

Lectures on Economic Inequality

Warwick, Summer 2016, Supplement 1 to Slides 2

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- Overview: Convergence and Divergence
- [Inequality and Divergence: Economic Factors, Part 2](#)
- Inequality and Divergence: Psychological Factors
- Inequality, Polarization and Conflict
- Uneven Growth and Conflict

A General Model with Financial Bequests and Occupational Choice

- Why study this?
- Interplay of financial and human bequests
- No need for persistent inequality in two-occupation model
- Nonconvexities and rich occupational structure
- Now the “curvature” of occupational returns is fully endogenous.

- Production with capital and “occupations”.
- Population distribution on occupations λ (endogenous).
- Physical capital k .
- Production function $y = F(k, \lambda)$, CRS and strictly quasiconcave.
- Training cost function x on occupations:
 - incurred up front.
 - parents pay directly, or bequeath and then children pay.

Prices

- Perfect competition.
- Return on capital fixed at rate r (international k -mobility).
- Returns to occupational choice: “wage” vector $\mathbf{w} \equiv \{w(h)\}$.
- \mathbf{w} endogenous, together with r supports profit-maximization.

Households

- Continuum of households, each with one agent per generation.
- Starting wealth y ; $y = c + b + x(h)$.
- Child wealth $y' = (1 + r)b + \mathbf{w}_{t+1}(h)$.
- Parent picks (b, h) to max utility.
- No debt! $b \geq 0$.
- Child grows up; back to the same cycle.

Preferences and Equilibrium

- **Preferences:** mix of income-based and nonpaternalistic

$$U(c) + \delta[\theta V(y') + (1 - \theta)P(y')]$$

- **Equilibrium:**
 - Wages \mathbf{w}_t , value functions V_t , and occupational distributions $\boldsymbol{\lambda}_t$ such that at every date t :
 - Each family i chooses $\{h_t(i), b_t(i)\}$ optimally
 - Occupational choices $\{h_t(i)\}$ aggregate to $\boldsymbol{\lambda}_t$;
 - Firms willingly demand $\boldsymbol{\lambda}_t$ at prices (\mathbf{w}_t, r) .
 - **Note:** physical capital willingly supplied to meet any demand.

Steady State

- A **stationary equilibrium** with positive output and wages:
 - $\mathbf{w}_t = \mathbf{w} \gg 0$, and
 - $(k_t, \boldsymbol{\lambda}_t) = (k, \boldsymbol{\lambda})$ for all t , and $F(k, \boldsymbol{\lambda}) > 0$.

Divergence and History: Going Deeper

- Two notions of history-dependence.
 - Individual (household destinies depend on past events)
 - Economy-wide (multiple **distributions** of wealth)
 - Former endemic in this model. Latter is what we are after.
 - Literature usually studies a small number of occupations (two).
 - Steady-state conditions written as inequalities
 - Multiplicities are endemic (as we've seen).

