

# Lectures on Economic Inequality

Warwick, Summer 2016, Supplement 3 to Slides 2

Debraj Ray

---

- Overview: Convergence and Divergence
- Inequality and Divergence: Technical Progress
- Inequality and Divergence: Psychological Factors
- Inequality, Polarization and Conflict
- Uneven Growth and Conflict

## The Fourth Fundamental Law of Capitalism

- In the long-run, technical progress must displace labor.
- Large literature on induced technical change:
  - Classical: Hicks (1932), Drandakis-Phelps (1965), Kennedy (1964), Salter (1966)
  - Recent: Autor-Krueger-Katz (1998), Galor-Maov (2000), Acemoglu (1998, 2002)
- Largely taxonomy of technical progress, or reactions to shocks.
- But my comments here on the ultra-long run.

## Outline of the Argument

- Labor is fixed, or grows exogenously.
- Capital is endogenously accumulated.
- In growing economy, the price of capital relative to labor falls.
- Induces technical progress that **displaces labor**.
- “**Stability argument**”: But too fast, otherwise labor will become too cheap.
- That negates the initial incentive to substitute away from labor.
- So “in equilibrium,” displacement happens **gradually**. But it must happen.

## More Detail

- For simplicity, fix the labor supply at  $\bar{L}$ .  
(Can easily have exogenous growth.)
- Output produced by capital and labor using Cobb-Douglas:

$$Y = AK^\alpha L^{1-\alpha}$$

- Capitalists accumulate own capital, hire labor at wage  $w$ :

$$\max_L AK^\alpha L^{1-\alpha} - wL$$

- Indirect **linear profit function** defined on  $K$  alone:

$$\pi(\alpha, A, w)K \equiv \alpha A^{1/\alpha} \left( \frac{1-\alpha}{w} \right)^{(1-\alpha)/\alpha} K.$$

- $w$  is going to be endogenously pinned by  $\bar{L}$  (later).

- Capitalists choose an **accumulation path**  $\{K_t\}$  to maximize

$$\sum_{t=0}^{\infty} \delta^t (\text{Dividends})_t = \sum_{t=0}^{\infty} \delta^t [\pi(\alpha, A, w_t) K_t - K_{t+1} - c(I_t)],$$

- where  $K_{t+1} = (1 - d)K_t + I_t$  and  $I_t \geq 0$ ,
- $\{w_t\}$  is fully anticipated, and
- $c$  is strictly convex with  $c(0) = c'(0) = 0$  and  $c'(\infty) = \infty$ .
- **Equilibrium condition:**  $L_t = \bar{L}$  for all  $t$ .

- **In long run steady state,**  $K_t \rightarrow K^*$  such that

$$\delta \pi(\alpha, A, w^*) = \delta \alpha A^{1/\alpha} \left( \frac{1 - \alpha}{w^*} \right)^{(1-\alpha)/\alpha} = 1.$$

- which is the **modified golden rule**, and

$$(1 - \alpha) A (K^* / \bar{L})^\alpha = w^*,$$

- which is static profit maximization.
- Now introduce some exogenous growth just as in Solow:
- $A_t < A_{t+1}$  for all  $t$ , and  $A_t \rightarrow \infty$ .
- Then “in the long run”  $w_t$  chases  $A_t$  to maintain the equality

$$\delta \alpha A_t^{1/\alpha} \left( \frac{1 - \alpha}{w_t} \right)^{(1-\alpha)/\alpha} = 1,$$

- and in particular,  $w_t \rightarrow \infty$ .

- The “chasing relationship”:

$$\delta \alpha A^{1/\alpha} \left( \frac{1-\alpha}{w(A)} \right)^{(1-\alpha)/\alpha} = 1,$$

- **Proposition.** For each  $\alpha$ , there is a threshold  $A^*(\alpha)$  such that
  - $\pi(\alpha, A, w(A))$  is decreasing in  $\alpha$  when  $A < A^*(\alpha)$ , and
  - $\pi(\alpha, A, w(A))$  is increasing in  $\alpha$  when  $A > A^*(\alpha)$ .
- **Intuition.**  $w(A)$  rises faster than  $A$ , benefiting from both technical progress and capital accumulation.

- **Implication of Proposition.**

- If  $\alpha$  endogenously chosen at some cost, the share of capital income in total income will go to 1. [Limit Robotic Economy.]
- (But not too fast . . .  $w$  still has to climb on transition path.)
- Unbounded functional inequality, can translate into unbounded personal inequality if combined with earlier arguments.