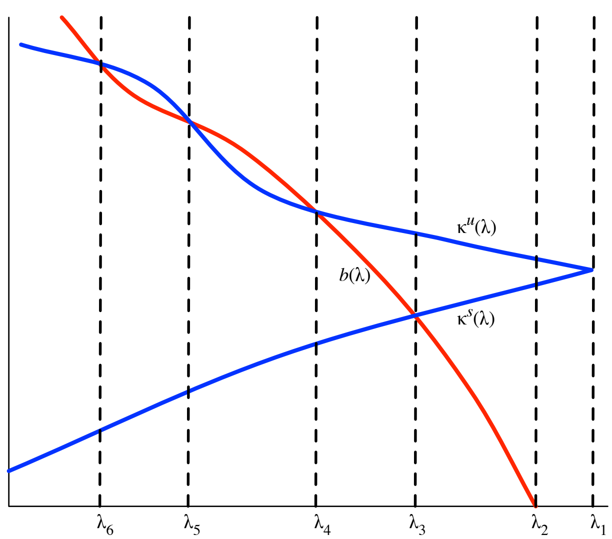
## Equilibrium

- A sequence $\{\lambda^t, w_s^t, w_u^t\}$ such that
  - $w_s^t = w_s(\lambda^t)$ and $w_u^t = w_u(\lambda^t)$ for every $t$.
  - $\lambda^0$ given and the other $\lambda^t$'s agree with utility maximization.

## Steady State

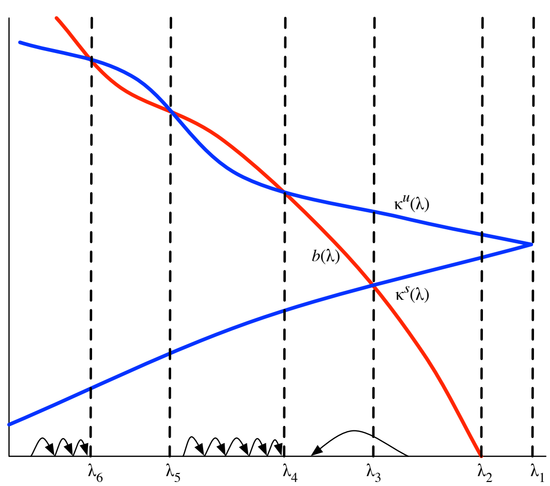
- A stationary equilibrium with positive output and wages.
  - $\kappa^j(\lambda) \equiv U\left(w_j(\lambda)\right) - U\left(w_j(\lambda) - X\right)$ (investment cost in utils)
  - $b(\lambda) \equiv V\left(w_s(\lambda)\right) - V\left(w_u(\lambda)\right)$ (investment gain in utils)
- Steady state condition:

$$\kappa^u(\lambda) \geq b(\lambda) \geq k^s(\lambda).$$



- Two-occupation model useful for number of insights:
- No convergence; persistent inequality in utilities.
- Symmetry-breaking argument.
- Multiple steady states must exist.
- See diagram for multiple instances of $\kappa^u(\lambda) \geq b(\lambda) \geq k^s(\lambda)$.
- Steady states with less inequality have higher net output.
- Net output maximization: $\max_l F(l, 1-l) - X$. Say at $l^*$.
- So $F_1(l^*, 1-l^*) - F_2(l^*, 1-l^*) = X$.
- All steady states to left of this point: inequality ↑, output ↓.

- Can get an exact account of history-dependence (dynamics).



## Applications

- The Cottage and the Factory

- Banerjee and Newman (1993)

- Each person can set up factory at cost $X$.

- Gets access to production function $g(L)$, hire at wage $w$.

- Otherwise work as laborer.

- Multiple steady states in factory prevalence.

- To embed this story into two-occupation model:

- Define $u$ = laborer, $s$ = entrepreneur. Let

$$f(l, 1-l) \equiv lg\left(\frac{1-l}{l}\right).$$

- Then

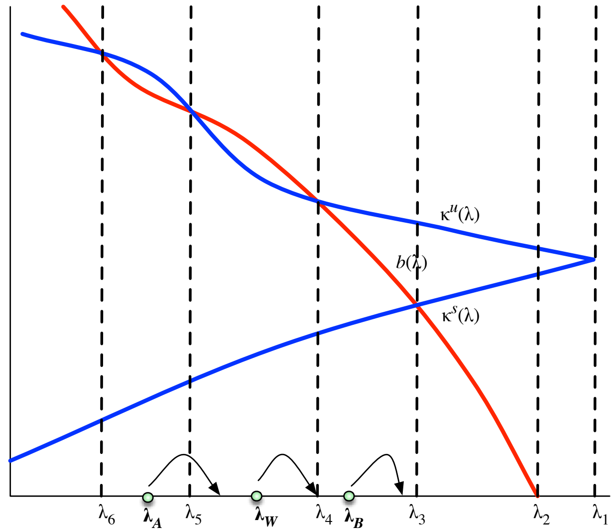
$$w^u(l) = f_2(l, 1-l) = g'\left(\frac{1-l}{l}\right) = w,$$

- and

$$w^s(l) = f_1(l, 1-l) = g\left(\frac{1-l}{l}\right) - \frac{1-l}{l}g'\left(\frac{1-l}{l}\right) = \text{profits}.$$

- Inequality and Comparative Advantage
  - Tanaka (2003), Zakarova (2006)
- Two ex-ante identical countries
  - differ only in initial inequality: their autarkic $\lambda$s.
- Nested production function:
  - aggregate output made from two intermediates
  - each intermediate produced with skilled and unskilled labor
  - One intermediate relatively intensive in skilled labor.