Rich Occupational Structure

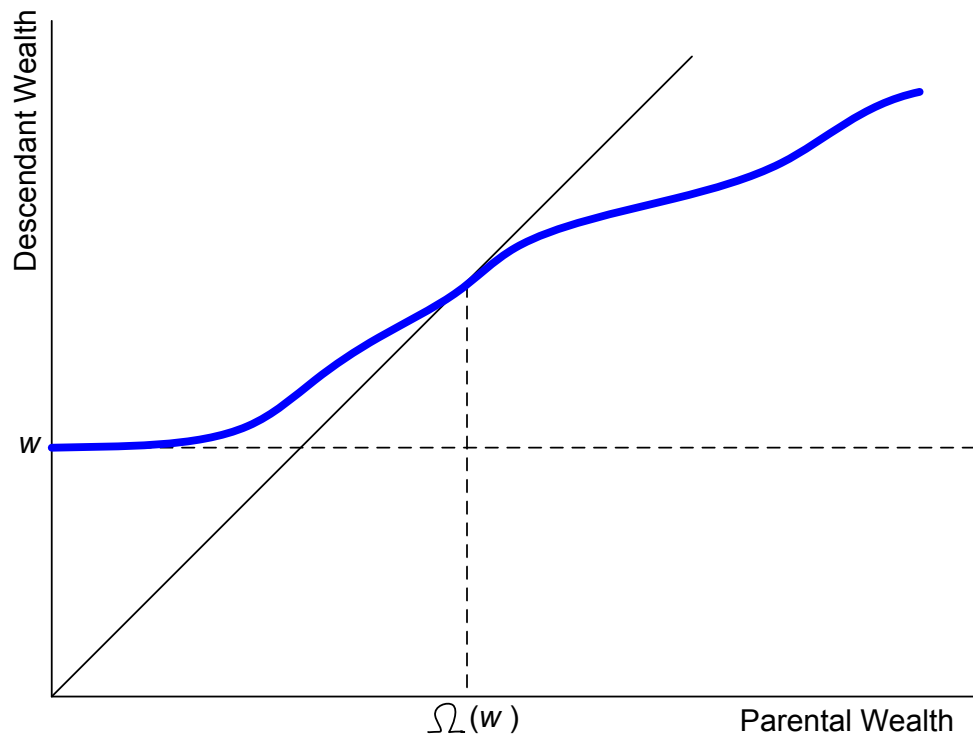
- Try the other extreme:
- The set of all training costs is a compact interval $[0, X]$.
- If λ is zero on any positive interval of training costs, then $y = 0$.
- Jointly the [richness](#) assumption [R].
- Want to investigate economy-wide history-dependence under this assumption.

A Benchmark With No Occupational Choice

- Financial bequests (at rate r) + just one occupation (wage w).
- Parent with wealth y selects $b \geq 0$ to

$$\max U(c) + \delta[\theta V(y') + (1 - \theta)P(y')].$$

- Child wealth $y' \equiv w + (1 + r)b$.
- Depends on (y, r, w) ; increasing in y .
- Limit wealth $\Omega(w, r)$: intersections with 45° line (or ∞).
- [U] $\Omega(\hat{w}, \hat{r})$ independent of initial conditions for all (\hat{w}, \hat{r}) .
- [F] $\Omega(\hat{w}, r) < \infty$ for all \hat{w} .

