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Problem Set 5

[1] Countries that have a high international debt face what is called the *debt overhang*. The prospect of having to service a large debt from investment returns discourages investment. In such situations, some amount of debt forgiveness can help. To see this, consider the following example:

A country needs to make $2 billion of service payments on its debt. To do so, its government can attempt to raise additional resources, say by extra taxation or reduced expenditures, in order to make investments of $10 billion dollars. Only an investment with a net return of 10% or more (to the government) will be undertaken. These investments pay off $2.5 billion dollars in additional revenue: a 25% return.

(a) Show that if debt service payments have to be made from the returns, these investments will not be undertaken.

(b) Show that a certain amount of forgiveness (or postponement) of the debt service will make both the government and the creditors better off. Calculate the minimum amount of forgiveness that's necessary.

(c) Suppose a whole range of investments (at different levels) are available. Creditors might forgive some of the debt in order to induce these investments (as above) or might participate more actively in the investments themselves, asking for a fixed percentage of the returns as debt service. That is, debt is converted into equity. Compare these two options.

[2] "Conditionality" refers to the specification of certain policies that a government must follow in order to receive loans or assistance from international organizations such as the International Monetary Fund or the World Bank. Here is an example to illustrate one useful aspect of conditionality.

A loan of $10 billion is to be given to a government. Just as in the previous question, the loan may be used in investments, and the government will undertake investments if they pay off at least 10%, but not otherwise. Suppose that the ongoing debt service is $2 billion (and that debt service can only be done if these investments are made). The organization making the loan is happy with a 3% return (especially if the loan helps to service the debt).

(a) Show that if the loan is made without any conditions, it will not be used in investments but consumed, and therefore that the loan will not be made in the first place. In contrast, show that an appropriate commitment from the government in return for the loan will make the international organization, the borrowing government, and the private creditors all better off.

(b) Modify this example to create a case in which *both* conditionality and some debt forgiveness is required to create a satisfactory solution.

[3] Consider the debt-overhang problem studied in class and described in Ghosh, Mookherjee and Ray [2000]. There we showed that if $w < L$ and if p is increasing *and strictly concave*, then equilibrium effort is lower than first best. Suppose I made the weaker assumption instead that p is strictly increasing and that there is a unique solution to the borrower's effort choice problem for every value of collateral and repayment. Prove that this is enough to show that equilibrium effort must be less than first best.

[4] Collateral and Debt Overhang. Similar (but not identical) to the model studied in class.

We study a case in which collateral is more profitable to the borrower than to the seller. Suppose that a borrower has a divisible asset whose total per-period flow value equals A. Assume that the value to the lender equals βA, where $0 < \beta < 1$.

The borrower has income today of w_0, and anticipated income tomorrow of $w_1 > w_0$. But the latter occurs with probability p; with probability $1 - p$ his income is zero. At the moment assume that p is exogenous.

The borrower has a strictly concave utility function u, and both borrower and lender have a common discount factor δ. The only reason loans are taken in this example is to smooth consumption over time. If the borrower borrows B, needs to repay R ($< w_1$), and puts up an amount C as collateral, his net two-period utility is

$$W \equiv u(w_0 + B + A) + \delta[pu(w_1 - R + A) + (1 - p)u(A - C)],$$

while the lender's utility is

$$\Pi \equiv -B + \delta[pR + (1 - p)\beta C].$$

(a) First, think about the effects of increasing collateral. There is one that makes the use of collateral favorable, and another that makes it unfavorable. What are they? Having thought about this, work out the "competitive solution" to the problem above: maximize W subject to $\Pi = 0$, for some *given* C. What are the features of this solution so far as consumption smoothing goes? Now what happens to this maximum as W changes? Can you provide conditions under which $dW/dC < 0$?

(b) Now introduce the debt overhang as we did in class. That is, in period 1 assume that p depends on effort put in by the borrower. Show that collateral now acquires a new function in addition to the features observed in part (a). Describe this as precisely as you can.

[5] We review the model of credit with limited enforcement that we studied in class. A *borrower* takes loans L as working capital; these are converted to output by means of a production function $F(L)$ satisfying standard assumptions. A *lender* advances L and specifies a repayment R: the implicit rate of interest on the loan is then $(R/L) - 1$.

Assume that the borrower cannot be asked to repay more than the total output produced from the loan; thus a *contract* is a pair (L, R) with $R \leq F(L)$. The borrower's payoff under a contract is $F(L) - R$, while the lender's payoff is $R - (1 + r)L$, where r represents the opportunity interest rate per unit of funds advanced.

Both borrower and lender have a common discount factor $\delta \in (0, 1)$. At any date the borrower can default by not repaying the loan. Let $v > 0$ be the (normalized) present value of the borrower's outside option if he defaults.

(a) Assume that the same contract is offered period after period. It is *incentive compatible* if the lender gets nonnegative return and the borrower always repays. Prove that an incentive-compatible contract exists *if and only if* the following maximization problem

$$\max_{x \geq 0}[F(x) - \frac{1+r}{\delta}x]$$

has a value that's at least as large as v.

(b) For each contract (R, L), define the concepts of *credit rationing* and *loan pushing*. Paying careful attention to the relative bargaining power of lender and borrower, discuss when equilibrium incentive-compatible contracts are likely to satisfy one property or the other.

