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Economic Development, Fall 2002

Problem Set 2

P.S. Some of these problems are not easy. But don't be disheartened; do what you can because you are exploring new research papers and ideas. For instanec, the last two problems are not typical exam questions for this particular course (because I am asking you read and explore variations ona paper that I have not taught in class).

[1] This exercise studies the breakdown of traditional societies. Suppose there are two societies, a "traditional society" and a "market society". There is a continuum of agents uniformly on $[0, 1]$, each of whom decide at the beginning of any date to be "traditional" or "market" members. Each individual has a transaction need at each date (which is met by traditional or market sources depending on his membership at that date).

In each society, the probability that a particular date's need will be met is an increasing function $p(s)$ of the total size s (membership) of that society. Assume p is given by the simple form $p(s) = s$ in each society.

If the need is met in the market, then the individual receives a payoff of M at that date. If it is met in the traditional society, the payoff is T. [If a need is not met in the chosen market, the payoff is zero.] Because of greater trust in the traditional society, $T > M$, but the deal can be broken giving a one-time payoff of D to the individual, where $D > T$. If this happens, the individual is excluded forever from the traditional society and joins the market.

In the market there is no possibility of breaching a transaction.

Assume that agent i has a discount factor $\delta(i)$. We arrange people so that people with a lower index are more patient, and we assume that $\lim_{i\to 0} \delta(i) = 1$ while $\lim_{i\to 1} \delta(i) = 0$.

(a) Suppose that the number of people in each sector is constant over time; say at s for the traditional sector and $1 - s$ for the market. Give the necessary and sufficient condition for person i to stay in the traditional sector without exclusion. Use this to argue that if person i is in the traditional sector, then so is j for $j < i$.

(b) Use part (a) to construct a mapping which has s — the expected size of the traditional sector — on the horizontal axis, and the fraction of the population that *can credibly want to be* (without exclusion) in the traditional sector as a "best response" to s. Argue that this mapping represents a complementarity. Describe this complementarity precisely and explain the various sources of the complementarity (in words).

(c) Prove that it cannot be an equilibrium for the market to shut down entirely. Prove that it can be an equilibrium for the traditional sector to shut down entirely. Explain the asymmetry in words.

(d) Provide (and interpret) conditions under which the traditional sector is (partially) active in at least one equilibrium.

[2] Consider the following two-player game:

	L	R
U	$a+\theta, a+\theta$	$0, 0$
D	$0, 0$	$b-\theta, b-\theta$

where a and b both lie strictly between 0 and 1, and θ is a random variable distributed uniformly on $[-1, 1]$. Use the Carlsson-van Damme / Morris-Shin construction: θ is observed with some uniform noise on $[\theta - \epsilon, \theta + \epsilon]$, where ϵ is a tiny positive number. The noise is iid across the two players.

[a] Solve the equilibrium strategy of the perturbed game using the techniques studied in class. Find the limit value of the switch point θ^* as $\epsilon \to 0$ and evaluate this limit relative to the values of a and b.

By the way, take special note of this: in the Morris-Shin world, a lot of the argument works because of reasoning like this: player i thinks that player j thinks that player k thinks that ... But in this model there are only two players! Explain why the above sort of reasoning still matters.

[b] Apply the same logic to the game

	L	R
U	θ, θ	$\theta, 0$
D	$0, \theta$	$4, 4$

where θ is a random variable on some interval $[\underline{\theta}, \bar{\theta}]$, with $\underline{\theta} < 0$ and $\bar{\theta} > 4$. Is it true that (as $\epsilon > 0$) the critical switch point involves the Pareto-dominant equilibrium being played? Is this in contrast to the game of part [a], and why or why not?

[3] This is not so much a problem, but a plea for you to read a substantial variant on the Morris-Shin idea, which is Frankel and Pauzner (QJE 2001). In that model, there are two sectors, A and B. In Sector A the unit return is normalized to zero. In Sector B the return depends on the number of units of capital K in that sector (as in Adserà-Ray) but also on some exogenous state variable z. Write this as $f(K, z)$. Assume that f is continuous and increasing in both arguments.

If you think of z moving in continuous time, think of it as following a stochastic process which can move up or down with equal probability at each instant (an example is Brownian motion with no drift).

As before, there is a continuum of individuals with one unit of capital each (the total being $\bar{K}$). Assume they can migrate back and forth costlessly between the two sectors. (We will return to this assumption later and change it a bit.) Let ρ be the (common) discount rate of all the agents.

[a] Explain carefully the meaning of the following two assumptions: there exist $\bar{z}$ and $\underline{z}$ in the support of the random variable z such that

$$\mathbb{E}_{\{z_t\}} \left[\int_{t=0}^{\infty} e^{-\rho t} f(0, z_t) | z_0 = \bar{z} \right] \geq 0 \tag{1}$$

and

$$\mathbb{E}_{\{z_t\}}\left[\int_{t=0}^{\infty} e^{-\rho t} f(\bar{K}, z_t)|z_0 = \underline{z}\right] \leq 0. \qquad (2)$$

Relate these to the corresponding assumptions made by Morris and Shin.

[b] Using my notes and the expository part of the Frankel-Pauzner paper, make sure you understand the main arguments of that paper.

[4] Here is yet another variation on equilibrium selection in coordination games. Let us suppose that agents live only for two periods. When they are young they make a choice of one of two sectors to move to, A or B. When old they are forced to enjoy the payoffs of their youthful choice; they cannot change their choices. Let us assume that in Sector A, the return in normalized to 1, while in Sector B, the return at any date is given by

$$n + s,$$

where n is the number of *youthful* agents joining that sector at that date, and s is the realization of some Markov process which has nice properties, no drift, and moves up and down from its current value with equal probability. That is, $E(s_{t+1}|s_t) = s_t$, and $P(s_{t+1} > s_t|s_t) = P(s_{t+1} < s_t|s_t) = 1/2$.

An individual places weight α on his youth and $1-\alpha$ on his old age. The value of α will depend on how inflexible are his career choices later: the more inflexible they are and the more determined by youth, the smaller is the value of α. So the payoff from choosing A is just $\alpha + (1-\alpha)$, or 1, while the payoff from choosing B is

$$\alpha(s+n) + (1-\alpha)(s' + n'),$$

where unprimed variables denote current values and primed variables denote corresponding values when our individual is at old age.

Finally, assume that the total population of youths is always of size 2.

(a) Suppose for the moment that s is forever fixed. Then show that if $s > 1$, then B is a dominant choice, while if $s < -1$, A is a dominant choice. Show that between these two values there are all always multiple equilibria, depending on expectations (about the choices of other individuals, in the same and in future generations).

(b) Now go back to the case where s unfolds as a stochastic process. Show that the thresholds in part (a) now change from 1 and -1 to α and $-\alpha$. You should supplement your analysis by intuition and explanation, and you can use informal reasoning.

(5) Let us combine some ingredients from the Adserà-Ray and Frankel-Pauzner papers. Begin again with the standard two-sector model with complementarities studied in class. That is, there are two sectors A and B, with A's return normalized to zero and B's return dependent on the amount of capital in that sector. Introduce lags in the rate of return as in Adserà-Ray, but keep the model as in Frankel-Pauzner in the sense that people occasionally get (Poisson) opportunities to move (at zero cost). In all other senses the model is the basic one (in particular, there are no other state variables).

Explore the following idea: for each lag function describing the movement in the rate of return: if the opportunites to move are frequent enough, then equilibria are exclusively history-dependent, while if they are infrequent enough, there will be multiple equilibria.