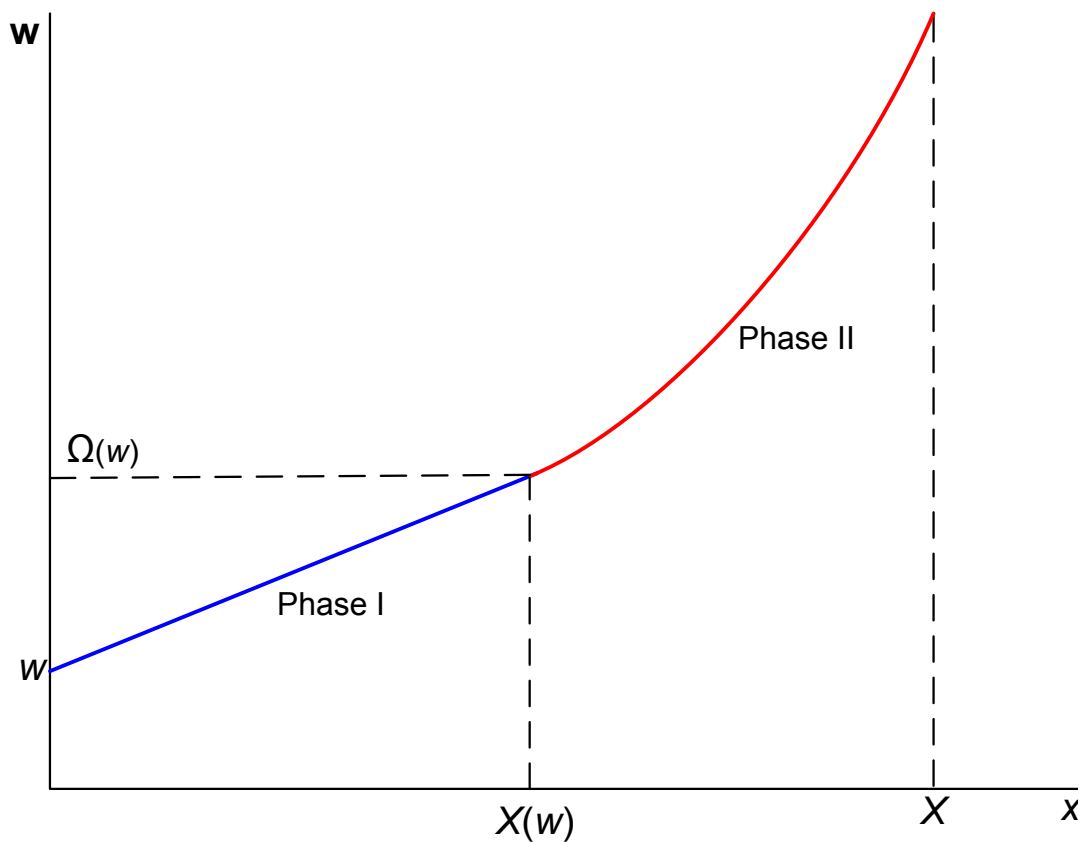


Remarks on [U] and [F]

- Related to **limited persistence** (cf. Becker and Tomes).
- [U] requires some degree of paternalism in preferences:
 - Recall $U(c) + \delta[\theta V(y') + (1 - \theta)P(y')]$
 - Need $\theta < 1$.
- Yet our results will generally extend to the dynastic case.

Back to Occupational Choice

- **Theorem.** Assume [R], [U] and [F].
- Every steady state has wage function w continuous in x .
- w is fully described by a **two-phase property**:



- In Phase I w is linear in x : there is $w > 0$ such that

$$w(x) = w + (1 + r)x \text{ for all } x \leq \theta.$$

- All families in Phase I have the same **overall** wealth $\Omega(w, r)$.

- In Phase II, w follows the differential equation

$$w'(x) = \frac{U'(w(x) - x)}{\delta[\theta U'(w(x) - x) + (1 - \theta)P'(w(x))]}$$

- with endpoint to patch with I: $w(x) = w + (1 + r)x$ at $x = X(w)$.

- Families located in Phase II will have different wealths.

$$w'(x) = \frac{U'(w(x) - x)}{\delta[\theta U'(w(x) - x) + (1 - \theta)P'(w(x))]}$$

- Note that the shape of a steady state wage function

- depends fundamentally on preferences

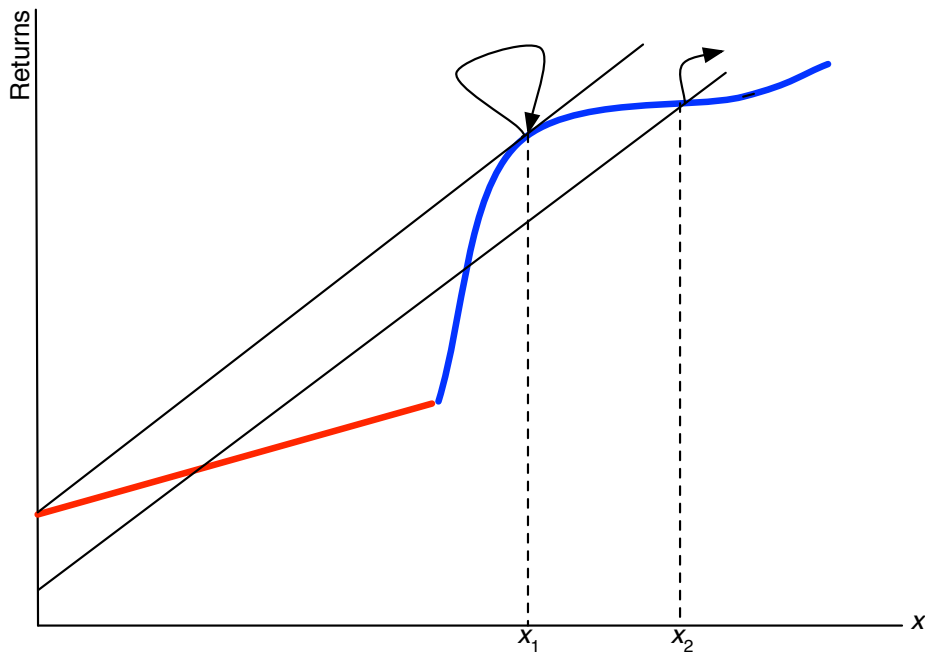
- is independent of technology apart from baseline w

