Game Theory Fall 2003

Problem Set 4

[1] (a) Give an example of a game in which the one-shot deviation principle fails, and explain precisely why the failure occurs.

(b) Prove that the one-shot deviation principle is satisfied in any repeated game with discounting, provided that each person has a finite number of actions at each date.

(c) Consider a growth model in which there is an initial stock every period, y, to be divided between consumption c for that period and a capital investment k. Thus $y_t = c_t + k_t$ for each t. There are two agents, A and B. A moves in every even period and chooses (c, k) for that period; B does the same in every odd period. Outputs across time are linked by the production function $y_{t+1} = f(k_t)$, where f is some increasing, smooth, concave function with $f'(\infty) < 1$. A has a discount factor $\alpha \in (0, 1)$ and continuous utility function a, so gets infinite-horizon payoffs $\sum_t \alpha^t a(c_t)$, while the corresponding objects for B are β and $b(.)$.

Prove that the one-shot deviation principle is satisfied for this game.

(d) Consider a finite game tree with 2 players. The only twist is that player 1 receives two-dimensional vector payoffs. Strategy profiles are defined exactly as in class. Say that a strategy (σ_1, σ_2) is a subgame perfect equilibrium if at the initial node and at every subgame, player 1 cannot *unambiguously* gain; that is, there is no alternative strategy which will improve his payoffs along *both* dimensions. Show by example that such games do not satisfy the one-shot deviation principle, in general.

[2] Give an example of a three-player bargaining game with discounting in which history-dependent strategies can be used to support an *inefficient* payoff division of the cake (i.e., some amount in actually thrown away in equilibrium).

[3] Prove that the random-proposer version of the n-person bargaining model has a unique stationary equilibrium. Examine its history-dependent equilibria just as we did in class for the "rejector-proposes" model.

[4] Bargaining with linear time costs (check OR).

[5] Prove that $v(n)/n \geq v(s)/s$ for all s is equivalent to balancedness for a symmetric characteristic function.

[6] A characteristic function is *convex* if for every pair of coalitions S and T, $v(S \cup T) + v(s \cap T) \geq v(S) + v(T)$. By using either the Bondareva-Shapley theorem or directly, prove that every convex game has a nonempty core.

[7] Consider the following public goods game. Each person gets the sum of linear utility from private consumption c and concave utility from a pure but local public good g for that person's coalition: $c + u(g)$, where $u'' \leq 0$. To produce g requires a cost of $c(g)$, payable from the private endowments of individuals in the coalition. Person i has endowment w_i. Assume private consumption (endowment minus contribution) can be positive or negative.

Construct the characteristic function for this game, and prove that its core is nonempty.

[8] Prove that an exchange economy with transferable utility is balanced, when viewed as a characteristic function game. (See Osborne-Rubinstein, Proposition 264.2).

[9] The core has an interesting asymmetry. It demands that allocations not be blocked for the grand coalition, but does not test the credibility of the blocking allocations themselves. One way to get around this is to define the *credible core* as follows:

The credible core of any coalition S, $C(S)$, is the set of all allocations in $V(S)$ that are unblocked by any subcoalition T using only allocations from *its* credible core $C(T)$.

[a] Why is this informal definition not a formally correct definition? Reformulate it using recursion.

[b] Prove that the credible core *equals* the traditional core.

[c] Speculate on how you would proceed if the number of players were (countably) infinite.

[10] Give an example of a superadditive game (in the sense that every coaition is superadditive, not just the grand coalition) which fails condition [M], discussed in class. Give another example of a non-superadditive game which satisfies condition [M].

[11] In class we defined a vector m^* and showed that the condition $\sum_{i=1}^{n} m_i^* > v(N)$ is sufficient for the inefficiency of Rubinstein bargaining. Show by means of an example (satisfying condition [M]) that it is not necessary. [Such an example appears in Chatterjee et al (1993, RES), but try it yourself before you look at this paper.]