Development Economics

Slides 4

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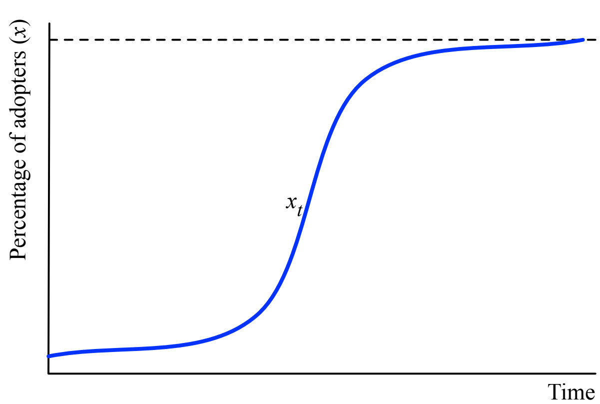
- Transitions between one equilibrium and another
- Higher-order beliefs
- Lags in adjustment
- An empirical example

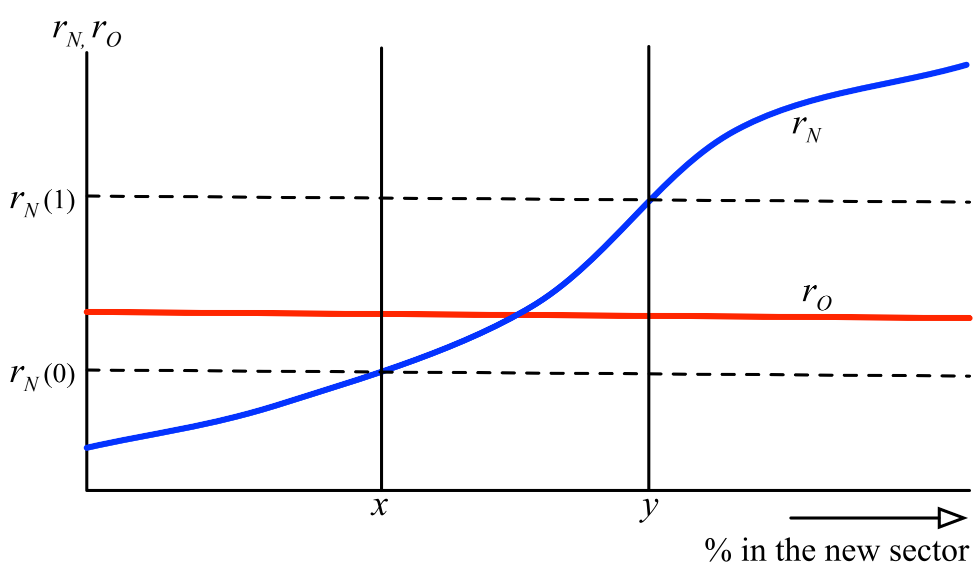
Questions of Transition

- Problem: these theories are way too nondeterministic.
- Why does "yesterday's state" affect "today's state"?
- Why is QWERTY stickier than fashion?
- Why does a transition follow a logistic path?



Canonical Multiple Equilibrium Model

- Two regions or sectors, A and B.
- Total endowment of $\bar{K}$ split between the regions:
  - K_A in A, $K_B = \bar{K} - K_A$ in A.
- Each person has one unit of endowment:
  - Deliberately chooses sector A or B.
- Capital in A has fixed rate of return, say 0.
- In B, $r = f(K)$, continuous, increasing, and $f(0) < 0 < f(\bar{K})$.



- Alternative initial conditions: $K_B = x$, $K_B = y$.
  - Most of our examples fit this canonical model well.

History Versus Expectations

- Myopic adjustment: history matters completely
- Farsighted adjustment: history doesn't matter at all.
  - Can one allow for expectations, but retain the weight of history?

Analysis of the Canonical Model

- Capital free to move but there is a switching cost:
  - From B to A: $\hat{c}_A(K_A)$.
  - From A to B and $\hat{c}_B(K_B)$.
  - Congestion in sector j if $\hat{c}_j$ is increasing.
- Path of prices $\gamma \equiv \{r(t), c_A(t), c_B(t)\}_{t=0}^{\infty}$ given for the individual.
  - Discount rate ρ.
  - Value function $V(\gamma, i, t)$.
  - Switch sectors at time t if $V(\gamma, i, t) < V(\gamma, j, t) - c_j(t)$.
- Equilibrium price path: γ generated by optimal decisions of agents in response to γ.