- Define the **average return** to occupational investment x by

$$\rho(x) \equiv \frac{w(x) - w}{x}.$$

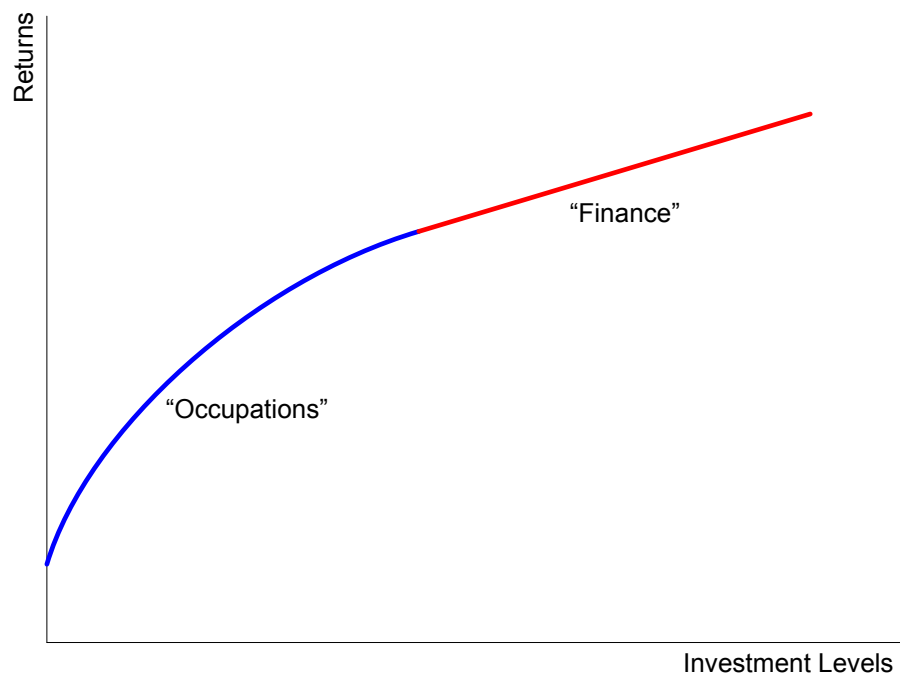
- **Theorem.** The average return to occupational investment is strictly increasing in x on $[z, X]$.

■ **Proof.** Suppose not; then:

