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# Problem Set 1

You will benefit from doing these problems, but there is no need to hand them in. If you want more discussion in class on these problems, I will be happy to oblige. Problems 1–6 are from my textbook. They are simply meant to give you an idea of complementarities in various situations.

(1) Suppose that fax machines are made newly available in NeverNeverLand. Companies are deciding whether or not to install a machine. This decision partly depends on how many *other* companies are expected to install fax machines. Think of a graph that describes how many companies *will* install fax machines as a function of how many companies are *expected* to install fax machines.

(a) If there are complementarities in the adoption of fax machines, describe the shape of this graph.

(b) Now suppose that even if no companies in NeverNeverLand are expected to install fax machines, $A$ companies will indeed do so (because of communication to the outside world). If $x$ companies are expected to install, then an additional $(x^2)/1000$ companies (over and above the number $A$) will install machines. This occurs up to a maximum of one million companies (which is the total number of companies in NeverNeverLand). Plot this relationship as a graph.

(c) Think of $A$ as the strength of contact with the outside world. Analyze the equilibrium adoption of fax machines in NeverNeverLand as $A$ varies. Pay attention to the possibility of multiple equilibria. For which values of $A$ does a *unique* equilibrium exist? Provide some intuition for your answer.

(2) Complementarities arise in all sorts of situations. Here is a tax evasion problem. Suppose that each of $N$ citizens in a country needs to pay a tax of $T$ every year to the government. Each citizen may decide to pay or to evade the tax. If an evader is nabbed, the law of the country stipulates payment of a fine of amount $F$, where $F > T$. However, the government's vigilance is not perfect, because it has limited resources to detect evaders. Assume that out of all the people who evade taxes, the government has the capacity to catch only one, and this person is chosen randomly. Thus, if $n$ people have decided to evade taxes, each has probability $1/n$ of being caught. In what follows, we assume that people simply calculate the expected losses from each strategy and choose the strategy with the lower expected loss.

(a) If the number of evaders is $m$, show that the average (expected) loss to an evader is $F/m$. This is to be compared with the sure loss faced by someone who complies, which is $T$.

(b) Why is this situation like a coordination game? Describe the complementarity created by one citizen's actions.

(c) Show that it is always an equilibrium for nobody in society to evade taxes. Is there another equilibrium as well? Find it and describe when it will exist.

(3) Consider a hypothetical economy in which each worker has to decide whether to acquire education and become a high-skilled worker or remain low-skilled. Education carries a cost of $C$. Assume that interest-free education loans are available to everybody. Let $I_H$ and $I_L$ denote the incomes earned by a high- and low-skilled worker respectively. These incomes are defined as $I_H = (1+\theta)H$ and $I_L = (1+\theta)L$, where $H$ and $L$ are constants $(H > L)$ and $\theta$ is the fraction of the population that decides to become high skilled. This formulation captures the idea that a person's productivity is positively linked not only to his own skills, but also to that of his fellow workers. Assume that all individuals simultaneously choose whether or not to become skilled.

(a) Explain why this is like a coordination problem. What is the complementarity?

(b) Show that if $H - L < C < 2(H - L)$, there are three possible equilibria: one in which everybody acquires skills, one in which nobody does, and a third in which only a fraction of the population becomes high-skilled. Give an algebraic expression for this fraction in the last case, and argue intuitively that this equilibrium is "unstable" and is likely to give way to one of the two extreme cases.

(c) Change the preceding example slightly. Suppose the return to low-skilled occupations is now given by $I_L = (1 + \lambda\theta)L$, where $\lambda$ is some constant. The return to high-skilled jobs is the same as before. Show that if the value of $\lambda$ is sufficiently high, there is only one possible equilibrium.

(d) Explain why multiple equilibria arise in the first case but not in the second.

(e) Consider another variation. Incomes from different occupations are independent of the number of high-skilled people in the economy. Specifically, $I_H = H$ and $I_L = L$. However, the cost of education is variable, and is given by $C(\theta) = (1 - \theta)/\theta$ (the idea here is that it is easier to learn if there are more educated people around). Show that once again, there are three possible equilibria. Describe them.

(4) Here are other examples of coordination problems. Discuss them and think of other situations which can be modeled in this way.

(a) *International debt*: Suppose that a country considers default on its international debt to a creditor country. In case of default, the creditor country can stop trade with the defaulter, even though this action may be costly for the creditor country to do. Hence, the more potential defaulting countries there are, the more difficult it becomes for the creditor country to "punish" a defaulter. Show that this situation gives rise to a coordination problem among the defaulters, and describe precisely the complementarity among borrower countries.

(b) *Investing in the stock market*: Suppose that a stock yields returns that exceed the average market rate if no one sells the stock in panic. Now imagine that for every person who panics and sells the stock, the rate of return on the stock decreases. Show why this is a coordination problem and describe the possible equilibria.

(c) *Cities*: Think of the emergence of cities as an outcome of coordination games. What would we mean by multiple equilibria in this context? Discuss this answer with respect to the concentration of certain types of industries in certain locations: for example, computer companies in Silicon Valley.

(5) Consider a society where people throw garbage on the streets. A person who does so inflicts negative externalities on others: suppose that the (psychic) dollar cost to any one person is $an$, where $a > 0$ is a positive constant and $n$ is the number of (other) people who throw garbage on the streets. Assume that your own act of throwing garbage on the street gives you a (psychic) dollar "convenience gain" of $G$ (because you don't have to wait for the next trash can) and a (psychic) dollar "shame loss" of $c/n$, where $c > 0$ and $n$, again, is the number of other garbage throwers (the idea is that the shame is smaller the more the number of other garbage throwers).