- Generating γ.

- Fix a path of capital allocation $\{K_A(t), K_B(t)\}_{t=0}^{\infty}$.

- Assume that at date 0, $r(0)$ is precisely $f(K_B(0))$.

- Thereafter: increasing function g, with $g(0)=0$, such that

$$\dot{r}(t) = g(f(K_B(t)) - r(t)).$$

- That is, $r(t)$ "chases" the "appropriate" rate at every date.

- The steeper is g, the faster the adjustment.

- Finally, $c_A(t) = \hat{c}_A(K_A(t))$ and $c_B(t) = \hat{c}_B(K(t))$ for all t.

- This is how $\{K_A(t), K_B(t)\}$ generates $\gamma(t) = \{r(t), c_A(t), c_B(t)\}$.

- If g is "infinitely steep," then multiplicity of equilibrium paths.

- In contrast, an equilibrium is exclusively history dependent there is migration only to the sector that is initially profitable.

- Theorem. Assume $f(K(0)) \neq 0$. Barring congestion, every equilibrium must be exclusively history dependent.

- Outline of proof. Assume (wlog) $f(K(0)) < 0$.

- Claim. $K(t) \leq K(s)$ for all (s,t) with $t \geq s$.

- Suppose not, then there are t and s with $t > s$, such that:

a. $K(t) > K(s)$.

b. $r(\tau) < 0$ for all $\tau \in [s,t]$.

c. Some agent moves to sector B at date s.

- That agent should postpone her move, unless congestion.

- Implications of the lagged externalities model:

  - Independent of the magnitude of discounting
  - Independent of how quickly returns adjust, as long as not instantaneous.
  - Why congestion matters.
  - Back to question: why is QWERTY different from fashion?
  - Economic mavericks: those who don't mind making losses.

Global Games and Equilibrium Transition

- Carlsson and Van Damme (1993), Morris and Shin (1998)
- Coordination game indexed by a state variable.
- Sometimes multiple equilibrium, and sometimes not.
- State variable observed publicly but with a bit of individual noise.

- Example: Pegged exchange rate e (overvalued)

  - Fundamentals: $f(\theta)$, with $e > f(\theta)$.
  - θ is the state (interest rate, oil price).
  - Arrange so that $f(\theta)$ increasing. High θ is good state.

- Speculators (of total measure 1).
  - Each can sell one unit of the currency; transactions cost t.
  - If peg holds, payoff is $-t$.
  - If peg abandoned, payoff is $e - f(\theta) - t$.
- Government has only one decision: abandon or retain peg.
  - Abandons if attacks exceed $a(\theta) < 1$, increasing function.
- End-points.
  - There is $\underline{\theta}$ such that $a(\theta) = 0$ for $\theta \in [0, \underline{\theta}]$.
  - Then $a(\theta)$ rises but always stays less than one by assumption.
  - There is $\bar{\theta}$ s.t. $e - f(\bar{\theta}) - t = 0$ (no one wants to sell at $\theta > \bar{\theta}$).
  - $\underline{\theta} < \bar{\theta}$.

- Benchmark: assume that θ perfectly observed.

- Case 1. $\theta \leq \underline{\theta}$. Abandon. Everyone sells; currency crisis.

- Case 2. $\theta \geq \bar{\theta}$. No speculator attacks; peg holds.

- Case 3. $\underline{\theta} < \theta < \bar{\theta}$. Multiple equilibria.
  - One equilibrium: no one attacks, peg holds.
  - One equilibrium: everyone attacks, peg abandoned.
  - Prototype of the second-generation financial crises model, in which expectations — over and above fundamentals — play an important role (see Obstfeld (1994, 1996)).

- Now we drop common knowledge of realizations of θ.

- How to model higher-order beliefs

- Say θ uniform on $[0,1]$ (with $0 < \underline{\theta} < \bar{\theta} < 1$).

- Each individual sees x uniform on $[\theta - \epsilon, \theta + \epsilon]$.

- This additional noise is iid across agents.

- Theorem. There is a unique value of the signal x such that an agent attacks the currency if $x < x^*$ and does not attack if $x > x^*$.

- Extraordinary result: a tiny amount of noise refines the equilibrium map considerably.

- As $\epsilon \to 0$, we're practically at common knowledge limit, yet no multiplicity zone.

- An "infection argument" central to the proof.

- Proof of the theorem.

- Suppose that someone receives a signal $x \leq x_0 \equiv \underline{\theta} - \epsilon$.