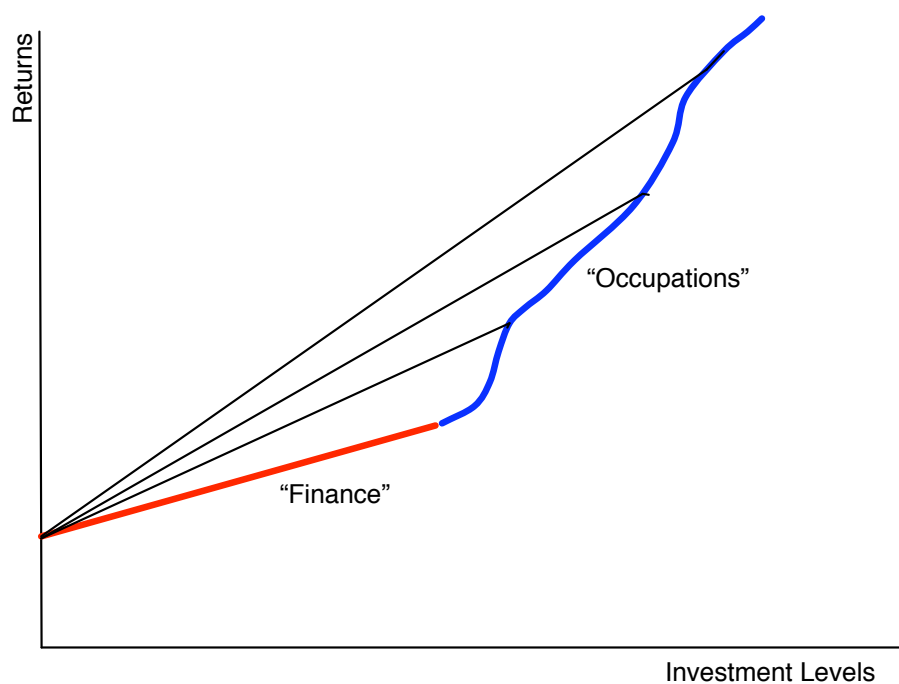


■ Contradiction to unique limit wealth in the benchmark model.

■ Theorem stands the usual literature on its head. Compare:



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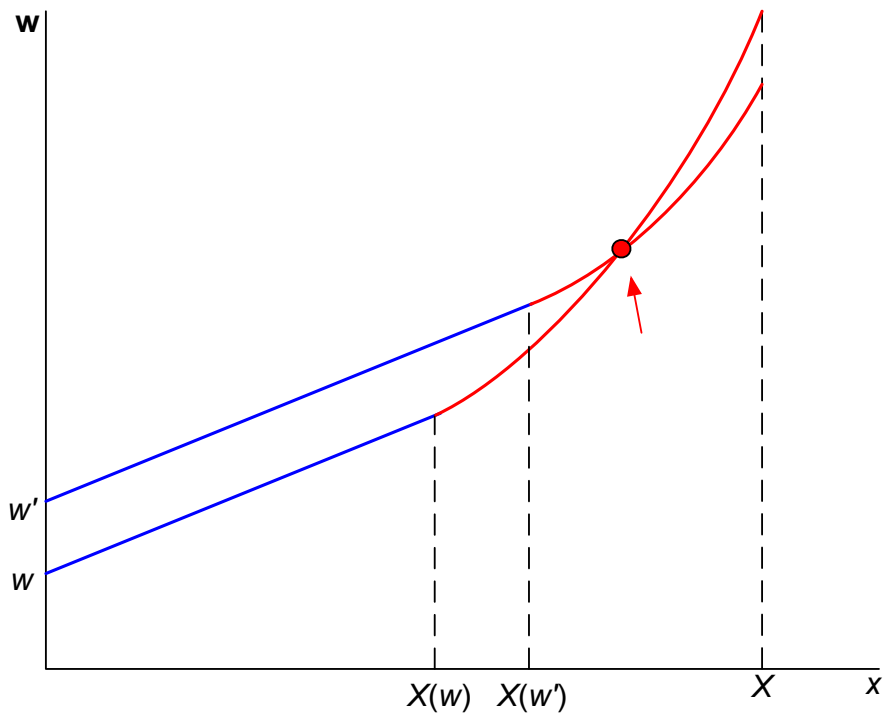
- Increasing occupational returns a (central) testable implication.

Unique Steady State with Rich Occupational Structure

- Now a fundamental difference from two-occupation case:
- **Theorem.** Assume $[R]$, $[U]$ and $[F]$. Then there is at most one steady state.
- **Proof** rests on the fact that two members of the two-phase family cannot cross.
- See succeeding slides.
- Once that is settled, then only one intercept wage is possible that supports profit maximization with positive output.
- (For all wages must climb along with intercept wage.)