- In autarky, our model applies to each country
  - (simply integrate out the intermediate goods)
- In trade, our model applies to the world as a whole.
  - (with convex combination of two $\lambda$'s as initial condition)
- By theorem on dynamics, converges to edge steady state.
- Implies incomplete convergence across identical countries.




- Specific National Infrastructure
- Interpret occupations as labor for a specific bundle of goods.
- Interpret $f$ as (common) utility function from the goods
- $X$ as (utility) cost of producing those goods.
- National goods-specific infrastructure to facilitate production.
- Then countries segregate, each dominant in different activities.
- Initial conditions $\Rightarrow$ choice of infrastructure (Sokoloff-Engerman).
- If utility is nonhomothetic, composition will change with growth.

## Occupational Choice for Children

- Say parents can choose to skill or not skill their kids.

$$u(c) + \delta n^{\theta}\left[eV^s + (1-e)V^u\right].$$

  - $e$ = fraction of skilled kids, $V^i$ = value function, and $0 < \theta < 1$.

- Cost of raising kids: $r_0(w)$ for unskilled, $r_1(w)$ for skilled.
  - Define $r(w,e) \equiv er_1(w) + (1-e)r_0(w)$.
  - Then total cost can be written as $z \equiv r(e,w)n$.

- Parent consumption $c = w - z = w - r(w,e)n$.

## Maximize above utility function subject to these constraints.

- Proposition. $e$ is optimally set either to zero or 1.

- Proof.
  - Define extra "education cost" $X \equiv r_1(w) - r_0(w)$.
  - Differentiate parental utility with respect to $e$:

$$\frac{\partial \text{Utility}}{\partial e} = \delta n^{\theta}(V^s - V^u) - u'(w - r(w,e)n)nx,$$

  - Now use the first-order condition (w.r.t $n$) to eliminate $u'$:

$$\frac{\partial \text{Utility}}{\partial e} = \frac{\delta n^{\theta} x}{r(w,e)}\left[\left\{\frac{r_0(w)}{x} + (1-\theta)e\right\}(V^s - V^u) - \theta V^u\right].$$

  - Proves that parental utility is strictly quasiconvex in $e$, given $w$.

- So no interior solution to $e$ can ever maximize parental utility.

- Proposition. At any transition point to a higher occupation, fertility must jump down.

  - Remark. Contrast ambiguity on income vs substitution effects.

- Proof. Given $e$ look at optimal choice of $n$:

$$u'(w-z)\,r(w,e) = \delta\theta n^{\theta-1}\left[eV^s + (1-e)V^u\right].$$

- Substitute this into utility function $u(c) + \delta n^\theta \left[eV^s + (1-e)V^u\right]$

  - so that parent effectively chooses $e$ to max

$$u(w-z) + \frac{1}{\theta}u'(w-z)\,z.$$

  - Monotone in $z$. (Why?) So find an extremal value of $z$.

  - At transition, both $e=0$ and $e=1$ optimal, so have same $z$.

  - Therefore $e=1$ must have lower $n$.

- So in steady state, a combination of Barro-Becker over the range in which no occupational transition, plus a fertility drop at the occupational transition.

- Theorem: Net effect is always a fertility drop over observed wage rates in steady state.

- Corollary. Steady states exhibit upward population drift from unskilled to skilled, roughly at the rate of difference in fertility.

  - For more, see Mookherjee, Prina and Ray (AEJ Micro 2010)

## Conditionality in Educational Subsidies

- Recall that higher $\lambda$ associated with higher net output.

- So there is a role for educational subsidies.

- Assume all subsidies funded by taxing $w_s$ at rate $\tau$.

- Unconditional subsidies: give to unskilled parents.

$$T_t = \frac{\lambda_t \tau}{1 - \lambda_t} w_s(\lambda_t).$$

- Add this to the unskilled wage: $w_u(\lambda_t) + T_t$.

- Conditional subsidies: give to all parents conditional on educating children.

$$Z_t = \frac{\lambda_t \tau}{\lambda_{t+1}} w_s(\lambda_t).$$

## Theorem.

- With unconditional subsidies, every left-edge steady state declines, lowering the proportion of skilled labor and increasing pre-tax inequality, which undoes some or all of the initial subsidy.

- With conditional subsidies, every left-edge steady state goes up, increasing the proportion of skilled labor. In steady state, no direct transfer occurs from skilled to unskilled, yet unskilled incomes go up and skilled incomes fall.

- Conditional subsidies therefore generate superior macroeconomic performance (per capita skill ratio, output and consumption) and welfare (Rawlsian or utilitarian).

- For more, see Mookherjee and Ray (Economic Record 2008)