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Sketches of Answers to Problem Set 1.

The answers below are brief and try to give you the basic idea of how to approach these problems. You will gain a lot more from studying these answers if you spend some time independently trying to work on the problems.

(1) (a) Imagine that companies have some different costs of installing fax machines (perhaps because of different degrees of liquidity or access to credit) but that they all face a return from installing the machines that depends positively on how many *other* companies are installing. Then the graph that you draw will be upward-sloping. The more companies that are expected to install the machines, the more will actually do so. [Important: note that the intersections of this graph with the 45^0 line describes the equilibrium outcomes. Can you tell which intersections might be stable and which unstable, in the sense discussed in class?]

(b) The information given tells us that the number of companies y who actually install (as a function of the number of companies x who are expected to install) is given by the equation:

$$y = A + \frac{x^2}{1000},$$

provided that the upper bound of one million is not overstepped for either x or y (in this case, simply replace the corresponding values from the equation by 1 million).

Now let us calculate equilibria. First note that if x equals 1 million, then y as given by the equation will be way over 1 million, which simply means that *everybody* installing fax machines is always an equilibrium.

The remaining equilibria (if any) can be calculated by setting $y = x$, because this is where expected number and actual number coincide. Doing this, we get the equation

$$x = A + \frac{x^2}{1000},$$

(This is the same as looking at the intersection with the 45^0 line.)

The positive solutions are the other equilibria. Can you show that if A exceeds the value 250, there are *no* other equilibria? Use the graph, or your ability to solve quadratic equations.

(2) (a) If I am an evader, then I will be caught with probability $1/m$ where m is the total number of evaders. E.g., if $m = 3$, then there are three evaders and the chance of my getting caught is one out of three or 1/3. If I am not caught, then I pay nothing. But if I am caught, then I pay a fine of F. Thus my *expected* payout is $1/m$ times F, or simply F/m. As a potential evader, I will compare this loss with the sure payment of T (if I do not evade), and take the course of action that creates smaller losses.

(b) This situation is like a coordination game because if one person becomes an evader, she makes it *easier* for other people to evade. This is because the probability of getting caught comes down, so that the expected losses from evasion come down as well. In terms of part (a), m goes up if an additional evader enters the scene, so that F/m comes down. Thus an evader causes complementarities for other evaders.

(c) To see that "no evasion" is an equilibrium, suppose that nobody in the economy is evading. You are a potential evader. If you pay your taxes you will pay T. If you evade, then $m = 1$ (which is just another way of saying that you will be caught for sure), so that your expected loss is simply F. But $F > T$ by assumption. It follows that if nobody else is evading, you won't evade either. The same mental calculation holds for everybody, so that "no evasion" all around is an equilibrium.

What about everybody evading? Suppose that this is indeed happening, and you are considering evasion. If you do evade, then $m = N$, so that your expected losses are F/N. It follows that if $F/n < T$, you will jump on the bandwagon and evade as well. Thus "widespread evasion" is *also* an equilibrium provided that the consition $T > F/n$ holds.

(3) (a) Our formulation captures the following idea: a person's productivity is positively linked not only to his own skills, but also to that of his fellow workers. But more than that is true: note that $I_H - I_L = (1 + \theta)(H - L)$, which means that the *difference between the incomes from low and high skills* widens with more people acquiring high skills. It follows that whenever a person chooses to acquire skills, he increases the return to skill acquisition by everybody else. This is precisely the complementarity that underlies any coordination problem.

(b) Assume that $H - L < C < 2(H - L)$. First let us see if "no skill acquisition" can be an equilibrium. To this end, suppose that no one in society is acquiring skills: then $\theta = 0$. If you are thinking of becoming high-skilled, then the gain in your income is $I_H - I_L$, which is just $H - L$ (because $\theta \simeq 0$). If $H - L < C$ (which is assumed —see above), then it is not worthwhile for you to acquire skills. We have thus shown that if everybody believes that everybody else will not acquire skills, then no one will acquire skills. These beliefs thus form a self-fulfilling prophecy.

