# Game Theory Fall 2003

## Problem Set 2

[1] Consider the following game, in which player 1 chooses rows, player 2 columns, and player 3 matrices. I have only written down player 1's payoffs.

| | $L$ | $R$ |
|---|---|---|
| $a$ | 4 | 4 |
| $b$ | 3 | 0 |
| $c$ | 0 | 4 |

| | $L$ | $R$ |
|---|---|---|
| $a$ | 4 | 0 |
| $b$ | 0 | 3 |
| $c$ | 4 | 4 |

Prove that $b$ is not dominated, and find a belief for which it is is best response. Now prove that *every* such belief must involve correlation between the actions of players 2 and 3.

[2] Rationalizability is defined in a manner analogous to strict dominance. Set $\Sigma_i^0 \equiv \Sigma_i$ for all $i$. For any set $A$, let $\bar{A}$ denote its convex hull. Recursively, having defined $\Sigma_j^k$ for all $j$ (for some $k$), define

$$\Sigma_i^{k+1} \equiv \{\sigma_i \in \Sigma_i^k | \text{ there is } \sigma_{-i} \in \prod_{j \neq i} \bar{\Sigma}_j^k \text{ such that } \sigma_i \text{ is a best response to } \sigma_{-i}\}.$$

(a) Could we equivalently have taken all the pure strategy best responses and then all mixed strategies from these, to construct $\Sigma_i^{k+1}$? Why or why not?

(b) What is the idea behind using $\bar{\Sigma}_j^k$ instead of just $\Sigma_j^k$?

(c) What would be the interpretation of using $\overline{\prod_{j \neq i} \Sigma_j^k}$ instead of $\prod_{j \neq i} \bar{\Sigma}_j^k$?

(d) Prove that the following yields an equivalent recursion:

$$\Sigma_i^{k+1} \equiv \{\sigma_i \in \Sigma_i | \text{ there is } \sigma_{-i} \in \prod_{j \neq i} \mathcal{M}(S_j^k) \text{ such that } \sigma_i \text{ is a best response to } \sigma_{-i}\},$$

where for each $k$ and $j$, $S_j^k$ is just the set of pure strategies in the support of $\Sigma_j^k$.

To complete the definition, the rationalizable (mixed) strategies are taken to be the infinite intersection of the $\Sigma_i^k$'s; i.e., for each $i$, define $R_i \equiv \cap_k \Sigma_i^k$.

(e) Prove that for each $i$, $R_i$ is nonempty and contains at least one pure strategy.

The pure strategies in $R_i$ are called the *rationalizable strategies* for player $i$.

[3] A family of nonempty pure strategy subsets $\{\tilde{S}_1, \ldots, \tilde{S}_n\}$ is called a *rationalizable* family if for each $i$

$$\tilde{S}_i \subseteq \{s_i \in S_i | \text{ there is } \sigma_{-i} \in \prod_{j \neq i} \mathcal{M}(\tilde{S}_j) \text{ such that } s_i \text{ is a best response to } \sigma_{-i}\}.$$

(a) Prove that a pure strategy for $i$ is rationalizable if and only if it is an element of $i$'s subset in some rationalizable family.

(b) Recall and write down the corresponding characterization proved in class for strategies that survive strict iterated dominance. Compare the two conditions, to understand why rationalizability is equivalent to iterated strict dominance when correlated beliefs are permitted, but yields a smaller set when only independent beliefs are permitted.

[4] Read Sections 5.1.–5.3. in Osborne-Rubinstein and do the exercises there. Then read the rest of the chapter.