Development Economics Fall 2002

Problem Set 3

[1] Make sure you understand why a stochastic *noninteractive* model of intertemporal bequests cannot explain why two different steady state distributions of income *with overlapping supports* can coexist in the same model.

[2] In class we assumed that the credit market is missing so that all investments must be made upfront. Let us explore in a very elementary way an imperfect credit market rather than a missing credit market.

Consider a country with only two occupations. You can work as a laborer or you can become an entrepreneur, who hires labor and makes profits. To become an entrepreneur, you need a loan of $20,000. With this money you can set up a factory that hires 10 workers, to each of whom you must pay an income of w over the year. Together, they produce for you an output of $30,000. At the end of the year, you must sell your factory (for $20,000) and repay the loan. The rate of interest is 10% per year.

[i] Suppose you ran away instead. Imagine that you would be caught, fined $5000, and that 20% of your business profits would be seized. You would also lose whatever collateral you put up with the bank (plus interest), but you would get to keep the $20,000 (plus interest that you also owe). Find a formula that describes how much collateral the bank would ask for before it would advance you a loan. Examine this required collateral for different levels of the wage income: $w = 1000$, $w = 2000$, $w = 2,500$. Does the required collateral go up or down with the wage? Explain your answer.

[ii] Let's suppose that the minimum wage is fixed by law at $500 per annum. Find the collateral required to get a loan if w is at the minimum wage. Consider the following statement: "If more than x% of the people are to be unable to put up this collateral, then some people would be unable to get employment, whether as laborers or entrepreneurs."

Calculate x.

[iii] Using a model suggested by this example, show that if there are only two occupations, the equilibrium wage rate in this society will be determined by the distribution of initial wealth.

[iv] If a given fraction of total consumable assets plus income is bequeathed, prove that there might be multiple steady states in your model. Go back to question [1]. Any problem with multiplicity and overlapping supports here?

[3] Here is a variant on [2]. Now we shut down the credit market altogether but allow for a more interesting production structure. There is a continuum of individuals, indexed on $[0, 1]$. We start with a distribution of wealth on $[0, 1]$. Individuals must decide to become entrepreneurs or workers.

If the former, they must make an *upfront investment* of I from their own resources, whereupon they can open a firm. *There are no capital markets by assumption, so this is* *possible only if their inheritance exceeds I.* Output is a strictly concave function $f(L)$ of labor input L. The net income of entrepreneurs is the (variable) profit of their firm, plus leftover inheritance, if any, after their investment. *Of this, a given fraction is consumed, and the rest bequeathed to a single successor.*

If the latter, no investment need be made. Workers enter the labor market. Each worker supplies inelastically one unit of labor, and receives a wage. They consume the same given fraction of their wage plus inheritance (if any), and bequeath the rest.

There is no uncertainty.

(i) Set up this model formally. Be sure to include the following details: (a) for each set of choices in each period, describe the market clearing wage, (b) describe in each period the equilbrium occupational choices given a wage, (c) combine (a) and (b) to get the equilibrium in each period, and (d) along the equilibrium, describe the evolution of the inheritance distribution over time.

(ii) Use your model to show the interaction between distributional effects and aggregate outcomes (such as wages, occupational choices, aggregate income). You may restrict yourself to steady state inheritance distributions. Describe how the absence of capital markets drives this kind of interaction by discussing (in words, if you like) how such interactions would disappear if investment could be financed by a perfect capital market.

[3] Now consider the model of inequality and development studied in class. I want you to revisit the case in which $\mathcal{H}$, the set of occupations, is the entire interval $[0, 1]$. Fix any steady state, and assume that all occupations are indeed occupied (that is, there is a strictly positive density over $[0, 1]$ describing the occupational distribution). Let w be the wage function and x be the cost function in that steady state. Assume these are both differentiable. Arrange the occupations such that $x(h)$ is increasing.

(i) Prove that if we have a steady state, then for every occupation h, the expression

$$u\left(w(h) - x(h')\right) + \frac{\delta}{1-\delta} u\left(w(h') - x(h')\right)$$

(viewed as a function of h') must be maximized at $h' = h$. Interpret this in words.

(ii) Prove that a necessary and sufficient condition for the maximization problem in part (i) to work is that

$$w'(h) = \frac{1}{\delta} x'(h) \tag{1}$$

for every occupation h (where w' and x' denote derivatives). [Hint: For necessity write down the foc. For sufficiency show that the soc holds whenever the foc holds — even though the relevant function may not be concave.]

(iii) Notice that the expression (1) in part (ii) pins down a steady state condition that is independent of technology, training technology, or the utility functions, as long as all the occupations are occupied? Try and intuitively explain why.