Now let us see if "universal skill acquisition" can be an equilibrium. Suppose that you believe that everybody else will acquire skills: then $\theta = 1$. Thus, if you are thinking of becoming high-skilled, then the gain in your income is $I_H - I_L$, which is $2(H - L)$ (because $\theta = 1$). If $2(H - L) > C$ (which is assumed —see above), then it is worthwhile for you to acquire skills. We have thus shown that if everybody believes that everybody else will acquire skills, then everyone will acquire skills. These beliefs also form a self-fulfilling prophecy.

Finally, there is a third equilibrium in which just the right amount of people invest in skill acquisition so that everybody is indifferent between acquiring or not acquiring skills. This is given by a fraction of skilled people θ^* such that $(1 + \theta^*)(H - L) = C$. This is an equilibrium because no one is doing anything suboptimal given his or her beliefs. But you can intuitively see why this equilibrium must be "unstable". If for some reason the fraction of skilled people exceeds θ^*, even by a tiny amount, then it becomes *strictly* preferable for everyone else to acquire skills, so that we rapidly move to the "universal skills" equilibrium. If on the other hand, θ falls below θ^* (if only by a tiny amount), everyone will desist from acquiring skills, so that we move towards the "no skills" equilibrium.

(c) and (d) If the returns to low-skilled occupations is now given by $I_L = (1+\lambda\theta)L$, what this means is that we are changing the "sensitivity" of low-skill income to the fraction of high-skilled people. A higher λ means that low-skill income is more and more responsive to the fraction of high-skilled people. Note that the *difference* between high and low skill incomes thus becomes *less* responsive. To see this, observe that $I_H - I_L = (1+\theta)H - (1+\lambda\theta)L = [H - L] + \theta[H - \lambda L]$. Now see that if λ exceeds the value H/L, the difference between the two incomes will actually *fall* as θ goes up. In this case there cannot be any multiple equilibrium, for exactly the same reason as the traffic congestion example in the text cannot exhibit multiple equilibria.

(e) In this case, note that the cost of acquiring skills becomes infinitely high as θ becomes close to zero, while the cost declines to near zero as θ approaches one. Thus we see again that there are three equilibria. In the first, there is no skill acquisition because everyone, expecting that there is no skill acquisition, feels that the cost of acquiring high skills will be very high, and so desist from doing so. At the same time, the expectation that everyone acquires skills is also a self-fulfilling prophecy, because in this case the cost of education is very low. And there is a third equilibrium where people are indifferent between the two options. Just as in part (a), this equilibrium must be described by the condition that $I_H - I_L = \frac{1-\theta^*}{\theta^*}$ (why?).

(4) Other examples of coordination problems. To show that a situation gives rise to a coordination problem, what one needs to do is check if there are complementarities between the various agents concerned. In the first case (part (a)) the agents in question are the potential defaulting countries. The more defaulters there are, the harder it is to punish any one of them simply because it is harder for the creditor to give up trade with several countries. Thus each defaulter creates complementarities for other defaultors. [This may be one reason why we observed a sudden wave of defaults and renegotiations during the debt crisis of the 1980s, instead of sporadic isolated instances of default. See Chapter 17 of DE, Section 17.4.2 for more on the debt crisis.]

You should be able to do part (b) on your own. By this time you should also be thinking harder about the term "complementarities". Even though complementarities are sometimes associated with positive externalities for other people, this is not always the case, as part (b) shows. In its most abstract form, the term simply means: if one individual carries out an action, it tends to increase the propensity for others to carry out the same action. The action could mean buying a new computer, not paying taxes, defaulting on debt, or selling a stock in panic, as we have already learnt.

In part (c), think about what leads to a particular region turning into a full-fledged city. To some extent it is a question of location, but there are positive externalities at work here as well. If an area already has a conglomeration of businesses, it makes it easier for other businesses to set up there as well, because of access to a variety of infrastructural services. Likewise, individuals are more keen on moving to sucha place to work, because they know that the amenities of life are more likely to be available. Thus setting up life in a city creates positive externalities (up to a point at least: later there is pollution, congestion, and high cost-of-living to worry about), in the sense that it raises to return to others of setting-up in the city as well.

Thus think about concentrations of high tech companies in Slicon Valley or along Route

128 in the Boston area. It is easier for a new company to locate here because it will be easier to hire trained personnel, to have access to the latest in technological knowhow, to take advantage of the ancillary activities that have grown up around these firms. This is clearly a case of complementarities (and in this case the externalities are positive as well: they are also beneficial to society).