(a) Show that nobody throwing garbage on the streets is always an equilibrium in this example.

(b) Show that there is a threshold population size such that if population exceeds this threshold, everybody throwing garbage on the streets is *also* an equilibrium. Why is the threshold dependent on the parameters $G$ and $c$, but *independent* of $a$? Explain intuitively.

(c) Starting from a situation where everybody is throwing garbage on the streets, assess the chances of moving to the good equilibrium of part (a) overnight. Why do you consider such a move unlikely? Discuss this with reference to the arguments in the text.

(d) Consider a policy that imposes a fine $F > 0$ on every garbage thrower. Show that the threshold population required to support the bad equilibrium must rise with the fine.

(e) Suppose that the fine is such that the actual population falls below the threshold. In this case, discuss why the fine policy may be removed after a few years with a permanent change implemented in social norms.

(6) Suppose that people's attitudes can take three possible positions: $L$, $M$, and $R$, where you can think of $L$ as leftist, $R$ as rightist, and $M$ as middle-of-the-road. Consider a society in which *it is known that* a fraction $\alpha$ are $M$ types, and the remaining fraction $1 - \alpha$ are divided equally between $L$ and $R$, but no individual is known to be $L$, $M$, or $R$ at first sight.

Suppose that each individual gets satisfaction $S$ from expressing his or her own true views, but feels a loss ("social disapproval") in not conforming to a middle-of-the-road position. The amount of the loss depends on the fraction $\alpha$ of $M$ types: suppose that it equals the amount $\alpha/(1 - \alpha)$.

(a) Show that there is a threshold value of $\alpha$ such that everybody in society will express their own view if $\alpha$ is less than the threshold, but will all express $M$-views if $\alpha$ exceeds the threshold.

(b) What happens if we change the specification somewhat to say that the "social disapproval loss" equals $\beta/(1 - \beta)$, where $\beta$ is the expected fraction of people who *choose* to express $M$ views (and not necessarily the true fraction of $M$ types)?

(c) Indicate how you would extend the analysis to a case in which there are potential conformist urges attached to each of the views $L$, $M$, and $R$, and not just $M$.

(7) Suppose that we introduce taxes into the Murphy-Shleifer Vishny framework. Show that a profits tax is incapable of creating mutiple equilibria but that multiple, Pareto-ranked equilibria can exist with output taxation. Use the intuition for multiple equilibria developed in class to construct the necessary model.

(8) Assume that capital is produced through a continuum of intermediate goods indexed over a range $[0, n]$, and that output is produced using labor. Explain the different ways in which an expansion in the range $n$ affects the demand for any one of the intermediate goods.

(9) The extension of the Ciccone-Matsuyama model studied in class gives rise to the following equilibrium equation for determining intermediate goods variety:

$$\frac{n\sigma}{A(n)} = \frac{T}{S}.$$

Using a constant-elasticity form for the final-output production function, examine the possibility of multiple solutions to $n$ and provide an economic interpretation.

(10) Suppose that there are a continuum of individuals on $[0, 1]$, with wealth distributed uniformly on the interval $[0, W]$. Suppose that an investment can be made at a setup cost $x$, which must be paid out of initial wealth (there are no capital markets). The total return to the investment is $R$, and this depends positively on the number of people $n$ who make the investment. An investment is made if (i) the cost can be financed out of own wealth and (ii) if the return exceeds the cost. Provide conditions under which (a) there is a unique equilibrium with no investment, and *simultaneously*, (b) with wealth redistribution to perfect equality, there are multiple equilibria, at least one of which Pareto-dominates the equilibrium in (a).

(11) The existence of complementarities generally produces Pareto-suboptimal outcomes. This is especially true if the space is actions is continuous. Indeed, externalities in general (and not just complementarities) will produce this suboptimality.

   To illustrate the point, suppose that there are two individuals who take actions drawn from the interval $[0, 1]$. Write the payoff function of the first individual as $f(a, b)$, and that

of the second as $g(a, b)$, where $a$ and $b$ are the actions taken. Assume these are smooth functions, and let subscripts denote the appropriate partial derivatives.

(a) Show that the game exhibits complementarities if $f_{ab} > 0$ and $g_{ab} > 0$. In what follows, we will assume that these conditions hold.

(b) Define a Nash equilibrium $(a^*, b^*)$ of this game precisely. Say that it is *interior* if both $a^*$ and $b^*$ lie inside $(0, 1)$.

(c) Prove that at any interior Nash equilibrium in which $f_b$ and $g_a$ are nonzero that there is some joint change in the action vector that makes *both* individuals better off (Hint: use necessary conditions for constrained optimization problems). Compare with Acemoglu-Zilibotti in the optional readings.

(d) Notice that (c) *does not mean* that efficient outcomes occur at the "edge" of the action space. Write down an example just to be sure.

(e) Show by means of another example that $c$ is false if the space of actions is some finite subset of $[0, 1]$ rather than a continuum. Compare with 1999 midterm, question 4.

(f) What if $f(a, b)$ and $g(a, b)$ can be written in the following form: $f(a, g)$ and $g(f, b)$? Does (c) hold? Why or why not? (Compare with Murphy-Shleifer-Vishny.)