- $\Rightarrow$ true state *cannot* exceed $\underline{\theta}$.

- So the government abandons peg for sure. Therefore sell.

- Now look at x slightly bigger than x_0.

- True θ must lie in $[x - \epsilon, x + \epsilon]$.

- For each such θ, Prob(others' $x' \leq x_0$) is $(1/2\epsilon)[x_0 - (\theta - \epsilon)]$.

- Integrating over θ, Prob(others' $x' \leq x_0 | x$) $= (1/2\epsilon)[x_0 - (x - \epsilon)]$.

- So even under best-case scenario that only sub-x_0's sell, half expected to sell.

- And $a(\theta)$ is low, so x-type will sell: the infection spreads.

- So proceed recursively. Suppose that for some index n:

- "Everyone sells if $x \leq x_n$." (Already know this for x_0).

- Let x_{n+1} be the largest value of the signal for which people will want to sell, knowing that all below x_n are selling.

- Calculate x_{n+1}.

- For any θ, government will yield (under the x_n-presumption) if

$$\frac{1}{2\epsilon}[x_n - (\theta - \epsilon)] \geq \alpha(\theta), \text{ or } \theta + 2\epsilon a(\theta) \leq x_n + \epsilon$$

- Rewrite above inequality as

$$\theta \leq h(x_n, \epsilon).$$

- That is, $h(x, \epsilon)$ solves

$$h(x, \epsilon) + 2\epsilon a(h(x, \epsilon)) = x + \epsilon.$$

- So if signal is x and our person attacks, expected payoff is

$$\frac{1}{2\epsilon}\int_{x-\epsilon}^{h(x_n,\epsilon)}[e - f(\theta)]d\theta - t.$$

- x_{n+1} defined by value of x such that above expression is 0.

- Start the recursion at $x_0 = \underline{\theta} - \epsilon$ and keep going.

- We've already seen that $x_1 > x_0$.

- Because h is increasing in x, recursion creates a strictly increasing sequence $\{x_n\}$, which converges up to x^*, where

$$\frac{1}{2\epsilon}\int_{x^*-\epsilon}^{h(x^*,\epsilon)}[e - f(\theta)]d\theta - t = 0.$$

- Solutions to this equality? Just one. In fact, can prove something stronger.

- Claim. If $x' > x^*$, then

$$\frac{1}{2\epsilon}\int_{x'-\epsilon}^{h(x',\epsilon)}[e-f(\theta)]d\theta - t < 0.$$

- Proof. Consider any $x' > x^*$.

- Then two things happen: first:

$$h(x',\epsilon) - x' < h(x^*,\epsilon) - x^*,$$

- so that the support over integral above narrows.

- In addition, the stuff inside the integral is also smaller when we move from x^* to x', because $f(\theta)$ is increasing. Claim proved.

- So far: every equilibrium involves attack when $x < x^*$.

- Claim. No signal above x^* can ever attack.

- Proof. Suppose not.

- Then some $x > x^*$ finds it profitable to attack.

- Take the sup of all such signals; call it x'.

- At x' it is weakly profitable to attack.

- If we pretend that everybody below x' attacks; this cannot affect the weak profitability of attack at x'. But the profit is

$$\frac{1}{2\epsilon}\int_{x'-\epsilon}^{h(x',\epsilon)}[e-f(\theta)]d\theta - t,$$

which is negative as per previous Claim. Contradiction!

- Summary. There is a unique equilibrium in the "perturbed" game, in which a speculative attack is carried out by an individual if and only if $x \leq x^*$.

- Take $\epsilon \to 0$ and calculate limit x^*:

$$\begin{aligned}[e - f(h(x^*,\epsilon))][1 - a(h(x^*,\epsilon)] &\leq \frac{1}{2\epsilon}\int_{x^*-\epsilon}^{h(x^*,\epsilon)}[e - f(\theta)]d\theta \\ &\leq [e - f(x^* - \epsilon))][1 - a(h(x^*,\epsilon)]\end{aligned}$$

- noting that $f(x^* - \epsilon) \leq f(\theta) \leq f(h(x^*,\epsilon))$ for all $\theta \in [x^* - \epsilon, h(x^*,\epsilon)]$.

- x^* and $h(x^*,\epsilon)$ go to a common limit, call it θ^*. This solves:

$$[e - f(\theta^*)][1 - a(\theta^*)] = t.$$

- Note: Careful when reading Morris-Shin; error in Theorem 2.

- See Heinemann (AER 2000).