(5) The same idea as in the tax problem.

(6) (a) The gain from being your own self is S. If you are an L-type or an R-type, however, you will also feel a loss equal to $\frac{\alpha}{1-\alpha}$. Therefore the *net* gain from being your own type (L or R) is

$$S - \frac{\alpha}{1-\alpha}.$$

This is negative if $\alpha > \frac{S}{1+S}$. Above this threshold value, everybosy will say that they are type M.

(b) In this case, there are two possibilities. First, assume that $\alpha > \frac{S}{1+S}$, the threshold derived in part (a). Note that a fraction α (the true M-types) will always say that they are type M, because they have nothing to gain by stating any other position. But by part (a), the other types will hide their identity, which raises the value of β (the *announced* M-types) above the value of α. This process can only stop when everybody announces that they are type M.

On the other hand, if $\alpha < \frac{S}{1+S}$, there is an equilibrium in which everybody announces their true type, and so $\beta = \alpha$. You can check that nobody will want to deviate from their announcements. But at the same time, there is another "conformity" equilibrium in which everybody announces that they are type M (and in which β takes on the value of one).

(c) If there are potential conformist urges attached to each of the views L, M, and R (and not just M), then other equilibria appear. There may be conformist equilibria in which everybody announces L, or in which everybody announces R (try and provide a simple algebraic example of this).

(7), (8) and (9): Discussed in class.

(10) Suppose that n people invest; then the return to investment is $R(n)$, an increasing function. Let F be the cdf of initial wealth.

For no investment to be a unique equilibrium, it must be the case that for every value of $n > 0$, we have either $R(n) < x$ (so that investors, even if able, are unwilling to invest), *or* $1 - F(x) < n$ (so that all n investors, even if willing, are unable to invest).

Now suppose there is wealth equalization with average wealth w. the first condition for there to be any equilibrium with positive investment is $w \geq x$. We must also have $R(0) \leq x \leq R(1)$ for multiple equilibria. Finally, we need the full-investment equilibrium here to Pareto-dominate the no-investment equilibrium *without redistribution.* This means that $R(1) - x - w > \bar{w}$, where $\bar{w}$ is the upper limit of the support of F.

Now satisfy yourself that these conditions are indeed potentially consistent with one another. That is, make sure you can write down at least one example in which all the conditions are simultaneously satisfied.

(11) (a) Define a correspondence by

$$\Gamma(b) \equiv \arg\max_a f(a, b).$$

[Note: this is generally a correspondence and not just a function.] To show that the game exhibits complementarities, we must show that if $b' > b$, and $a \in \Gamma(b)$ and $a' \in Gamma(b')$, then $a' \geq a$. Suppose this is false for some b and b' with $b' > b$, and for some $a \in \Gamma(b)$ and $a' \in Gamma(b')$. Then $a > a'$. Now observe that

$$f(a', b') \geq f(a, b')$$

while

$$f(a, b) \geq f(a', b).$$

Adding the two and transposing terms, we see that

$$f(a, b) - f(a', b) \geq f(a, b') - f(a', b')$$

In words, this means that f increases faster across a' to a when the second argument is bm, which is smaller than b'. This contradicts the assumption that $f_{ab} > 0$, which implies exactly the opposite.

(b) Standard.

(c) Let (a^*, b^*) be an interior Nash equilibrium. Then by the first-order condition for best responses, we know that

$$f_a(a^*, b^*) = 0 \text{ and } g_b(a^*, b^*) = 0.$$

Now suppose that we are given that f_b and g_a are nonzero, as we are. Think of a tiny change in a and b, mimc by da and db. Taking total derivatives around (a^*, b^*), we see that

$$df = f_b(a^*, b^*)db$$

where we've already used the fact that $f_a(a^*, b^*0 = 0$. Similarly,

$$dg = g_a(a^*, b^*)da$$

Now, depending on the signs of f_b and g_a, we may choose da and db to be positive or negative to assure ourselves that both payoffs go up.

(d)—(e). Easy. Try them yourself. ask me if you cannot do them.