(c) [extension] We can easily extend the definition of incentive-compatibility for a *sequence* of contracts which might vary over time. Do so. Show by means of an example that it is possible to Pareto-dominate every stationary incentive-compatible contract by means of a (nonstationary) sequence of incentive-compatible contracts.

[6] Look at the limited enforcement model studied in class and introduce the following twist. Suppose that with probability p, a borrower is an "intrinsic defaulter": that is, he has a discount factor of zero. [With probability $1 - p$, he is a "normal" borrower with a discount factor δ.] An individual lender does not know whether the borrower is an intrinsic defaulter or not, but can try to find out by offering a "testing loan". Clearly any *one* loan will reveal the true characteristics of the borrower (why?).

(a) Assume that all normal borrowers have an outside opportunity of v per period (the outside opportunities of the intrinsic defaulters are irrelevant because they will default anyway). Assume further that the lender has all the bargaining power. Solve for the optimal (incentive constrained) testing loan in period 1 and the optimal stationary contract thereafter.

(b) Informally discuss what you would expect to see if borrowers had not just two possible discount factors but a whole array of them.

[7] Here is a variant on what we did (or will do) in class for the insurance model.

Consider a society with a large number of people, indexed on $[0, 1]$. Each person produces output 0 or 1, the "good" value occuring with probability p (i.i.d. across individuals). Because there is a large number of people, aggegate output in this economy will be p for sure. An *insurance scheme* prescribes a transfer t for those who have produced the good output, the aggregate transfer being divided among the have-nots in that period. Each person has a strictly concave utility function u defined on consumption. The society seeks to maximize the sum of individual utilities.

(i) Calculate the optimal insurance scheme.

Now suppose that individuals can refuse to pay their share of the transfer in the event of producing a good output. In that case we assume that society ostracizes them, so that they cannot participate in the scheme from the next period onwards. Assume a common discount factor δ, and suppose that each person seeks to maximize the expected value of the discounted sum of utilities.

(ii) Find the threshold discount factor that permits the optimal insurance scheme of part (i) to be supported as an equilibrium in this repeated situation.

(iii) If this is not supportable, describe an approach to the optimal *stationary* second-best scheme, using the methods taught in class.

[8] Consider a village economy with a large number of identical individuals. Suppose that each farmer can produce one of two levels of a *nonstorable* output: H and L, with $H > L$. Let the probability of H be given by $p \in (0, 1)$. Assume that realizations are all independent across farmers, so that you can take village level output to be deterministic at $pH + (1-p)L$ (think of farmers as being on the continuum $[0, 1]$). Finally, assume that each farmer has a von-Neumann utility function u described on consumption (c), which displays risk aversion.

An (static) *insurance contract* is a function that assigns payments to a village insurance fund, depending on produced output, such that the integral over all payments sums to zero.

[i] If the whole village can commit to a contract, describe the best insurance contract.

[ii] Suppose that each farmer needs to exert an unobservable effort of at least E to ensure that the high output is produced with probability p. Otherwise the probability is q, with $q < p$. [No other probability values are involved.] The cost E is simply subtracted from utility (so that the utility function is now $u(c) - E$). Now find the best contract compatible with exerting an effort of E. Is this necessarily *the* best contract?

[9] Consider the following employer-employee relationship, and keep in mind throughout the relationship with the credit market model with limited enforcement. At each date an employer demands a certain number of hours of work h from an employee, and pays out a total income of w. The employee suffers disutility $e(h)$ from complying with this request; assume $e(h)$ satisfies standard assumptions. Write the employee's (current) payoff as

$$w - e(h).$$

Suppose that h hours of work produces h units of monetary output. Then the employer's (current) payoff is

$$h - w.$$

Intertemporal payoffs are obtained by summing these current payoffs using a common discount factor of δ.

Now, at any date, the employee is free to enjoy his current income but shirk his duties. Assume that shirking is detected at the end of the period. Denote the per-period equivalent of the punishment payoff thereafter by v.

Finally, suppose that at each date, the employer must make some nonnegative "present-value profit" from his dealings with the employee.

Study the (stationary) second-best equilibria of this model, and discuss whether it involves "hours restrictions" or "overworking" relative to the equilibrium wage offered. Once again, keep the parallel with credit in mind.

[10] Consider a different employer-employee relationship. Suppose that there are two seasons of equal length in every year: a slack season with equilibrium wage w_* and a peak season with

equilibrium wage w^*. Assume $w^* > w_*$. An infinitely-lived laborer has strictly concave utility function u and an inter-season discount factor of δ.

(i) Assuming that wages cannot be saved, write down the laborer's lifetime utility from the above sequence.

Now suppose that a landlord-employer with a linear payoff function offers the laborer a contract (x_*, x^*), which is a vector of slack and peak payments.

(ii) Even assuming that the offer is made in the slack season, there are still two constraints that must be respected for the laborer to accept and (later) honor this contract. What are they? [Assume that in the event of noncompliance the laborer is thrown into the spot market and never gets a special contract again.]

(iii) Use the constraints in part (ii) to prove that a mutually profitable contract exists if and only if

$$\delta^2 u'(w_*) > u'(w^*). \tag{1}$$

Notice that "fluctuation-aversion" — which is just $u'(w_*) > u'(w^*)$ would be enough to guarantee a mutually profitable contract had there not been enforcement constraints (in fact, it would have been enough to guarantee full smoothing of consumption). If (1) fails, why isn't *any* smoothing — however small — profitable?