(iv) Using the expression (1), and remembering that occupations are arranged so that $x(h)$ is increasing, prove that a steady state *must* contain inequality; i.e., prove that $w(h) - x(h)$ is an increasing function of h.

(v) Now we pin down the steady state even further (note that (1) gives us a differential equation without an initial condition). To do this, we specify a training technology. Let us suppose that to go from $h - dh$ to h needs $\alpha(h)$ teachers of level h: this corresponds to the training cost function

$$x(h) = \int_0^h \alpha(h')w(h')dh'. \tag{2}$$

Using (1) and (2), prove that the wage function equals

$$w(h) = w(0) \exp\left(\int_0^h \frac{\alpha(h')}{\delta} dh'\right). \tag{3}$$

(vi) To go the final step, we need to specify the production technology. Suppose that it is Cobb-Douglas:

$$\log Y = \int_0^1 a(h) \log l(h) dh \tag{4}$$

where $\int_0^H a(h) = 1$ (this is CRS). Using the labor demand equation

$$w(h) = \frac{a(h)Y}{l(h)}$$

for all h, prove that the following equation must hold:

$$\int_0^1 a(h) \log w(h) dh = \int_0^1 a(h) \log a(h) dh. \tag{5}$$

(vii) Equations (3) and (5) together guarantee that there is a *unique steady state*. Why? Also, explain why we don't get uniqueness of steady states in the two-occupation model. What is the main difference?

[4] Now go to the two-occupation model studied in class. Assume that the skilled occupation costs a fixed amount x, to acquire, while the unskilled profession costs nothing. Recall the discussion of Pareto-efficiency.

(i) Prove that if a steady state has the property that $w(2) - w(1) > x/\delta$, then such a steady state cannot be Pareto-efficient.

(ii) Assume the Mookherjee-Ray result that if the opposite inequality holds in (i), then such a steady state must be Pareto-efficient (though give some thought to trying to prove it for this two-occupation case). Using this and part (i) prove that there is a continuum of efficient steady states and a continuum of inefficient steady states.

(iii) Recall that in this model, net consumption always rises with λ (the proportion of skilled labor) in the steady state. That means that people in higher-λ steady states all have uniformly higher lifetime consumption (and therefore utility). Why doesn't this contradict what you were asked to prove in part (ii)?

[5] Suppose that a production function uses three types of labor in production. Type 1 needs no cost to train, type 2 needs x units of the final good, and type 3 needs z units of the final good, where $z > x > 0$.

[i] Following the two-occupation analysis done in class, write down a characterization of the set of steady states in which all occupations are occupied.

[ii] Prove that there are steady states such that

$$\frac{w(2) - w(1)}{x} \neq \frac{w(3) - w(2)}{z - x}.$$

[ii] In the previous question we showed that a comparison of $[w(2)-w(1)]/x$ with the discount factor is the appropriate check for Pareto-optimality in the two-occupation case. Show that the above condition yields Pareto-inefficiency regardless of any comparison with the discount factor.

[6] Suppose that there were n occupations, with training costs $0 = x_1 < x_2 < \ldots < x_n$ and a production function based on these n occupations. Show that a steady state characterization (in which all occupations are occupied) can be achieved by simply checking that the holders of these occupations don't want to slide *just one occupation up or down.*

[7] There are n individuals on a family farm, and they produce output with joint effort. Each person supplies one unit of labor time inelastically, but the effective units of labor supplied by i is $\lambda(c_i)$, where c_i is that person's consumption. Make the following assumptions on λ. Assume it is flat at zero for all $c \leq \hat{c} > 0$, and thereafter follows a smooth strictly concave shape. This λ is called the *capacity curve* in the nutrition-efficiency literature. (You don't need to know anything more about that literature to answer this question.)

Total output is just $f\left(\sum_i \lambda(c_i)\right)$, where f is a standard strictly concave function, so that we have the constraint

$$f\left(\sum_i \lambda(c_i)\right) \geq \sum_i c_i.$$

The farm seeks to maximize

$$\sum_i u(c_i),$$

where u (identical across everyone) is a standard concave utility function.

Assume that the capacity curve has the shape discussed in class: zero up to a threshold, (locally) strictly concave and increasing thereafter.

(a) Prove that at the optimal solution, there can not be two *strictly positive, distinct* levels of consumption assigned to two family members.

(b) In the light of (a), reformulate the problem fully as one of choosing a consumption level c and a number of people m so as to maximize $\ldots$, subject to $\ldots$. Try and find conditions under which m must be less than n.

(c) Part (b) proves that under some conditions, unequal division may be optimal (given the welfare function). But it does not show that equal division of consumption is necessarily *Pareto-dominated*. Examine whether this is possible.