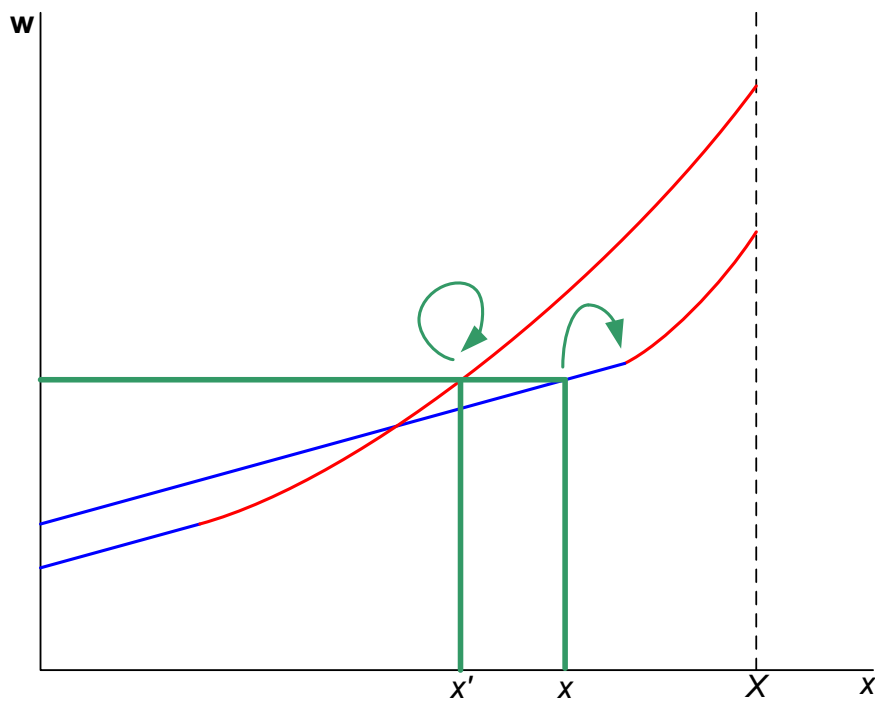
■ No-crossing argument, part I

- Theory of differential equations won't allow this:



■ No-crossing argument, part II

- Revealed preference argument rules this out:



But What About Divergence?

- In Phase I, there is perfect equality of overall wealth.
- (All families in Phase I must have wealth equal to $\Omega(w, r)$.)
- Families at different occupations in Phase II **cannot** have the same wealth.
- Thus, “most” inequality comes from nonalienable capital.

“Labor income inequality is as important or more important than **all other income sources combined** in explaining total income inequality”. [Fields (2004)]

- When is Phase II nonempty?
 - When there is a large occupation span relative to bequest motive.
- Can examine this condition for different situations/applications.
 - **Discounting**.
 - **Poverty**, via TFP differences.
 - **Growth** in TFP, lowers effective bequest motive
 - **World return on capital**.
 - **Globalization**: new occupations.

Divergence and History-Dependence

- At the macro-level, history-dependence depends on occupational richness.
- A lot of history-dependence at the individual level.
 - Individual dynasties have to occupy slots that are needed for aggregate production (or utility).
 - Recall the world-economy interpretation, with individuals as countries.
 - The distribution **as a whole** is pinned down, but not who occupies which slot.

Luck versus Markets: Philosophy of Inequality

- Two views on the evolution of inequality:
 - **Equalization**: Inequality an ongoing battle between convergence and “luck”
 - Brock-Mirman (1972), Becker-Tomes (1979, 1986), Loury (1981)...
 - **Disequalization**: Markets intrinsically create and maintain inequality
 - Ray (1990, 2006), Banerjee-Newman (1993), Galor-Zeira (1993), Ljungqvist (1993), Freeman (1996), Mookherjee-Ray (2000)...
- We’ve explored here the second view.
 - Fundamentally based on symmetry-breaking.
 - It remains to be seen if this is the right view of the world.